- Friday, May 04, 2012
- Speaker: Jian Ding
- Title: Maxima of two-dimensional discrete Gaussian free field
- Note taker: Xiaoqin Guo

1. Definition of the Gaussian free field.
   V_N: a 2-dimensional box of side N.
   $(\eta_v, v \in V_N)$: mean-zero Gaussian process with $\eta|_{\partial V_N} = 0$. Ways to define the GFF $(\eta.)$:
   - Markov field
   - Density $f(\eta_v) \propto e^{-\sum_{u\sim v}(\eta_u-\eta_v)^2/16}$.
   - Use the Green function of the simple random walks in V_N to define the covariance
   - $E(\eta_u - \eta_v)^2$:=effective resistance between u and v.

2. Review of results in BBM/BRW

   - (Bramson) *Precise estimate on the expectation of the maximum displacement (use truncated second moment method)
     *Connection to KPP equation (determine the limiting law)
   - (Bolthausen, Deuschel, Giacomin '01)
     *Asymptotics of the maximum displacement ($\sim 2\sqrt{\frac{2}{\pi}}\log N$)
     *A key ingredient in analyzing the repulsion in the presence of hard wall.
   - (Chatterjee '08)
     *Variance of the maximum displacement$\sim o(\log N)$
     *Super-concentration vs multiple peaks
     *Use abstract method: hypercontractivity.
   - (Bolthausen, Deuschel, Zeitouni '10)
     * $(M_N - EM_N)$ is tight along a subsequence;
     $$EM_N = 2\sqrt{\frac{2}{\pi}}(\log N - \frac{3}{8\log 2}\log\log N) + o(1).$$
     * "Expectation is the King"(–Zeitouni)
   - (D. '11)
     *$\forall 0 \le \lambda \le (\log N)^{2/3}$, $P(M_N \ge EM_N + \lambda) \sim e^{-c\lambda}$ and
     $$c\exp(-Ce^{C\lambda}) \le P(M_N \le EM_N - \lambda) \le C\exp(-ce^{c\lambda})$$
     $\Longrightarrow \mathrm{Var}(M_N) \asymp 1$.
     *Use "sprinkling" method (Ajtai,Komlos, Szemeredi '82)

Results on the maxima:

- BBM: Arguin-Bovier-Kistler, Aidekon-Berestycki, Brunet, Shi
- GFF: Daviaud '06: Geometry of the set of points $\geq \eta^m N$ for $\eta \in (0,1)$.

3. Results of D.-Zeitouni

(1) $\eta_u, \eta_v \geq EM_N - c \Rightarrow N/K(c) \leq |u-v| \leq K(c)$ with high probability.

(2) $\#\{u : \eta_u \geq M_N - \lambda\}$ is $\exp(\theta(\lambda))$.

(3) Gap of the largest two values has Gaussian decay for the right tail.