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# NOTETAKER CHECKLIST FORM

**(Complete one for each talk.)**

**Name:** BRANDEN STONE **Email/Phone:** bstone@bard.edu

**Speaker's Name:** DANIEL MURFET

**Talk Title:** THE BICATEGORY OF LANDAU-GINZBURG MODELS

**Date:** 2 / 12 / 13 **Time:** 2 : 00 am / pm (circle one)

**List 6-12 key words for the talk:** LANDAU-GINZBURG, MATRIX FACTORIZATION, HYPERSURFACE, ISOLATED SINGULARITY, TOPOLOGICAL FIELD THEORY, BICATEGORY

**Please summarize the lecture in 5 or fewer sentances:**

In two dimensions, topological field theories with defects are naturally related to bicategories with additional structure. An explaination of this connection and recent joint work with Nils Carqueville on the bicategory of Landau-Ginzburg models which is built out of isolated hypersurface singularities and matrix factorisations is given.

# CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

THE BICATEGORY OF LANDAU-GINZBURG MODELS 1

D. MURFET

JOINT w/ NILS CARQUEVILLE, ARXIV: 1208.1481

(I) DEFINE $\mathcal{LG}$

(II) APPLICATIONS

DEF: A MATRIX FACTORIZATION (MF) OF $W \in R$ IS A $\mathbb{Z}_2$-GRADED FREE MODULE

$$X = X^0 \oplus X^1$$

WITH DIFFERENTIAL $d_X : X \to X$, $|d_X| = 1$ s.t. $d_X^2 = W \cdot 1_X$

A MORPHISM $\varphi : (X, d_X) \longrightarrow (Y, d_Y)$ IS R-LINEAR $\varphi d_X = d_Y \varphi$, $|\varphi| = 0$

$$\text{hmf}(R, W) = \begin{cases} \text{f. rank MFs of } W \\ \text{w/ morphisms / htpy} \end{cases}$$

- STUDY $W \in \mathbb{C}[x_1, \dots, x_n]$, $\text{Sing}(W) \neq \emptyset$ LOOK AT $\text{hmf}(R, W)$

- FUNCTION $\text{hmf}(\mathbb{C}[x], W) \to \text{hmf}(\mathbb{C}[z], V)$ $\rightsquigarrow$ BICATEGORY

WHY? SYMMETRIES AND ORBIFOLDS $\longrightarrow$ 2D TFT w/ DEFECTS

DEF: A BICATEGORY $\mathcal{C}$ IS

- A CLASS OF OBJECTS $A, B, C, \dots$
- $\forall A, B$ A CATEGORY $\mathcal{C}(A, B)$ ~~WHOSE OBJECTS~~ WHOSE OBJETS $X, Y, Z, \dots$ ARE CALLED 1-MORPHISMS AND

MORPHISMS $\alpha, \beta, \gamma$ ARE CALLED 2-MORPHISMS

WRITE e.g. $X : A \to B$

AND FOR ALL $A, B, C$, A FUNCTOR

$$\mathcal{C}(A,B) \times \mathcal{C}(B,C) \xrightarrow{\otimes} \mathcal{C}(A,C)$$

$$(Y, X) \longmapsto X \otimes Y$$

AND FOR ALL $A$ A 1-MORPHISM $\Delta_A \in \mathcal{C}(A,A)$

AND 2-ISOS

$$(X \otimes Y) \otimes Z \simeq (X \otimes Y) \otimes Z$$

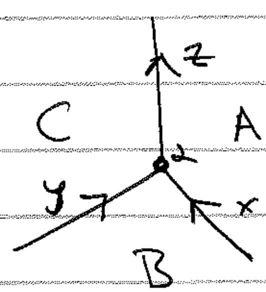
IF $X : A \to B$, $\Delta_B \otimes X \simeq X \simeq X \otimes \Delta_A$

SAT. COHERENCE RELATIONS

<u>NOTE</u>: $(\mathcal{C}(A,A), \otimes, \Delta_A)$ IS MONOIDAL

<u>EX</u>: RINGS, BIMODULES, BIMOD MAPS

<u>STRING DIAGRAMS</u>



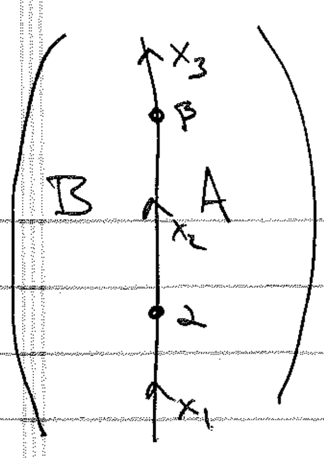

$X : A \to B$

$Y : B \to C$

$Z : A \to C$

$\alpha : Y \otimes X \to Z$

THE <u>VALUE</u> OF THIS DIAGRAM

VALUE  $= \beta \circ \alpha$



**DEF:** $\mathcal{LG}$ HAS ($n$ EVEN; ~~$n$ ODD~~)

$\boxed{\text{OBJ}}$ PAIRS $(\mathbb{C}[x_1, \dots, x_n], W)$ WITH $W \in \mathbb{C}[x]$ AND $\dim_{\mathbb{C}} \mathbb{C}[x]/\partial W < \infty$

$\boxed{\text{MOR}}$ $\mathcal{LG}\big((\mathbb{C}[x], W), (\mathbb{C}[z], V)\big)$

$$= \mathrm{hmf}\big(\mathbb{C}[x] \otimes \mathbb{C}[z],\ 1 \otimes V - W \otimes 1\big)^{\omega}$$

IDEMPOTENT COMP

i.e. $X : W \to V$ IS A MF OF $V - W$

**COMPOSITION:**

GIVEN $Y : W \to V$, $X : V \to U$, $U \in \mathbb{C}[y]$

$X \otimes_{\mathbb{C}[z]} Y$ IS A FREE $\mathbb{C}[x,y]$-MODULE

WITH $d_{X \otimes Y} = d_X \otimes 1 + 1 \otimes d_Y$

$$d_{X \otimes Y}^2 = U - V + V - W = U - W$$

Prop $(X \otimes Y, d_{X\otimes Y}) \in \mathrm{hmf}(\mathbb{C}[x,z], V - W)^{\omega}$

$$\cap\!|$$

HMF i.e. $\infty$-RANK

UNIT $\quad \Delta_W \in \mathcal{LG}\big((\mathbb{C}[x], W), (\mathbb{C}[x], W)\big)$

$$\| $$

$$\mathrm{hmf}\big(\mathbb{C}[x] \otimes \mathbb{C}[x],\ W \otimes 1 - 1 \otimes W\big)$$

$$(x_i \otimes 1 - 1 \otimes x_i)_{1 \le i \le n}$$

Prop: $\mathcal{LG}$ IS A BICATEGORY

Example: $(\mathbb{C}, 0) \in \mathcal{LG}$

$$\mathcal{LG}\big((\mathbb{C}, 0), (\mathbb{C}[x], W)\big) = \mathrm{hmf}(W)^{\omega}$$

i.e. Given $X : W \to V$ GET A FUNCTOR

$$\mathrm{hmf}(W)^{\omega} = \mathcal{LG}(0, W) \xrightarrow[\substack{\| \\ \Phi_X}]{X \otimes -} \mathcal{LG}(0, V) = \mathrm{hmf}(V)^{\omega}$$

Duality:

Given $X : (\mathbb{C}[x], W) \to (\mathbb{C}[z], V)$ DEFINE

$$X^{\vee} : (\mathbb{C}[z], V) \longrightarrow (\mathbb{C}[x], W)$$

~~[illegible]~~ BY

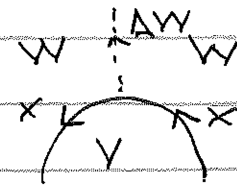
$$X^{\vee} = \mathrm{Hom}_{\mathbb{C}[x,z]}\big(X, \mathbb{C}[x,z]\big)$$

$$d_{X^{\vee}}(\alpha) = -(-1)^{|\alpha|}\, \alpha \circ d_X$$

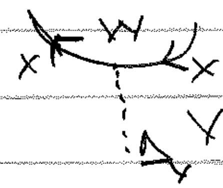
**Thm** $X^\vee$ is left and right adjoint to $X$

i.e. $\exists$ 2-morphisms

$$\mathrm{ev}: X^\vee \otimes X \longrightarrow \Delta_W$$




$$\mathrm{coev}: \Delta_V \longrightarrow X \otimes X^\vee$$




$$\widetilde{\mathrm{ev}}: X \otimes X^\vee \longrightarrow \Delta_V$$

$$\widetilde{\mathrm{coev}}: \Delta_W \longrightarrow X^\vee \otimes X$$

sat. some relations

Formulas: Atiyah classes, Residues

Proof: Commutation relations $\square$

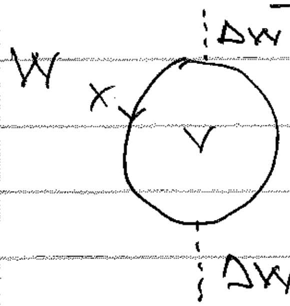
**Thm:** $\mathcal{LG}$ is pivotal. $X: W \to V$, $Y: V \to U$

$$(Y \otimes X)^\vee \cong X^\vee \otimes Y^\vee$$

i.e. two canonical isos agree.

**Def.** Given $X: (\mathbb{C}[x], W) \longrightarrow (\mathbb{C}[z], V)$

The left qdim $\mathrm{qdim}_\ell(X) \in \mathbb{C}[x]/\partial W$




$$\begin{array}{c} \Delta W \\ \uparrow \mathrm{ev} \\ X^\vee \otimes X \\ \uparrow \widetilde{\mathrm{coev}} \\ \Delta W \end{array}$$

$\mathrm{End}(\Delta W)$ is $\mathbb{C}[x]/\partial W$

$$\mathrm{qdim}_\ell(X) = -\,\mathrm{qdim}_\ell(X[1])$$

# II. Applications

Let $G$ be a finite group acting linearly on $R = \mathbb{C}[x_1, \ldots, x_n]$, $W \in R^G$

$$\Delta_W \in \mathcal{LG}(W, W) = \mathrm{hmf}(R \otimes R, W \otimes 1 - 1 \otimes W), \quad g \in G$$

$$(1 \otimes g) : R \otimes R \longrightarrow R \otimes R$$

$$\Delta_g := (1 \otimes g)^* \Delta_W$$

- $\Delta_g \otimes \Delta_h \cong \Delta_{gh}$
- $\Delta_g^\vee = \Delta_{\bar{g}}$
- $\mathrm{qdim}(\Delta_g) = \det(g)$

Define $A_G := \bigoplus_{g \in G} \Delta_g$

Prop: $A_G$ is an algebra, coalgebra, is Frobenius, separable.

$$\mathrm{mod}(A_G) \simeq \mathrm{hmf}(R, W)^G$$

i.e. a MF $X$ of $W$ with action

$$A_G \otimes X \longrightarrow X$$

Prop: $A_G$ is symmetric $\Longleftrightarrow$ $G \leq SL(n, \mathbb{C})$

**DEF:** A GENERALIZED ORBIFOLD OF $W$ IS A SEPARABLE, SYMMETRIC, FROBENIUS ALG IN $\mathcal{LG}(W,W)$.

ARXIVE. 1210.6363 CARQUEVILLE, RUNKEL

THERE IS A G.O. OF $W = u^{2d} - v^2 \in \mathbb{C}[u,v]$
$A \in \mathcal{LG}(W,W)$, $A_{2d-1}$

$$\mathrm{mod}(A) \cong \mathrm{hmf}(\mathbb{C}[x,y],\, x^d - xy^2)^\omega$$

(c.f. REITEN - RIEDTMANN '85, DEMONET '10)

**CONJECTURE:** $\exists$ G.O. $A \in \mathcal{LG}(W,W)$, W SOME A TYPE s.t.

$$\mathrm{mod}(A) \simeq \mathrm{hmf}(E_j)^\omega, \quad j = 6,7,8.$$