

## NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: BRANDEN STONE Email/Phone: bstone@bard.edu

Speaker's Name: ~~ALICE~~ ~~DAVE~~ DAVE BENSON

Talk Title: MODULES FOR ELEMENTARY P-GROUPS AND HYPERSURFACE SINGULARITIES.

Date: 2/14/13 Time: 9:00 (am/pm (circle one))

List 6-12 key words for the talk: P-GROUPS, HYPERSURFACE, ORLOV CORRESPONDENCE, MATRIX FACTORIZATION, STABLE MODULE CATEGORY, SINGULAR CATEGORY

Please summarize the lecture in 5 or fewer sentences: \_\_\_\_\_

Concerning the Orlov correspondence and its relation to the stable module category with the singularity category of a certain hypersurface.

## CHECK LIST

(This is **NOT** optional, we will **not** pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3<sup>rd</sup> floor.
  - **Computer Presentations:** Obtain a copy of their presentation
  - **Overhead:** Obtain a copy or use the originals and scan them
  - **Blackboard:** Take blackboard notes in black or blue PEN. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.  
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

# MODULES FOR ELEMENTARY ABELIAN $p$ -GROUPS AND HYPERSURFACE SINGULARITIES



D. Benson

$G$  FINITE GROUP,  $k$  FIELD OF CHAR  $p$   
 $kG =$  GROUP ALGEBRA

$$E = \underbrace{\mathbb{Z}/p \times \dots \times \mathbb{Z}/p}_{r \text{ COPIES}} \quad r = \text{RANK OF } E$$

CROWFORD: A  $kG$ -mod is projective iff its  
restriction to each elem. ab.  $p$ -subgroup  $E \leq G$  is  
projective.

Quillen:  $H^*(G, k) \longrightarrow \varprojlim_{E \leq G} H^*(E, k)$

$F$ -isomorphism

i.e. isom on spec

Goal: Understand  $\text{stmod}(kG)$

Obj: f.g.  $kG$ -mod

mor:  $\underline{\text{Hom}}_{kG}(M, N) = \text{Hom}_{kG}(M, N) / \text{P Hom}_{kG}(M, N)$

$\downarrow \uparrow$   
 $P$

This is a triangulated category

$$D^b(kG) / \text{Perf}(kG) \cong \text{stmod}(kG)$$

$$\begin{array}{ccccccc}
 0 & \rightarrow & M_i & \rightarrow & M_{i+1} & \rightarrow & \dots \rightarrow M_j \rightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \rightarrow & I_i & \rightarrow & I_{i+1} & \rightarrow & \dots \rightarrow I_j \rightarrow I_{j+1} \rightarrow \dots \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 M & & I'_1 & \rightarrow & I'_0 & \rightarrow & \dots \\
 & & & & & & \dots \rightarrow I'_j \rightarrow I_{j+1} \rightarrow
 \end{array}$$

Def.  $D_{sg}(R) = D^b(R) / \text{Perf}(R)$

$\text{stmod}(kE) = D_{sg}(R \otimes kE)$

eg.  $k[x, y] / (x^2, xy, y^2) = R$

$0 \rightarrow k \oplus k \rightarrow R \rightarrow k \rightarrow 0$

$k \xrightarrow{\sim} k(1) \oplus k(1) \in D_{sg}(R)$

Example:  $R = k[x_1, \dots, x_r] / (f)$

$D_{sg}(R) \cong \underline{\text{MCM}}(R)$

$\cong$  REDUCED MATRIX FACTORIZATIONS OF  $f$

If  $R$  is GRADUATED,  $X = \text{Proj } R$ ,  $\text{Coh}(X) = \frac{\text{mod}(R)}{\text{tors}(R)}$

$D_{sg}(X) = \underline{\text{MCM}}(R) / \underline{\text{MCM}}$   
APPROX. OF  $\text{TORS}(R)$

Orlov's Theorem:

$E = \langle g_1, \dots, g_r \rangle \cong (\mathbb{Z}/p)^r$  char  $k = p$

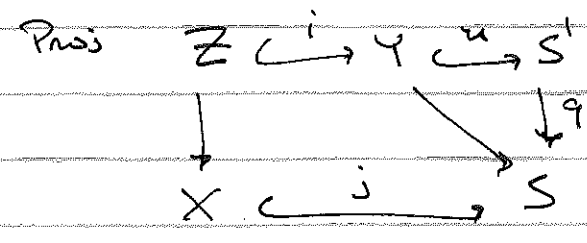
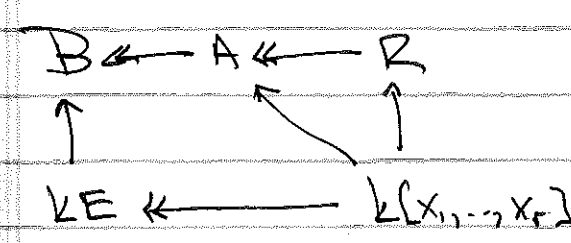
$x_i = g_i^{-1} \in kG, x_i^p = 0$

$kE = k[x_1, \dots, x_r] / (x_1^p, \dots, x_r^p)$

$R = k[y_1, \dots, y_r, x_1, \dots, x_r]$

$f = y_1 x_1^p + \dots + y_r x_r^p$

$A = R / (f), B = R / (x_1^p, \dots, x_r^p) = k[y_1, \dots, y_r] \otimes_k kE$



Orlov:

$$D_{sg}(X) \simeq D_{sg}(Y)$$

is

$$\text{struc}(kE)$$

Buchs:

$$D^b(kE) \simeq D_{sg}(A)$$

$$\begin{array}{ccc} \text{Struc}(kE) & \longrightarrow & D_{sg}(Y) \\ D^b(kE) & \longrightarrow & D_{sg}(A) \\ M & \longmapsto & k[y_1 \rightarrow \dots \rightarrow y_r] \otimes M \end{array}$$

$$D_{sg}(A) \longrightarrow D^b(kE)$$

$$F \begin{array}{c} \xrightarrow{u} \\ \xleftarrow{v} \end{array} F' \quad uv = vu = f \cdot I$$

$$F_0 \otimes_R B \xrightarrow{u} F'_0 \otimes_R B$$

Gives an object in  $D^b(kE)$ .

$$F \otimes_R B \xrightarrow{u} F' \otimes_R B$$

Example:  $r=2$

$$kE = k[x_1, x_2]$$

$$R = k[y_1, y_2, x_1, x_2], \quad A = R / (y_1 x_1 + y_2 x_2^p)$$

$$M = k$$

Resolve ~~the~~  $k[y_1, y_2]$  over  $A$ :

$$\begin{array}{c} \rightarrow \oplus \xrightarrow{A[-1]} \oplus \xrightarrow{A} \oplus \xrightarrow{A} A \xrightarrow{A} k[y_1, y_2] \\ A[-1] \begin{pmatrix} y_2 x_2^{p-1} & -y_1 x_1^{p-1} \\ x_1 & x_2 \end{pmatrix} A[-1] \begin{pmatrix} x_2 & y_1 x_1^{p-1} \\ -x_1 & y_2 x_2^{p-1} \end{pmatrix} A \end{array} \quad (x_1, x_2)$$

Example:

$$\begin{array}{c} \oplus \xrightarrow{A[-1]} \oplus \xrightarrow{A} \oplus \xrightarrow{A} A \xrightarrow{A} k[y_1, y_2] \\ A[-1] \begin{pmatrix} y_2 & -y_1 \\ x_1^p & x_2^p \end{pmatrix} A \begin{pmatrix} x_2^p & y_1 \\ -x_1^p & y_2 \end{pmatrix} A \end{array} \quad (x_1^p, x_2^p)$$

$$\oplus \xrightarrow{A[-1]} \oplus \xrightarrow{A} A \xrightarrow{A} \frac{k[x_1, x_2]}{(x_1^p, x_2^p)}$$

Example:

$$\begin{array}{c} \oplus \xrightarrow{A[-1]} \oplus \xrightarrow{A} \oplus \xrightarrow{A} A \xrightarrow{A} k[x_1, x_2] \\ A[-1] \begin{pmatrix} y_2 & -y_1 \\ x_1^p & x_2^p \end{pmatrix} A[-1] \begin{pmatrix} x_2^p & y_1 \\ -x_1 & y_2 \end{pmatrix} A[-1] \begin{pmatrix} y_2 & -y_1 \\ x_1 & x_2 \end{pmatrix} A \end{array}$$

## RELATED CONSTRUCTION (FRISZDLANDER-PROTSOVO, B-P)

5

Let  $\mathcal{O} =$  STRUCTURE SHEAF OF  $\mathbb{P}^{n-1} = \text{Proj } k[y_0, \dots, y_{n-1}]$

GIVEN A  $kE$ -MODULE  $M$

$$\tilde{M} = \mathcal{O} \otimes M$$

$$\forall j \quad \theta: \tilde{M}(j) \rightarrow \tilde{M}(j+1), \quad \theta(f \otimes m) = \sum_{i=1}^r y_i f \otimes x_{i,m}$$

$$1 \leq i \leq p \quad F_i(M) = \frac{\ker \theta \cap \text{Im } \theta^{i-1}}{\ker \theta \cap \text{Im } \theta^i} \quad \text{AS SUBG OF } \tilde{M}$$

RECIPE: GIVEN A SHEAF  $F$  ON  $\mathbb{P}^{n-1}$  COMING FROM  
A  $k[y_0, \dots, y_{n-1}]$ -MODULE, INFLATE TO AN  $A$ -MOD  
AND APPLY THE ABOVE FUNCTOR TO GET A  
 $kE$ -MODULE IN  $\text{stmod}(kE)$ . THEN

$$F_i(M) = F^*(F)$$

WHERE  $F$  IS THE FROBENIUS MAP ON  $\mathbb{P}^{n-1}$

AND  $F_i(M) = 0$ ,  $z \leq i \leq p-1$