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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: BRANDEN STONE **Email/Phone:** bstone@bard.edu

Speaker's Name: ~~Ahlaos~~ ~~Dag~~ DAVE BENSON

Talk Title: MODULES FOR ELEMENTARY P-GROUPS AND HYPERSURFACE SINGULARITIES.

Date: 2/14/13 **Time:** 9:00 am / pm (circle one)

List 6-12 key words for the talk: P-GROUPS, HYPERSURFACE, ORLOV CORRESPONDENCE, MATRIX FACTORIZATION, STABLE MODULE CATEGORY, SINGULAR CATEGORY

Please summarize the lecture in 5 or fewer sentances:

Concerning the Orlov correspondence and its relation to the stable module category with the singularity category of a certain hypersurface.

CHECK LIST

(This is **NOT** optional, we will **not pay for incomplete** forms)

- ☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULES FOR ELEMENTARY ABELIAN p-GROUPS AND HYPERSURFACE SINGULARITIES

D. BENSON

G FINITE GROUP, k FIELD OF CHAR p

kG = GROUP ALGEBRA

$$E = \underbrace{\mathbb{Z}/p \times \cdots \times \mathbb{Z}/p}_{r \text{ copies}} \qquad r = \text{RANK OF } E$$

<u>CHOUINARD</u>: A kG-MOD IS PROJECTIVE IFF ITS RESTRICTION TO EACH ELEM. AB. p-SUBGROUP $E \leq G$ IS PROJECTIVE.

<u>QUILLEN</u>: $H^*(G,k) \longrightarrow \varprojlim_{E \leq G} H^*(E,k)$

F-ISOMORPHISM

i.e. ISOM ON SPEC

<u>GOAL</u>: UNDERSTAND $\mathrm{stmod}(kG)$

<u>OBJ</u>: f.g. kG-MOD

<u>MOR</u>: $\underline{\mathrm{Hom}}_{kG}(M,N) = \mathrm{Hom}_{kG}(M,N) / P\mathrm{Hom}_{kG}(M,N)$ $\searrow P \nearrow$

THIS IS A TRIANGULATED CATEGORY

$$D^b(kG)/\mathrm{Perf}(kG) \simeq \mathrm{Stmod}(kG)$$

$$\begin{array}{ccccccccccc}
 & & 0 \to & M_i & \to & M_{i+1} & \to \cdots \to & M_j & \to 0 & & \\
 & & & \downarrow & & \downarrow & & \downarrow & & & \\
M & & 0 \to & I_i & \to & I_{i+1} & \to \cdots \to & I_j & \to & I_{j+1} & \to \cdots \\
I'_{-1} \to I'_0 \to \cdots & & & & & & \cdots \to & I'_j & \to & I_{j+1} & \to
\end{array}$$

Def. $D_{sg}(R) = D^b(R) / \mathrm{Perf}(R)$

$$\mathrm{stmod}(kE) = D_{sg}(kE)$$

e.g. $k[x,y]/(x^2, xy, y^2) = R$

$$0 \to k \oplus k \to R \to k \to 0$$

$$k \xrightarrow{\simeq} k[1] \oplus k[1] \quad \text{in } D_{sg}(R)$$

Example: $R = k[x_1, \ldots, x_n]/(f)$

$$D_{sg}(R) \simeq \underline{MCM}(R)$$

$$\simeq \text{reduced matrix factorizations of } f$$

If R is graded, $X = \mathrm{Proj}\, R$, $\mathrm{Coh}(X) = \dfrac{\mathrm{mod}(R)}{\mathrm{tors}(R)}$

$$D_{sg}(X) = \underline{MCM}(R) \Big/ \text{MCM approx. of } \mathrm{tors}(R)$$

Orlov's Theorem:

$E = \langle g_1, \ldots, g_r \rangle \simeq (\mathbb{Z}/p)^r \quad \mathrm{char}\, k = p$

$x_i = g_i - 1 \in kG, \quad x_i^p = 0$

$kE = k[x_1, \ldots, x_r]/(x_1^p, \ldots, x_r^p)$

$R = k[y_1, \ldots, y_r, x_1, \ldots, x_r]$

$f = y_1 x_1^p + \cdots + y_r x_r^p$

$A = R/(f), \quad B = R/(x_1^p, \ldots, x_r^p) = k[y_1, \ldots, y_r] \otimes_k kE$

$$\begin{tikzcd}
B & A \arrow[l, two heads] & R \arrow[l, two heads] \\
kE \arrow[u] & & k[x_1, \ldots, x_r] \arrow[ll, two heads] \arrow[ul] \arrow[u]
\end{tikzcd}
\qquad \mathrm{Proj} \qquad
\begin{tikzcd}
Z \arrow[r, hook, "i"] \arrow[d] & Y \arrow[dr] & S' \arrow[l, hook', "u"'] \arrow[d, "q"] \\
X \arrow[rr, hook, "j"] & & S
\end{tikzcd}$$

Orlov:

$$D_{sg}(X) \simeq D_{sg}(Y)$$

$$\text{IS}$$

$$\text{stmod}(kE)$$

Burke:

$$D^b(kE) \simeq D_{sg}(A)$$

$$\text{stmod}(kE) \longrightarrow D_{sg}(Y)$$

$$D^b(kE) \longrightarrow D_{sg}(A)$$

$$M \longmapsto k[y_1, \ldots, y_r] \otimes M$$

$$D_{sg}(A) \longrightarrow D^b(kE)$$

$$F \underset{v}{\overset{u}{\rightleftarrows}} F' \qquad uv = vu = f \cdot I$$

$$[1]$$

$$F_0 \otimes_R B \xrightarrow{u} F_0' \otimes_R B$$

$$F_1 \otimes_R B \xrightarrow{u} F_1' \otimes_R B$$

Gives an object in $D^b(kE)$.

Example: $r=2$

$$kE = k[x_1,x_2]$$
$$R = k[y_1,y_2,x_1,x_2], \quad A = R/(y_1x_1^p + y_2x_2^p)$$
$$M = k$$

Resolve $k[y_1,y_2]$ over A:

$$\to \begin{matrix} A[-1] \\ \oplus \\ A[-1] \end{matrix} \xrightarrow{\begin{pmatrix} y_2x_2^{p-1} & -y_1x_1^{p-1} \\ x_1 & x_2 \end{pmatrix}} \begin{matrix} A \\ \oplus \\ A[-1] \end{matrix} \xrightarrow{\begin{pmatrix} x_2 & y_1x_1^{p-1} \\ -x_1 & y_2x_2^{p-1} \end{pmatrix}} \begin{matrix} A \\ \oplus \\ A \end{matrix} \xrightarrow{(x_1,x_2)} A \to k[y_1,y_2]$$

Example:

$$\begin{matrix} A[-1] \\ \oplus \\ A[-1] \end{matrix} \xrightarrow{\begin{pmatrix} y_2 & -y_1 \\ x_1^p & x_2^p \end{pmatrix}} \begin{matrix} A \\ \oplus \\ A[-1] \end{matrix} \xrightarrow{\begin{pmatrix} x_2^p & y_1 \\ -x_1^p & y_2 \end{pmatrix}} \begin{matrix} A \\ \oplus \\ A \end{matrix} \xrightarrow{(x_1^p,x_2^p)} A \to \begin{matrix} k[y_1,y_2] \\ \otimes \\ k[x_1,x_2]/(x_1^p,x_2^p) \end{matrix}$$

Example:

$$\xrightarrow{\begin{pmatrix} y_2 & -y_1 \\ x_1^p & x_2^p \end{pmatrix}} \begin{matrix} A[-1] \\ \oplus \\ A[-2] \end{matrix} \xrightarrow{\begin{pmatrix} x_2^p & y_1 \\ -x_1 & y_2 \end{pmatrix}} \begin{matrix} A[-1] \\ \oplus \\ A[-1] \end{matrix} \xrightarrow{(y_2 \; -y_1)} A \to k[x_1,x_2]$$

RELATED CONSTRUCTION (FRIEDLANDER-PEVTSOVA, B-P)

LET $\mathcal{O}$ = STRUCTURE SHEAF OF $\mathbb{P}^{r-1} = \text{Proj}\, k[Y_1, \ldots, Y_r]$

GIVEN A kE-MODULE M

$$\tilde{M} = \mathcal{O} \otimes M$$

$$\forall j \quad \theta : \tilde{M}(j) \to \tilde{M}(j+1), \quad \theta(f \otimes m) = \sum_{i=1}^{r} Y_i f \otimes X_i m$$

$$1 \le i \le p \quad F_i(M) = \frac{\ker\theta \cap \operatorname{Im}\theta^{i-1}}{\ker\theta \cap \operatorname{Im}\theta^{i}} \quad \text{AS SUBQ OF } \tilde{M}$$

RECIPE: GIVEN A SHEAF $\mathcal{F}$ ON $\mathbb{P}^{r-1}$ COMING FROM A $k[Y_1, \ldots, Y_r]$-MODULE, INFLATE TO AN A-MOD AND APPLY THE ABOVE FUNCTOR TO GET A kE-MODULE IN $\text{stmod}(kE)$. THEN

$$F_1(M) \simeq F^*(\mathcal{F})$$

WHERE F IS THE FROBENIUS MAP ON $\mathbb{P}^{r-1}$

AND $F_i(M) = 0, \quad 2 \le i \le p-1$