Lech's Inequality

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For a zero-dimensional ideal I in a Noetherian local ring R of dimension d, Lech has shown that $e(I) \leq d!\,\lambda(R/I)e(R)$, where $e(-)$ denotes the Hilbert-Samuel multiplicity and $\lambda(-)$ stands for the length. One can argue that asymptotically this inequality is sharp, but it gives a very weak bound for $e(I)$ in general. In a joint work with H. Hariharan and C. Huneke, we explore refinements of the Lech's inequality and the consequent inequalities on the Hilbert coefficients of I.