Socles, quasi-socles, and integral dependence

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A quasi-socle (or iterated socle) of an Artinian local ring (R, m) is an ideal of the form $0 : m^s$ for some s. If R contains the field of rational numbers, we give formulas for the generators of $0 : m^s$ in a certain range of s. These formulas use suitably defined derivatives and the minimal free resolution of R over a power series ring mapping onto it. This extends earlier work by Juergen Herzog, who had treated the case $s = 1$ for graded algebras.

Quasi-socles have been applied to construct part of the integral closure of zero-dimensional ideals I in regular local rings (S, n). We use our structural results about quasi-socles, and in particular the connection with free resolutions, to prove that $I : n^s$ is integral over I in a wide range of cases. This generalizes and unifies work by Corso-Polini-Vasconcelos, Goto, Wang, Watanabe-Yoshida and others.

This is a report on joint work with Alberto Corso, Shiro Goto, Craig Huneke, and Claudia Polini.