Recent results on the grading of local cohomology modules

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After a brief introduction to local cohomology I am going to discuss the result of my student Yi Zhang on the grading of local cohomology modules in characteristic p>0 and its recent extension to characteristic 0 by Linquan Ma and Wenliang Zhang. Namely, if R is a polynomial ring in n variables over a field and $m\subset R$ is the maximal ideal generated by the variables then it is well-known that $H^n_m(R)$ with its natural grading is isomorphic to $E(n)$, i.e. the naturally graded injective hull E of R/m degree-shifted downward by n. It has also been well-known that if $I\subset R$ is any ideal, then the local cohomology module $H^i_m(H^j_I(R))$ is isomorphic to a direct sum of a finite number of copies of E. Yi, Linquan and Wenliang sharpened this result by showing that if $I\subset R$ is any homogeneous ideal, then the local cohomology module $H^i_m(H^j_I(R))$, with its natural grading is isomorphic to a direct sum of a finite number of copies of $E(n)$. Some other related recent results will also be discussed.