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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Neil Epstein **Email/Phone:** nepstei2@gmu.edu

Speaker's Name: James McKernan

Talk Title: ACC for the log canonical threshold and the termination of flips

Date: 05 / 06 / 2013 **Time:** 3:30 am / pm (circle one)

List 6-12 key words for the talk: ______

Please summarize the lecture in 5 or fewer sentences: Along with coauthors, the speaker has established the ACC property for log canonical thresholds in certain contexts. In addition to this result, he lays out six related conjectures and explains what progress has been made toward each of them.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

J. McKernan – ACC for the log canonical threshold and termination of flips

(Convention: We work over $\mathbb{C}$)

Def: A log pair (X,Δ); X normal variety, $\Delta \geq 0$ divisor, $K_X+\Delta$ is $\mathbb{Q}$-Cartier ($\mathbb{R}$-Cartier).

nicest possibility **Def** (X,Δ) is "log-smooth" (i.e. X smooth and $\operatorname{Supp}\Delta$ has simple normal crossings)

(**Def** A log resolution is a birational morphism $\pi: Y \longrightarrow X$ s.t. $(Y, \Gamma+E)$ is log smooth.

Γ = strict transform of Δ, $E = \sum$ exceptionals.

$$K_Y + \Gamma + E = \pi^*(K_X+\Delta) + \sum a_i E_i$$

[**Def**: a_i is the log discrepancy of E_i w.r.t. (X,Δ). We write $a_i = a(X,\Delta,E_i) = a(X,\Delta,V_i)$.

Write $a = a(X,\Delta) := \inf_{V_i} a(X,\Delta,E_i)$]

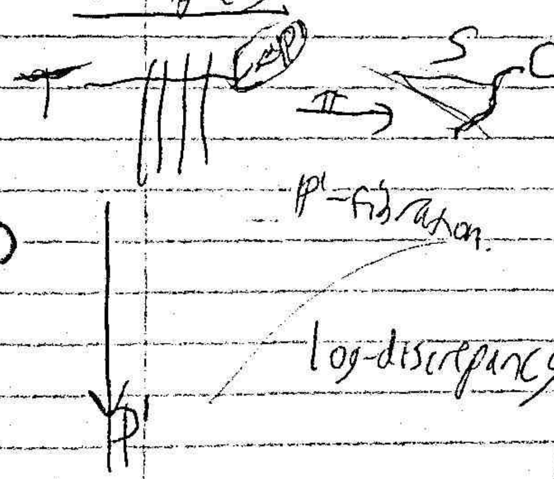
Remark: Either

$a = -\infty$

or $a \geq 0$ (Def: "log canonical")

or $a = a_i$ for some i.

Examples: S = cone over a rational curve of degree n



E (This is a log resolution of S).

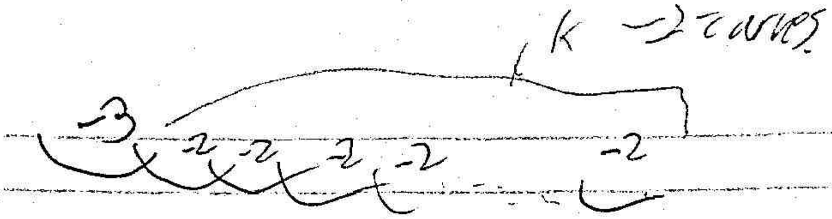
$$K_T + E = \pi^* K_S + a \cdot E.$$

Then dot both sides with E

$$-2 = K_{\mathbb{P}^1} = K_E = (K_T + E) \cdot E$$

$$= (\pi^* K_S + aE) \cdot E = aE^2 = -an.$$

Hence, $\boxed{a = \frac{2}{n}}$

$T =$ 

$\pi \downarrow$

$S =$ contract all these curves

$a(K_S, 0) = ?$

$K_T + E = \pi^* K_S + \sum_{i=0}^{k} a_i E_i$ (where E_0 represents the −3-curve)

Dot both sides with E_i

e.g. when $1 \le i \le k$,

LHS

$-2+1+1 = 0 + a_{i-1} - 2a_i + a_{i+1}$

Answer: $a_0 = \frac{k+2}{2k+3}$, $a_1 = \frac{k+3}{2k+3}$, $\dots$, $a_k = \frac{2k+2}{2k+3}$

(Note: $a_i \in [\frac{1}{2}, 1]$ $\forall i$)

(Example: $I = \{\frac{n-1}{n} \mid n \in \mathbb{N}\}$ satisfies DCC).

(6)

Conjecture A (local ACC for log pairs)

Fix n, two sets $I \subset [0,1]$, $J \subset \mathbb{R}_{\geq 0} = [0,\infty)$.

that satisfy the D.C.C.

$\exists I_0, J_0$ finite such that whenever you are given:

1. $x \in X$ affine, dim n,
2. (X, Δ) log canonical
3. $\mathrm{coeffs}(\Delta) \subseteq I$,
4. and $a(X, \Delta) \in J$

Then $\mathrm{coeff}(\Delta) \subseteq I_0$ and $a(X, \Delta) \in J_0$.

Conjecture B (ACC for the log discrepancy)

Same hypothesis, but the conclusion only asserts the existence of J_0.

Conjecture C: (ACC for thresholds).

Same conjecture but assume $J = \{a\}$.

Thm (Hacon, Xu): Conjecture C holds when $a=0$

Birkar-Shokurov: $B_n \Rightarrow C_n$

Alexeev: A_2 holds

Borisov: A_n holds for X toric.

$P \in X$

$a(X,\Delta,P) = \inf\{a(X,\Delta,v) \mid \text{center of } v = P\}$ (Ambro, Shokurov)

Conj D Let (X,Δ) be a log pair. Fix $a \in [0,\infty)$.
Then the set $\{P \in X \mid a(X,\Delta,P) \leq a\}$ is Zariski-closed

Thm (Shokurov): Assume B_n and D_n.
(X,Δ) dlt pair, $\dim X = n$, X $\mathbb{Q}$-factorial, projective.
If you run MMP, it always terminates (flips terminate)

Conjecture E (BAB) Fix $I \subset [0,1]$ which satisfies
DCC. Fix $n \in \mathbb{N}$, $\varepsilon > 0$
The set of all pairs (X,Δ) s.t.
(1) $\dim X = n$, (2) $\mathrm{coeffs}(\Delta) \subseteq I$, (3) $a(X,\Delta) > \varepsilon$ and (4) $-(K_X+\Delta)$ is ample) is bounded

[in other words, $\begin{array}{c} X \\ \varphi \downarrow \\ S \end{array}$ finite type, φ projective

$\forall$ elts of this set, $\exists$ fiber ...]

Alexeev: E_2 holds.

Borisov & Borisov hold in toric case.

~~Conjecture~~:

[ultimate goal: Reduce A_n to E_{n-1}]

Conjecture F: (Shokurov) Fix $I \subset [0,1]$ satisfying DCC.

Fix n. $\exists r \in \mathbb{N}$ s.t. given
① (X, Δ) klt, ② X $\mathbb{Q}$-factorial, ③ $\dim X = n$,
④ $\mathrm{coeff}(\Delta) \subseteq I$,

(He draws Dynkin diagrams)

$\exists$ log-canonical divisor $\Theta \geq \Delta$ s.t. $r(K_X + \Theta) \sim 0$ (local comp)

Thm (–, Coscini) Assume A_3. whenever the smallest accumulation pt. of J is ≥ 1. Assume C_3. Then A_3 holds.

Suppose not. Then $\exists\ (X_i, \Delta_i)$ s.t. $q_i = a(X_i, \Delta_i)$

For each i, pick Θ_i s.t. $r(K_{X_i} + \Theta_i)$ is Cartier

$\{a(X_i, \Theta_i, v_i) \in \{\frac{1}{r} \mid i \in N\}\}$

$\Phi_\lambda(i) = (1-\lambda)\Delta_i + \lambda\Theta_i$ $\qquad a(X_i, \Theta_i) \leq a(X_i, \Delta_i)$

May assume $a(X_i, \Theta_i)$ cst.

Cheat: Assume $a(X_i, \Theta_i) = a(X_i, \Theta_i, v_i)$ and $a(X_i, \Delta_i) = a(X_i, \Delta_i, v_i)$.

With that assumption,

$$a(X_i, \Phi_\lambda(i)) = (1-\lambda)a(X_i, \Delta_i) + \lambda \cdot a(X_i, \Theta_i)$$