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Name: Neil Epstein **Email/Phone:** nepstei2@gmu.edu

Speaker's Name: Emily Witt

Talk Title: F-pure thresholds of quasi-homogeneous polynomials

Date: 05 / 08 / 2013 **Time:** 2:00 **am / pm (circle one)**

List 6-12 key words for the talk: ______

Please summarize the lecture in 5 or fewer sentences: (see abstract)

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F-pure thresholds of quasi-homogeneous polynomials

Emily Witt
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The F-pure threshold is an invariant in characteristic $p>0$ measuring how ``bad" a singularity is; it is analogous to the log canonical threshold in characteristic zero. We describe the general form of an F-pure threshold of a homogeneous polynomial with isolated singularity in a polynomial ring endowed with a possibly non-standard grading. This is joint work with Daniel Hernández, Luis Núñez-Betancourt, and Wenliang Zhang.

F-pure thresholds of quasi-homog. polynomials (joint x3)

$f_{\mathbb{Q}} \in \mathbb{Q}[x_1, \ldots, x_n] \rightsquigarrow f_p \in \mathbb{F}_p[x_1, \ldots, x_n]$

$f_{\mathbb{Q}}(\underline{0}) = 0 \qquad f_p(\underline{0}) = 0.$

<u>log canon. threshold:</u>

$\mathrm{lct}(f_{\mathbb{Q}}) = \sup\{\lambda \mid \mathcal{J}(\mathbb{A}^n_{\mathbb{Q}}, f^{\lambda}) = \mathbb{Q}[\underline{x}]\}$

$\lambda > 0 \quad \mathbb{C}^n \longrightarrow \mathbb{R}$

$\underline{z} \longmapsto \frac{1}{|f(\underline{z})|^{2\lambda}}$

<u>F-pure threshold</u>

$\mathrm{fpt}(f_p) = \sup\{\lambda \mid \tau(f^{\lambda}, \mathbb{F}_p[\underline{x}]) = \mathbb{F}_p[\underline{x}]\}$

$= \lim_{e \to \infty} \frac{\max\{N \mid f_p^N \notin \mathfrak{m}^{[p^e]}\}}{p^e} \in (0,1] \cap \mathbb{Q}$

(Blickle-Mustaţă-Smith)

<u>Ex</u>: $f_{\mathbb{Q}} = x^2 - y^3$

$\mathrm{lct}(f_{\mathbb{Q}}) = 5/6$

$\mathrm{fpt}(f_p) = \begin{cases} 1/2, & p = 2 \\ 2/3, & p = 3 \\ 5/6, & p \equiv 1 \pmod 6 \\ 5/6 - 1/6p & p \equiv 5 \pmod 6 \end{cases}$

<u>EX</u> (Bhatt-Singh) $f_{\mathbb{Q}} \in \mathbb{Q}[x,y,z]$, deg 3 iso. sing at $\underline{0}$, $E \subseteq \mathbb{P}^2_{\mathbb{Q}}$. Then

$\mathrm{fpt}(f_p) = \begin{cases} 1, & E_p \text{ ordinary} \\ 1 - \frac{1}{p}, & E_p \text{ supersingular.} \end{cases}$

$f_{\mathbb{Q}}$ Calabi-Yau iso sing, deg n in n vars., $\mathrm{fpt}(f_p) = 1 - \frac{A}{p}$, $0 \le A \le n-2$.

<u>Thm</u> (Mustaţă-Zhang) If $f_{\mathbb{Q}} \in \mathbb{Q}[x_1, \ldots, x_n]$, $\exists C > 0$, $N \in \mathbb{N}$ s.t. $\frac{1}{p^N} \le \mathrm{lct}(f_{\mathbb{Q}}) - \mathrm{fpt}(f_p) \le \frac{C}{p}$ when $\mathrm{lct} \ne \mathrm{fpt}$ and $p \gg 0$.

<u>Hara-Yoshida</u>: $\mathrm{fpt}(f_p) \le \mathrm{lct}(f_{\mathbb{Q}})$, and $\lim_{p \to \infty} \mathrm{fpt}(f_p) = \mathrm{lct}(f_{\mathbb{Q}})$.

<u>Notation</u>: $\alpha \in (0,1]$, $\alpha = \sum_{e=1}^{\infty} \frac{\alpha_e}{p^e}$ (unique non-terminating base p expansion)

$\langle \alpha \rangle_d := \sum_{e=1}^{d} \frac{\alpha_e}{p^e}$, $\langle \alpha \rangle_\infty = \alpha$ (d^{th} truncation)

e.g. $\langle \frac{1}{p} \rangle_1 = 0$, $\langle \frac{1}{p} \rangle_2 = \frac{p-1}{p^2}, \ldots$ (since $\frac{1}{p} = \frac{0}{p} + \sum_{n \ge 2} \frac{p-1}{p^n}$.)

<u>Thm</u> (H-NB-W-Z): $f_{\mathbb{Q}} \in \mathbb{Q}[x,y]$ quasi-homog iso sing at $\underline{0}$

<u>Fact</u>: $\mathrm{lct}(f_{\mathbb{Q}}) = \min\{1, \frac{\deg(xy)}{\deg(f_{\mathbb{Q}})}\}$. Then $\mathrm{fpt}(f_p) = \langle \mathrm{lct}(f_{\mathbb{Q}}) \rangle_d$ for some $d \in \mathbb{N} \cup \{\infty\}$.

w/same hypotheses, but in $\mathbb{Q}[x_1, \ldots, x_n]$, $\mathrm{fpt}(f_p) = \langle \mathrm{lct}(f_{\mathbb{Q}}) \rangle_d - \frac{E}{p^d}$, $0 \le E \le n-2$.

<u>Rmk</u>: If $\mathrm{fpt}(f_p) \ne \mathrm{lct}(f_{\mathbb{Q}})$, write $\mathrm{lct}(f_{\mathbb{Q}}) = \alpha/\beta$ in lowest terms. Then $d \le$ order of p in $(\mathbb{Z}/\beta\mathbb{Z})^{\times}$

Ex: $f_{\mathbb{Q}} = x^{15} + xy^7$ $(\deg x = 1, \deg y = 2)$. $\mathrm{lct}(f_{\mathbb{Q}}) = \frac{3}{15} = \frac{1}{5}$.

(Hernández-Shibuta-Takagi):

p	11	13	17	19	23	29	31	$\cdots$	73
d	1	1	1	2	4	∞	1		3

Ex (Hara-Monsky): $f_{\mathbb{Q}} = l_1 l_2 l_3 l_4 l_5 \in \mathbb{Q}[x,y]$

linear forms

$p \equiv \square \bmod 5$	Possible $\mathrm{fpt}(f_p)$
1	$2/5$, $\frac{2p-1}{5p} = \langle 2/5 \rangle_1$
2	$\langle 2/5 \rangle_1$
3	$\langle 2/5 \rangle_{2n}$ $\langle 2/5 \rangle_3$
4	$2/5, \langle 2/5 \rangle_1, \langle 2/5 \rangle_2$