

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Neil Epstein Email/Phone: nepstei2@gmu.edu

Speaker's Name: Masayuki Kawakita

Talk Title: The index of a threefold canonical singularity

Date: 05/09/2013 Time: 10:30 am / pm (circle one)

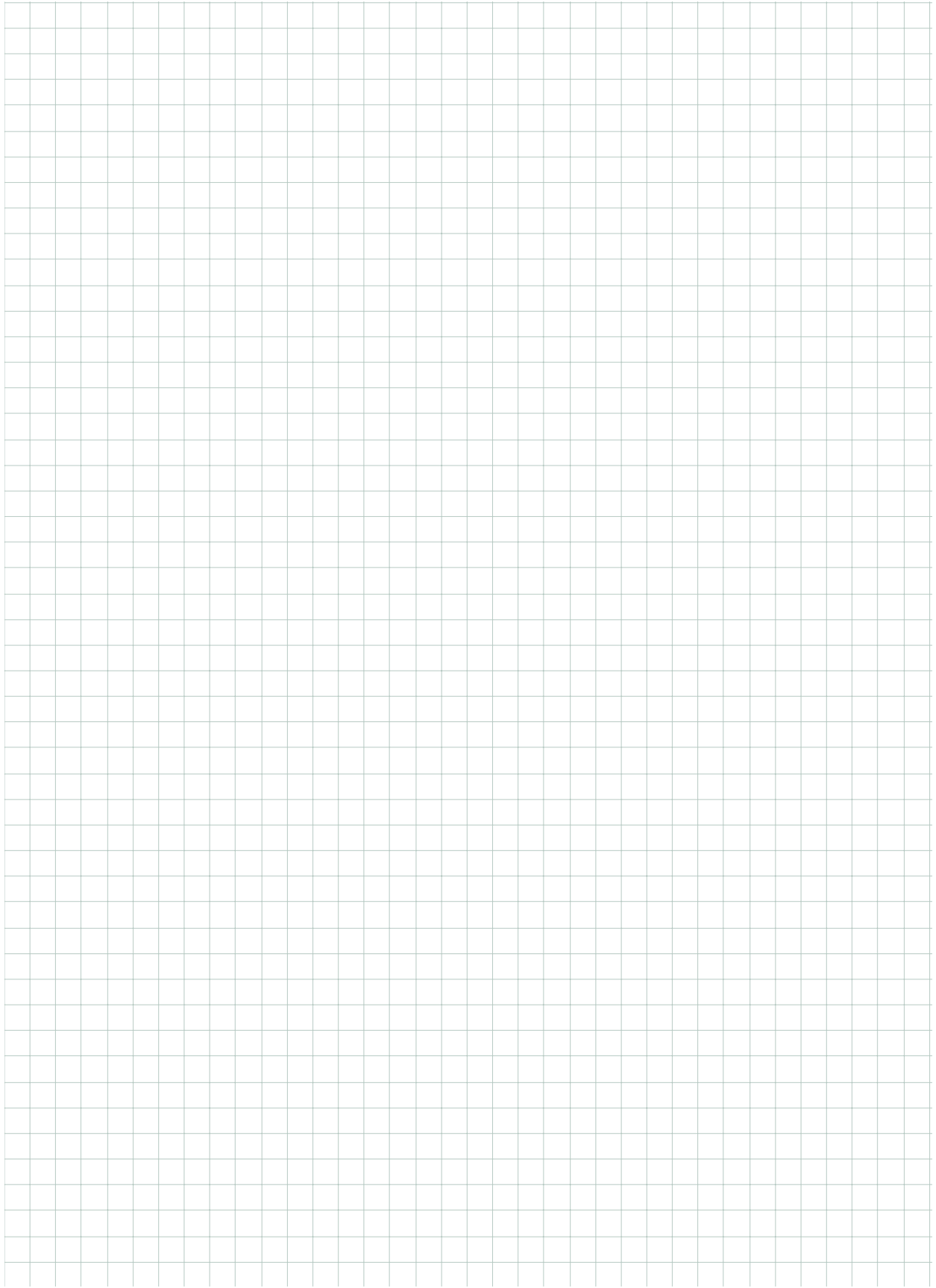
List 6-12 key words for the talk: _____

Please summarize the lecture in 5 or fewer sentences: (see abstract)

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
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(YYYY.MM.DD.TIME.SpeakerLastName)
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The index of a threefold canonical singularity

Masayuki Kawakita

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We will discuss the question raised by Shokurov: can one bound the index of a \mathbb{Q} -Gorenstein singularity in terms of the discrepancies over it? We will give an answer for 3-fold canonical singularities.

The index of a threefold canonical singularity

$P \in X/\mathbb{C}$ normal, \mathbb{Q} -Gor (i.e. $r \in K_X$ Cartier, index $r_P =$ the smallest $r > 0$)

Shokurov: Can bound r_P in terms of discrepancies of divisors over $P \in X$

$E \subset Y \xrightarrow{f} X$ proper birat $\text{discrep. } a_E(X) := \text{coeff of } E \text{ in } K_Y - f^*K_X$, center $\mathcal{C}_X(E) := f(E)$

X Thm. $\Leftrightarrow a_E(X) > 0 \forall E \text{ exc}$ $\xrightarrow{\text{dim 2}} \text{Gor}$ $\} r_P = 1$

cano	≥ 0
lt	> -1
lc	≥ -1

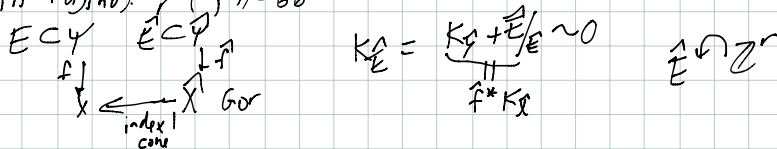
Want to bound r_P by fixing the min discrepancy $md_P(X) := \min_{\mathcal{C}(E)=P} a_E(X) \in \{-\infty\} \cup [0, \infty)$

$\frac{1}{r} (1, -2) \quad r_P = r \quad md_P = -1 + \frac{1}{r}$

Q: Fix $(d, a) \in \mathbb{N} \times [1, \infty)$
 $\exists? r(d, a)$ s.t. $\left[\begin{array}{l} P \in X \text{ dim } d, md_P X = a \\ \Rightarrow r_P \leq r(d, a) \end{array} \right. \rightarrow X \text{ lc.}$

typical case: $a = -1$ (i.e. P a lc center) $r(2, -1) = 6$

(Ichi-Fujino): $r(3, -1) = 66$



Modification to canonical sing:

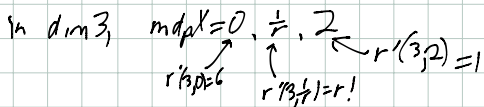
Q: fix $(d, a) \in \mathbb{N} \times [0, \infty)$

$\exists? r'(d, a)$ s.t. $\left[\begin{array}{l} P \in X \text{ cano. dim } d, md_P X = a \\ \Rightarrow r_P \leq r'(d, a) \end{array} \right.$

Typical case: $a = 0$, i.e. P is a crepant center. $r(3, 0) = 1$

Thm: $P \in X$ cano. dim 3. P is a crepant center $\stackrel{\text{def}}{\Leftrightarrow} md_P X = 0 \Rightarrow r_P \leq 6$.
 (in part, one can take $r(3, 0) = 6$)

Cor: \mathbb{Q} yes for $d = 3$.



Rmk: (1) Shokurov conjectured: $r_p \leq 6$ (presumably because for $P \in S \times \mathbb{A}^1 = X$, $r_p \leq 6$, $\text{md}_P X = 0$ not cano. ec.

We have examples only for $r_p \leq 4$:

(Morrison, Ishida-Iwashita): P cyc. quotient singularity $\Rightarrow r_p \leq 3$.
 $r_p \neq 1 \Leftrightarrow \frac{1}{24}(1, 2, 1, -2)_{r_p=2}, \frac{1}{14}(1, 3)_{r_p=2}, \frac{1}{4}(1, 4, 7)_{r_p=3}$

(Hayakawa-Takemuchi): $P \in (f=0) \subset \mathbb{C}^4 / \mathbb{Z}_r \Rightarrow r_p \leq 4$
 $r_p \neq 4 \Leftrightarrow (x_1^2 + x_2^2 + x_3^2 = 0) \subset \frac{1}{8}(1, 5, 3, 7)$

(2): need: P crepant center.

e.g. $0 \in (x_1^2 + x_2^2 = 0) \subset \mathbb{C} \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} / \mathbb{Z}_r$. cano, $\text{md}_P X = \frac{1}{r}$, crepant center x_4 -axis.

Sketch of pf. of theorem

Crepant partial resolution: $Y \xrightarrow{f} X$ (exists by min. model theory)

term. $K_Y = f^* K_X$
 $f^* P \quad \downarrow \quad P$

$\cdot \text{TAKE } E \subset f^{-1}P$ reduced divisor.

\Rightarrow characterization of r_p as follows:

$$f_* \mathcal{O}_Y(i(K_Y - E)) = \begin{cases} m_p \mathcal{O}_X(iK_X), & r_p | i \\ \mathcal{O}(iK_X), & r_p \nmid i \end{cases}$$

projection formula

If $r_p \nmid i$, $f_* \mathcal{O}(i(K_Y - E)) = \{u \in K_X \mid (u)_Y + iK_Y - E \geq 0\}$

$$\mathcal{O}(iK_X) = \{u \in K_X \mid (u)_X + iK_X \geq 0\}$$

\leftarrow not Cartier at $P \Rightarrow \exists D > 0$ s.t. $(u)_X + iK_X \geq D$

hence we get "U" containment above as well. That is, $\mathcal{O}(iK_X) = f_* \mathcal{O}(i(K_Y - E))$ (*)

\cdot find $E > 0$ satisfying (*) and vanishing $R^j f_* \mathcal{O}(i(K_Y - E)) = 0$ $\forall j \geq 1$ $\forall i$.

$$0 \rightarrow f_* \mathcal{O}(i(K_Y - E)) \rightarrow f_* \mathcal{O}(iK_Y) \rightarrow f_* \mathcal{O}(iK_Y|_E) \rightarrow 0$$

\parallel
 \mathcal{Q}_i

Then $\dim \mathcal{Q}_i = \chi(iK_Y) - \chi(iK_Y - E)$.

$$= \begin{cases} 1, & r_p | i \\ 0, & r_p \nmid i \end{cases}$$

explicitly described by Reid's singular RR^{red} (usual term + contribution term from $Q = \frac{1}{r_q}(1, -1, b_q)$ from Y .)

$\cdot r_p = \text{lcm}\{r_q\}$

$$\cdot \dim \mathcal{Q}_0 - \dim \mathcal{Q}_1 = \sum_q \frac{V_q(r_q - V_q)}{2r_q} \rightsquigarrow r_p \leq 9. \Rightarrow r_p \leq 6, \text{ geo}$$

\parallel
1 (unless $r_p = 1$)

Rmk: Y terminal $P \in X$ ec. $K_Y + D = f^* K_X$. Then sing RR $\rightsquigarrow 1 = \sum_q \frac{V_q(r_q - V_q)}{2r_q} + (\text{neg term})$
 \leftarrow can't control!

