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Name: Neil Epstein **Email/Phone:** nepstei2@gmu.edu

Speaker's Name: Shihoko Ishii

Talk Title: Singularities with respect to Mather-Jacobian discrepancies

Date: 05 / 09 / 2013 **Time:** 2:00 am / pm (circle one)

List 6-12 key words for the talk: ______

Please summarize the lecture in 5 or fewer sentences: (see abstract)

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Singularities with respect to Mather-Jacobian discrepancies

Shihoko Ishii
University of Tokyo

The Usual discrepancy of canonical divisors is defined for a normal Q-Gorenstein variety and used for classification of singularities from the view point of birational geometry. In the talk I will use Mather-Jacobian discrepancy instead of usual discrepancy and discuss ``canonical" and ``log-canonical" singularities with respect to this discrepancy. Our interest is also for non-Q-Gorenstein or even non-normal varieties.

Singularities with respect to Mather-Jacobian discrepancy

Shihoko Ishii

Graduate School of Mathematical Science
University of Tokyo

May 9, 2013, MSRI

Throughout this talk, X is always connected equidimensional reduced scheme of finite type over an algebraically closed field of characteristic 0. $d = \dim X$.

Outline

1. Motivation

X: normal and $\mathbb{Q}$-Gorenstein

- $f : \overline{X} \to X$: an appropriate resolution of the singularities of X
- $\Rightarrow$ “usual discrepancy divisor" $K_{\overline{X}/X}$ is defined
- $\Rightarrow$ canonical singularities, log canonical singularities
- $\Rightarrow$ multiplier ideal

X: not necessarily normal or $\mathbb{Q}$-Gorenstein ($\Rightarrow$ usual discrepancy is not defined.)

- $f : \overline{X} \to X$: an appropriate resolution of the singularities of X
- $\Rightarrow$ Mather-Jacobian discrepancy divisor $\widehat{K}_{\overline{X}/X} - J_{\overline{X}/X}$ is defined
- $\Rightarrow$ “canonical singularities", “log canonical singularities"
- $\Rightarrow$ “multiplier ideal"

Problem

What kind of singularities are these?

2. Definitions

$f : \overline{X} \to X$ a resolution of the singularities of X factoring through the Nash blow up

$$\begin{array}{ccc} f^* \wedge^d \Omega_X & \xrightarrow{df} & \wedge^d \Omega_{\overline{X}} = \omega_{\overline{X}} \\ & & \cup \\ & & Im(df) \quad \text{(invertible)} \\ & & \| \\ & & \mathcal{I}\omega_{\overline{X}} \end{array}$$

f through the Nash blow up $\Rightarrow$ $Im(df)$ is invertible.
$\mathcal{I}$ is an invertible ideal sheaf !!!
Define the divisor $\widehat{K}_{\overline{X}/X}$ as

$$\mathcal{O}_{\overline{X}}(-\widehat{K}_{\overline{X}/X}) = \mathcal{I}$$

$\widehat{K}_{\overline{X}/X}$ is an effective integral divisor on $\overline{X}$.
“Mather discrepancy divisor"

Jacobian ideal
$X \subset \mathbb{A}^N$
$I_X = (f_1, f_2, \ldots, f_r)$ the defining ideal of X in $\mathbb{A}^N$

$$\left(\frac{\partial f_j}{\partial x_i}\right) \quad \text{Jacobian matrix}$$

$$\mathcal{J}_X := \left((N-d) - minors\ of\ \left(\frac{\partial f_j}{\partial x_i}\right)\right)|_X$$

$\mathcal{J}_X$ is independent of the choice of embeddings $X \subset \mathbb{A}^N$.

$\mathcal{J}_X$ is the “Jacobian ideal of X"

Note

If $f : \overline{X} \to X$ is a log resolution of $(X, \mathcal{J}_X)$, then f factors through the Nash blow up.

Definition

Let $f : \overline{X} \to X$ be a log resolution of $(X, \mathcal{J}_X)$. Define a divisor $J_{\overline{X}/X}$ as

$$\mathcal{O}_{\overline{X}}(-J_{\overline{X}/X}) = \mathcal{J}_X \mathcal{O}_{\overline{X}}.$$

$J_{\overline{X}/X}$ is called the “Jacobian discrepancy divisor".

Definition

$f : \overline{X} \to X$ log resolution of $(X, \mathcal{J}_X)$, $\mathcal{O}_{\overline{X}}(-J_{\overline{X}/X}) = \mathcal{J}_X \mathcal{O}_{\overline{X}}$.
For a prime divisor E over X

$$\widehat{a}(E; X, \mathcal{J}_X) := ord_E \left(\widehat{K}_{\overline{X}/X} - J_{\overline{X}/X} \right) + 1$$

"Mather-Jacobian log discrepancy at E"

$$\widehat{a}(E; X, \mathcal{J}_X) := ord_E\left(\widehat{K}_{\overline{X}/X} - J_{\overline{X}/X}\right) + 1 \quad \text{(M-J)}$$

for a reduced equidimensional scheme X.

$$a(E; X) := ord_E\left(K_{\overline{X}/X}\right) + 1 \quad \text{(“usual")}$$

for a normal $\mathbb{Q}$-Gorenstein variety X.

Definition

X has MJ-canonical singularities
$\Leftrightarrow \widehat{a}(E, X, \mathcal{J}_X) \geq 1$ for every exceptional prime divisor E over X

Definition

X has log MJ-canonical singularities
$\Leftrightarrow \widehat{a}(E, X, \mathcal{J}_X) \geq 0$ for every exceptional prime divisor E over X

In [De Fernex, Docampo] these are called J-canonical and log J-canonical.

Remember the “usual" canonical / log canonical singularities.

Definition

X has canonical singularities
$\Leftrightarrow a(E, X) \geq 1$ for every exceptional prime divisor E over X

Definition

X has log canonical singularities
$\Leftrightarrow a(E, X) \geq 0$ for every exceptional prime divisor E over X

Why do we think of these singularities?

Because of good properties of Mather-Jacobian discrepancy.

First good property
Inversion of Adjunction of minimal MJ-discrepancy
([DD], [I-])
$\Rightarrow$ upper bound of minimal MJ-discrepancy
$\Rightarrow$ lower semi continuity of minimal MJ-discrepancy
$\Rightarrow$ ACC for MJ-log canonical threshold
(Better properties than usual discrepancy)

Second good property

We can define Mather-Jacobian multiplier ideal. This ideal has good properties (local vanishing, subadditivity, restriction theorem, Skoda type theorem..[EIM])

It is natural to think of MJ-canonical / log MJ-canonical singularities.

3. Deformations

How MJ-canonical and log MJ-canonical singularities behave under deformations?

Definition

Let D be a variety,
$0 \in D$ a closed point
and $\pi : X \to D$ a surjective morphism with equidimensional reduced fibers $X_t = \pi^{-1}(t)$ of common dimension for all $t \in D$.
Then $\pi : X \to D$ is called a deformation of X_0.

Theorem

Let $x \in X$ be a closed point and $\pi : X \to D$ a deformation of X_0 with $\pi(x) = 0$.
Assume that (X_0, x) is MJ-canonical (resp. log MJ-canonical) singularity.
Then, by replacing D and X by small neighborhoods of 0 and x respectively,
X_t has MJ-canonical (resp. log MJ-canonical) singularities for every $t \in D$.

Corollary

Let $0 \in X \subset \mathbb{A}^{d+1}$ *be an hypersurface defined by a polynomial f and* $\Gamma(f) \subset \mathbb{R}^{d+1}$ *its Newton polygon.*
If $(1, 1, \ldots, 1) \notin \Gamma(f)$ *(resp.* $(1, 1, \ldots, 1) \notin \Gamma(f)^0$*),*
then $(X, 0)$ *is not log MJ-canonical (resp. not MJ-canonical).*

4. Characterization of MJ-singularities of low dimension

Theorem

$\dim X = 1$

1. *X is MJ-canonical $\Leftrightarrow$ X is nonsingular.*
2. *X is log MJ-canonical $\Leftrightarrow$ X has at most ordinary double point.*

Note

$(X, 0)$ *is log MJ-canonical* $\Rightarrow$ *emb*$(X, 0) \leq 2d$
$(X, 0)$ *is MJ-canonical* $\Rightarrow$ *emb*$(X, 0) \leq 2d - 1$
Therefore
$(X, 0)$ *is 2-dimensional log MJ-canonical* $\Rightarrow$ *emb*$(X, 0) \leq 4$
$(X, 0)$ *is 2-dimensional MJ-canonical* $\Rightarrow$ *emb*$(X, 0) \leq 3$

Theorem

$\dim X = 2$

X is MJ-canonical $\Leftrightarrow$ X has at worst rational double point

Theorem

$\dim X = 2$ *and* $emb(X, 0) = 3$
$(X, 0)$ *is log MJ-canonical singularity if and only if* X *is defined by* $f(x, y, z) = 0 \in k[[x, y, z]]$ *as follows:*

1. $mult_0 f = 3$ *and* $V(in(f)) \subset \mathbb{P}^2$ *is reduced curve with at worst ordinary double point.*
2. $mult_0 f = 2$
   1. $f = x^2 + y^2 + g(z)$, $\deg g \geq 2$.
   2. $f = x^2 + g_3(y, z) + g_4(y, z)$, $\deg g_i \geq i$, g_3 *is homogeneous of degree 3 and* $g_3 \neq l^3$ *(l linear)*
   3. $f = x^2 + y^3 + yg(z) + h(z)$, $mult_0 g \leq 4$ *or* $mult_0 \leq 6$.
   4. $f = x^2 + g(y, z) + h(y, z)$, g *is homogeneous of degree 4 and it does not have a linear factor with multiplicity more than 2.*

Theorem

$\dim X = 2$, $emb(X, 0) = 4$

1. *If* $(X, 0)$ *is a complete intersection at 0, then* $(X, 0)$ *is log MJ-canonical* $\Leftrightarrow$ X *is defined by* f_1, f_2 *with* $\mathrm{mult}_0 f_i = 2$ *and* $V(in(f_1), in(f_2)) \subset \mathbb{P}^3$ *is a reduced curve with ordinary double points.*
2. *if* $(X, 0)$ *is not a complete intersection at 0, then* $(X, 0)$ *is log MJ-canonical* $\Leftrightarrow$ X *is a union of irreducible components of log MJ-canonical complete intersection scheme.*

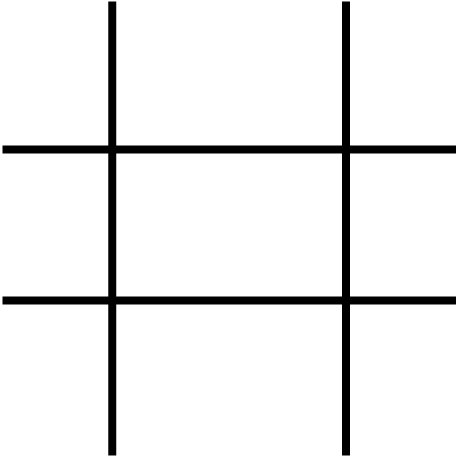
2-dimensional MJ-canonical singularities/log MJ-canonical singularities are determined.

Proposition

Assume X is S_2 and $\mathbb{Q}$-Gorenstein.
X is log MJ-canonical $\Rightarrow$ X is semi log canonical.

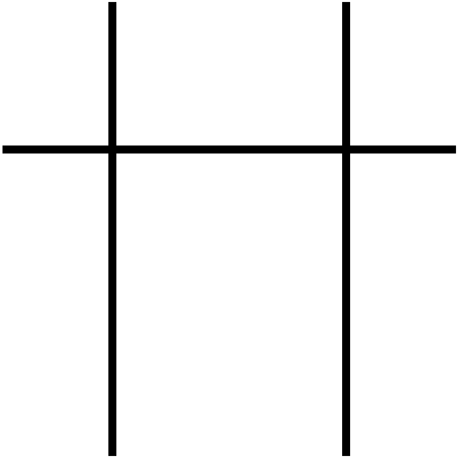
Note : There are log MJ-canonical singularities with non S_2 or non $\mathbb{Q}$-Gorenstein.

$X = V(xy, zw) \subset \mathbb{A}^4$, X = cone over



$\Rightarrow$ X is complete intersection log MJ-canonical

X= cone over



$\Rightarrow$ X is non $\mathbb{Q}$-Gorenstein log MJ canonical

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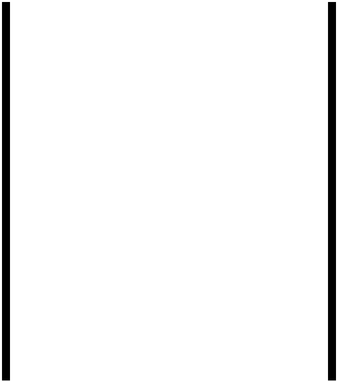

$\Rightarrow$ X is non S_2 non $\mathbb{Q}$-Gorenstein log MJ canonical singularities.

Ambitious Question

Is the class of log MJ-canonical singularities closed under every step in MMP?

If this holds true, it would be very nice.
Because MJ-discrepancy has good properties (Inversion of Adjunction, lower semi-continuity of the minimal MJ-discrepancy, ACC of log MJ-canonical thresholds...) and therefore proofs of MMP would be simpler.

Too Ambitious Question

There are examples of divisorial contractions such that the source varieties have only log MJ-canonical singularities and the target varieties have non log MJ-canonical singularities.

3-dimensional terminal cyclic quotient singularities

$$\frac{1}{r}(s,-s,1), \quad s<r, gcd(s,r)=1$$

Proposition ([Kawamata])

A divisorial contraction $Y \to Z$ to a point $\frac{1}{r}(s,-s,1)$ in Z is unique. It is a weighted blow up and Y has two singularities $\frac{1}{s}(-r,r,1)$ and $\frac{1}{r-s}(-r,r,1)$.
By the successive weighted blow ups one can resolve the singularities.

Example

If $s \neq 1$ or $s \neq r - 1$, then $\frac{1}{r}(s, -s, 1)$ is not log MJ-canonical.

Starting with non singular variety, one can obtain such a singularity by successive divisorial contractions.
$\Rightarrow$ counterexamples for the question.

T. De Fernex and R. Docampo [DD]
Jacobian discrepancies and rational singularities
arXiv: 1106.2172

L. Ein, S. Ishii and M. Mustaţă [EIM]
Multiplier ideals via Mather discrepancy
to appear Publ. RIMS, Proceeding of S. Mori's 60th birthday conference.

S. Ishii [I-]
Mather discrepancy and the arc spaces.
Annales de l'Institute Fourier, 63(1):89–111, 2013.

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& \| & \\
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\end{array}
$$

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$\mathcal{I}$ is an invertible ideal sheaf !!!
Define the divisor $\widehat{K}_{\overline{X}/X}$ as

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for a reduced equidimensional scheme X.

$$a(E; X) := ord_E\left(K_{\overline{X}/X}\right) + 1 \quad \text{(``usual")}$$

for a normal $\mathbb{Q}$-Gorenstein variety X.

Definition

X has MJ-canonical singularities
$\Leftrightarrow \widehat{a}(E, X, \mathcal{J}_X) \geq 1$ for every exceptional prime divisor E over X

Definition

X has log MJ-canonical singularities
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Corollary

Let $0 \in X \subset \mathbb{A}^{d+1}$ *be an hypersurface defined by a polynomial f and* $\Gamma(f) \subset \mathbb{R}^{d+1}$ *its Newton polygon.*
If $(1, 1, \ldots, 1) \notin \Gamma(f)$ *(resp.* $(1, 1, \ldots, 1) \notin \Gamma(f)^0$*),*
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X is MJ-canonical $\Leftrightarrow$ X has at worst rational double point

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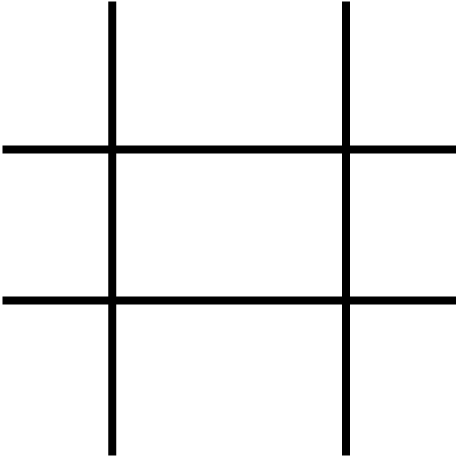

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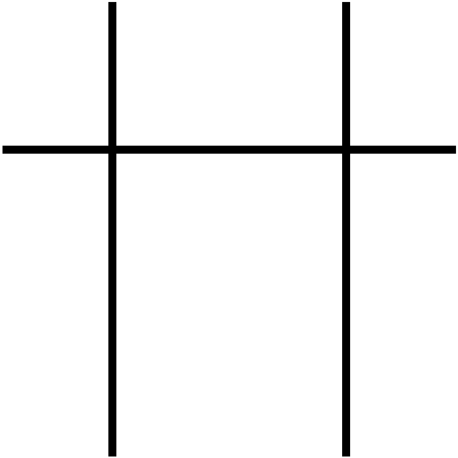

$\Rightarrow$ X is non $\mathbb{Q}$-Gorenstein log MJ canonical

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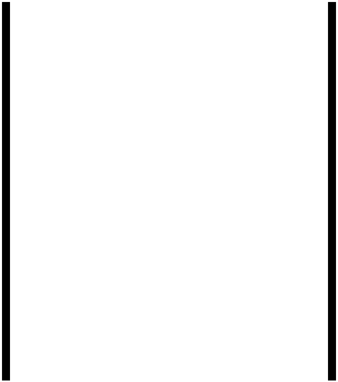

$\Rightarrow X$ is non S_2 non $\mathbb{Q}$-Gorenstein log MJ canonical singularities.

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