MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Neil Epstein **Email/Phone:** nepstei2@gmu.edu

Speaker's Name: Willem Veys

Talk Title: The monodromy conjecture for motivic and related zeta functions

Date: 05 / 09 / 2013 **Time:** 3:30 am / pm (circle one)

List 6-12 key words for the talk: ______

Please summarize the lecture in 5 or fewer sentences: The speaker introduces the "monodromy conjecture", discussing its interpretations in (and relevance to) complex analysis, resolution of singularities, analytic number theory, and topology.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

The monodromy conjecture for motivic and related zeta functions

① $f \in \mathbb{R}[x_1, \dots, x_d]$ (or $\mathbb{C}[x_1, \dots, x_d]$). $\int_{\mathbb{R}^d} |f(x)|^s \varphi\, |dx|$, where φ is C^∞ with compact support, $s \in \mathbb{C}$ with $Re(s) > 0$.

OR $\int_{\mathbb{C}^d} |f(x)|^{2s} \varphi\, |dx \wedge d\bar{x}|$. both called $Z(f, \varphi; s)$

Bernstein & Gel'fand, meromorphic cont. to $\mathbb{C}$
Atiyah, poles $\subset \mathbb{Q}_{>0}$

Thm 1 (Kashiwara-Malgrange? '83-84) $\forall \varphi$: s_0 pole of $Z(f, \varphi; s) \Rightarrow e^{2\pi i s_0}$ is an eigenvalue of local monodromy at some point of $\{f = 0\}$.

(At this point, he reveals a board of pictures explaining monodromy)

Thm 2 (Barlet): "Every eigenvalue is obtained in this way"

②: $f \in \mathbb{Q}_p[x_1, \dots, x_d]$, $\int_{\mathbb{Q}_p^d} |f|^s |dx|_p = Z_p(f; s)$, the "Igusa zeta function"
($s \in \mathbb{C}$, $Re(s) > 0$)

Thm (Igusa): $Z_p(f; s)$ is a rat'l function in p^{-s}.

Note: $Z_p(f; s)$, $f \in \mathbb{Z}[x_1, \dots, x_d]$
$\updownarrow$
$\#\{\text{solutions of } f = 0 \text{ in } (\mathbb{Z}/p^j\mathbb{Z})^d\}$, $j \geq 1$.

Monodromy Conjecture: $f \in \mathbb{Q}[x_1, \dots, x_d]$. For almost all primes p, s_0 is a pole of $Z_p(f; s) \Rightarrow e^{2\pi i s_0}$ is eigenvalue of local monodromy at some point of $\{f = 0\}$.

Rmks: $d = 2$, Loeser '88
Némethi-V. 2011

③ (resolution of singularities)

Given f, $\{f = 0\} \subset \mathbb{A}^d \xleftarrow[h]{\text{log resol.}} (E = \bigcup E_i \subset Y)$. For each i, define N_i by $\mathrm{div}\, h^* f = \sum_{i \in S} N_i E_i$
and $K_{Y/\mathbb{A}^d} = \mathrm{div}(\pi^* dx_1 \wedge \cdots \wedge dx_n) = \sum_i (\nu_i - 1) E_i$ defines ν_i.

$I \subset S$, $E_I^\circ := \bigcap_{i \in I} E_i \setminus \bigcup_{\ell \notin I} E_\ell$.

(*) [Denef] $Z_p(f; s) = \sum_{I \subset S} \#(E_I^\circ(\mathbb{F}_p)) \prod_{i \in I} \frac{p^{-1}}{p^{\nu_i + sN_i} - 1}$.

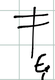
Note: $Re(\text{poles}) \subset \{-\nu_i/N_i \mid i \in S\}$, valid if h "has good reduction mod p", e.g. if f is defined over $\mathbb{Q}$, good reduction for almost all p.

⊗: alternating product of the characteristic polynomials of monodromy

$$= \prod_{i \in S} (t^{N_i} - 1)^{-\chi(E_i^\circ \cap h^{-1}(b))}$$

We expect: (in general): If $\chi(E_i^\circ) = 0$, then $-\frac{\nu_i}{N_i}$ is not real part of pole.

d=2: $\chi(E_i^\circ) = 0$



$\Updownarrow$

E_i intersects twice other E_ℓ.

d=3: Case

$E_i^\circ \cong \mathbb{P}^2 \setminus C$, $C = \bigcup$ irred components.

$\chi(\mathbb{P}^2 \setminus C)$ is typically highly positive. To find ≤ 0, ...

~~Conj~~ Thm: If $\chi(\mathbb{P}^2 \setminus C) < 0$, then all irred components of C are rational.

Q: If $\chi(\mathbb{P}^m \setminus C) = 0$, all irred components of C satisfy ??

④ Topological Zeta fn.

$$\lim_{p \to 1} Z_p(f; s) = \sum_{I \subset S} \chi(E_I^\circ) \prod_{i \in I} \frac{1}{\nu_i + s N_i} \quad (*)$$

1991, Denef & Loeser: $f \in \mathbb{C}[x_1, \ldots, x_d]$, $Z_{top}(f; s) := (*)$.

Q: (intrinsic definition that ends up with (*)).