

What are the noncommutative projective surfaces?

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Noncommutative algebraic geometers regard noncommutative rings as "coordinate rings of noncommutative spaces" and consider the fact that a noncommutative space does not actually exist to be a minor technical issue. I will ask the audience to follow me on this flight of fancy.

Let R be an N -graded domain with cubic growth, thought of as the "coordinate ring of a noncommutative projective surface." What are the possibilities for R ? This is the problem of "classification of noncommutative projective surfaces," an extremely active research area for the last 20 years. (The analogous problem for curves was solved by Artin and Stafford in 1995.)

Each "noncommutative projective surface" has an associated division ring, and understanding these is a natural first step towards classification. In 1996, Artin conjectured that these division rings come in only 4 (or $3\frac{1}{2}$) families. I'll explain Artin's conjecture, discuss evidence for and against it, and describe what is known about noncommutative projective surfaces in each of the 4 types.