Introduction to Derived Categories

Amnon Yekutieli

Ben Gurion University

We outline the construction of the derived category D(M) of an abelian category M. We then define left and right derived functors. We introduce K-projective, K-injective and K-flat resolutions, and prove existence of some derived functors when M is either the category of modules over a ring, or the category of sheaves of modules over a sheaf of rings. DG algebras and their derived categories will also be mentioned.

Next we discuss some more specialized topics: dualizing complexes (commutative and noncommutative), two-sided tilting complexes, derived Morita theory (for DG algebras), and perverse sheaves.