MSRI
Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Shuangjian Zhang **Email/Phone:** shuangjian.zhang@mail.utoronto.ca

Speaker's Name: Christina Sormani

Talk Title: Convergence of Riemannian manifolds and Metric Measure Spaces

Date: 08/22/2013 **Time:** 09:30 am / pm (circle one)

List 6-12 key words for the talk: Convergence, metric measure space, Riemannian Manifold

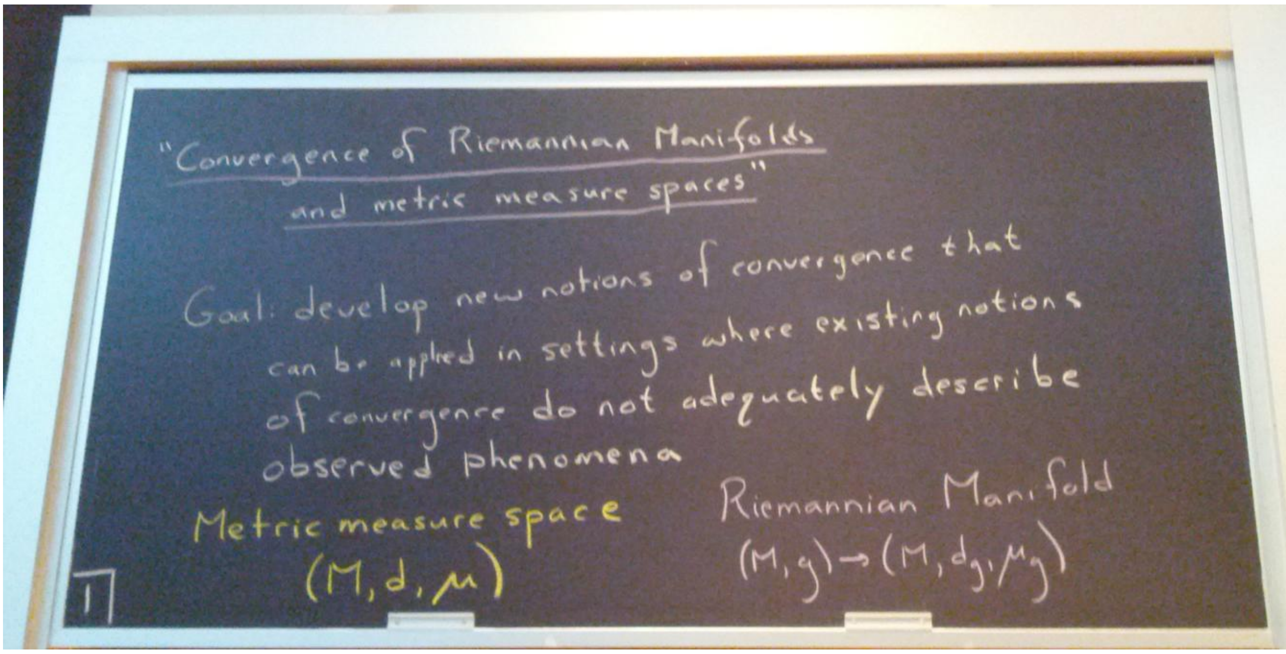
Please summarize the lecture in 5 or fewer sentances: Our goal is to develop new notions of convergence that can be appiled in settings where existing notions of convergence do not adequately describe observed phenomena.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker ~~as~~ to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Convergence of Riemannian Manifolds and metric measure spaces



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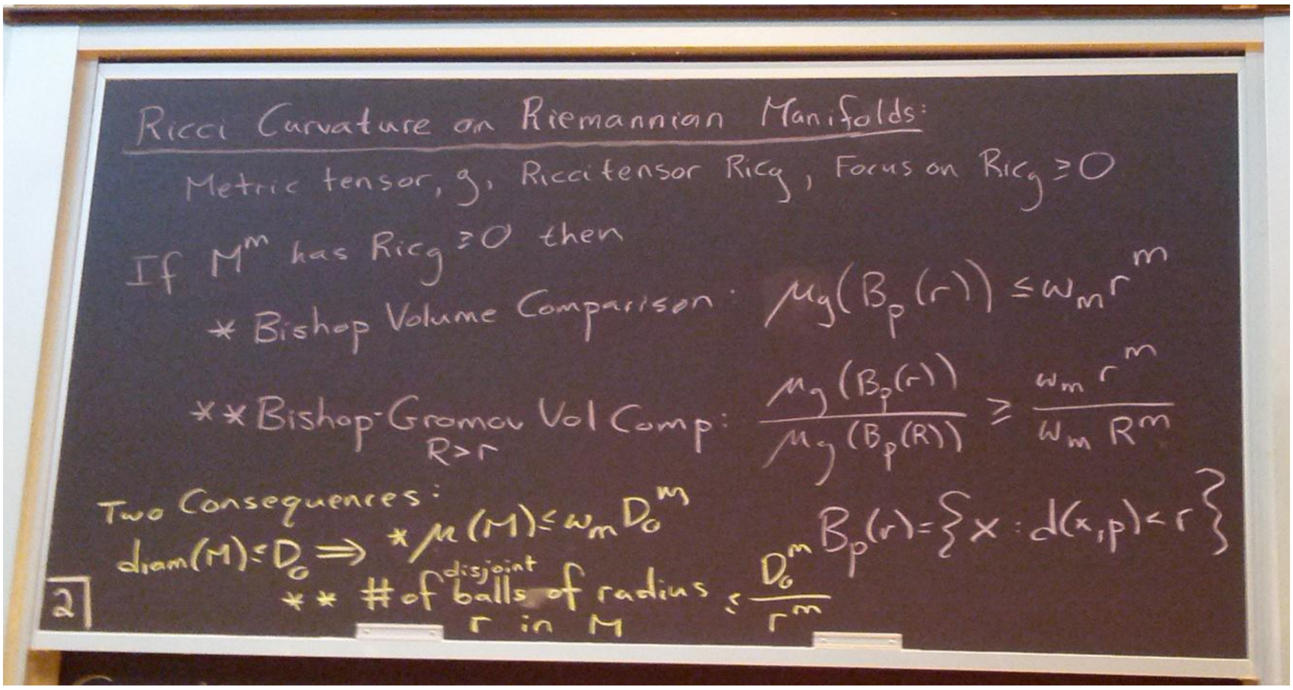

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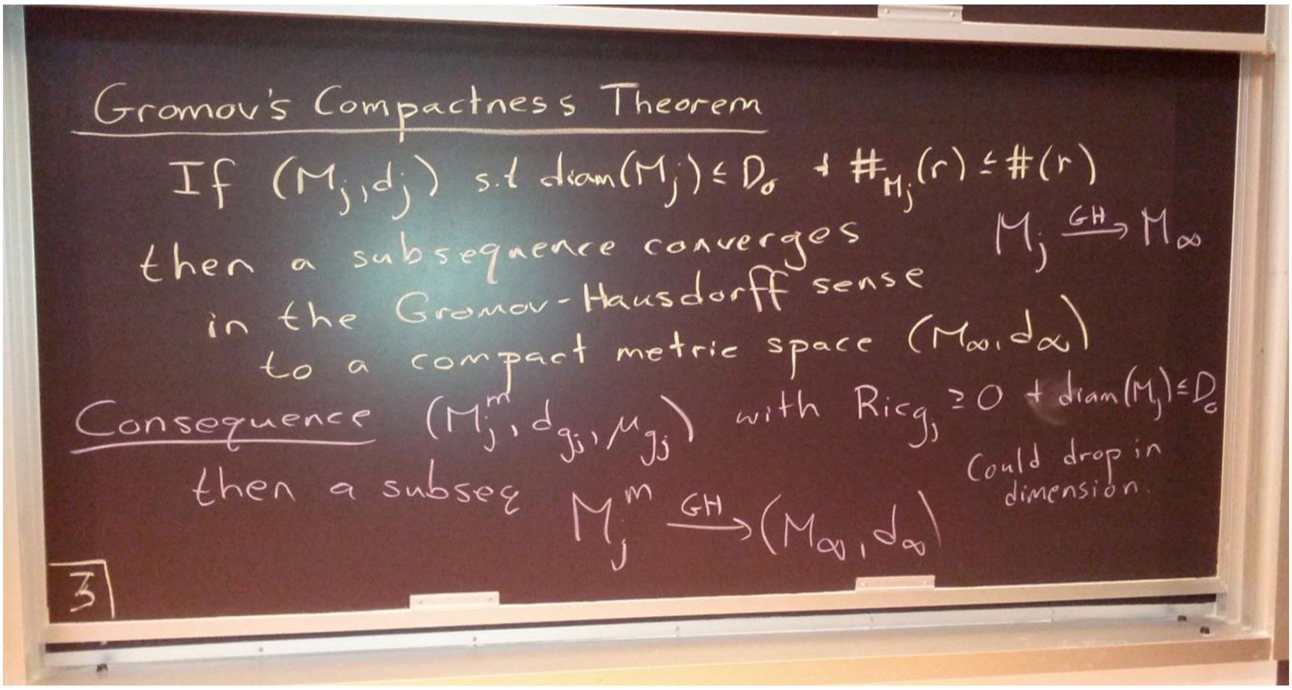


Gromov's Compactness Theorem

If (M_j, d_j) s.t. $\mathrm{diam}(M_j) \le D_0$ + $\#_{M_j}(r) \le \#(r)$

then a subsequence converges $\quad M_j \xrightarrow{GH} M_\infty$

in the Gromov-Hausdorff sense

to a compact metric space (M_∞, d_∞)

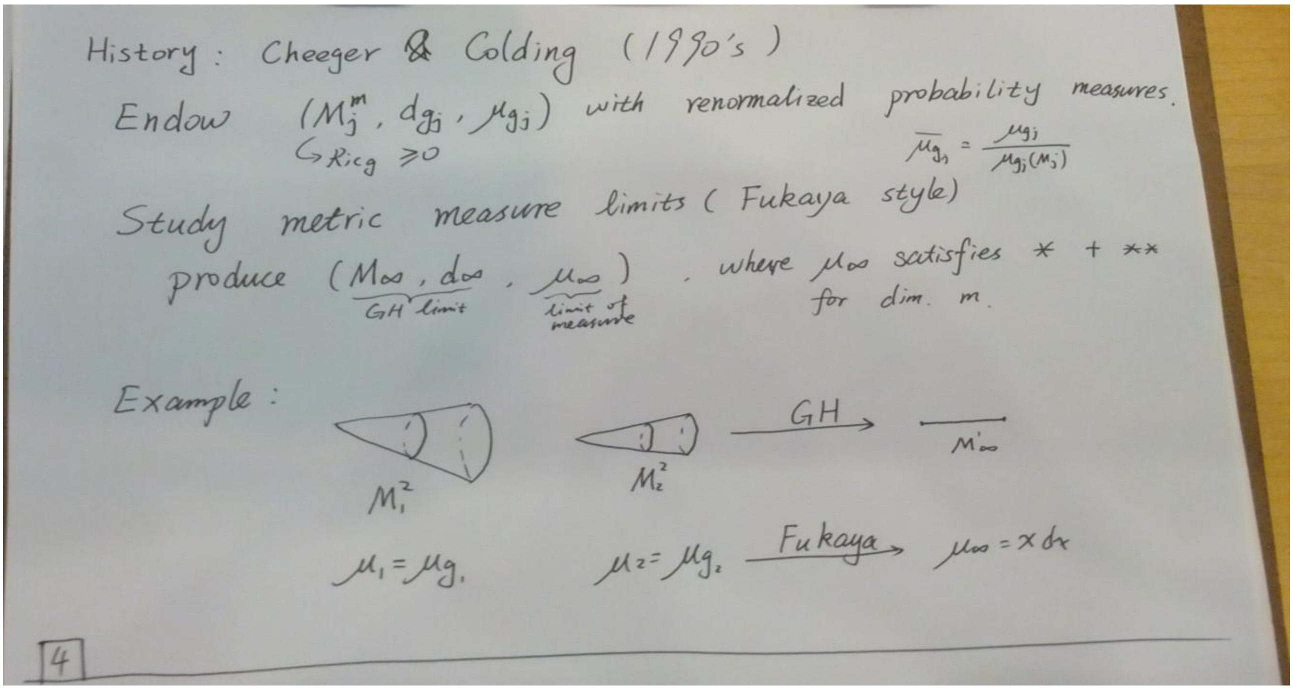
Consequence $(M_j^m, d_{g_j}, \mu_{g_j})$ with $Ric_{g_j} \ge 0$ + $\mathrm{diam}(M_j) \le D_0$

then a subseq $M_j^m \xrightarrow{GH} (M_\infty, d_\infty)$

Could drop in dimension.

$\boxed{3}$

3



History: Cheeger & Colding (1990's)

Endow $(M_j^m, d_{g_j}, \mu_{g_j})$ with renormalized probability measures.

$\hookrightarrow Ric_g \ge 0$ $\qquad \bar{\mu}_{g_j} = \frac{\mu_{g_j}}{\mu_{g_j}(M_j)}$

Study metric measure limits (Fukaya style)

produce $(\underbrace{M_\infty, d_\infty}_{GH\ limit}, \underbrace{\mu_\infty}_{limit\ of\ measure})$. where μ_∞ satisfies * + ** for dim. m.

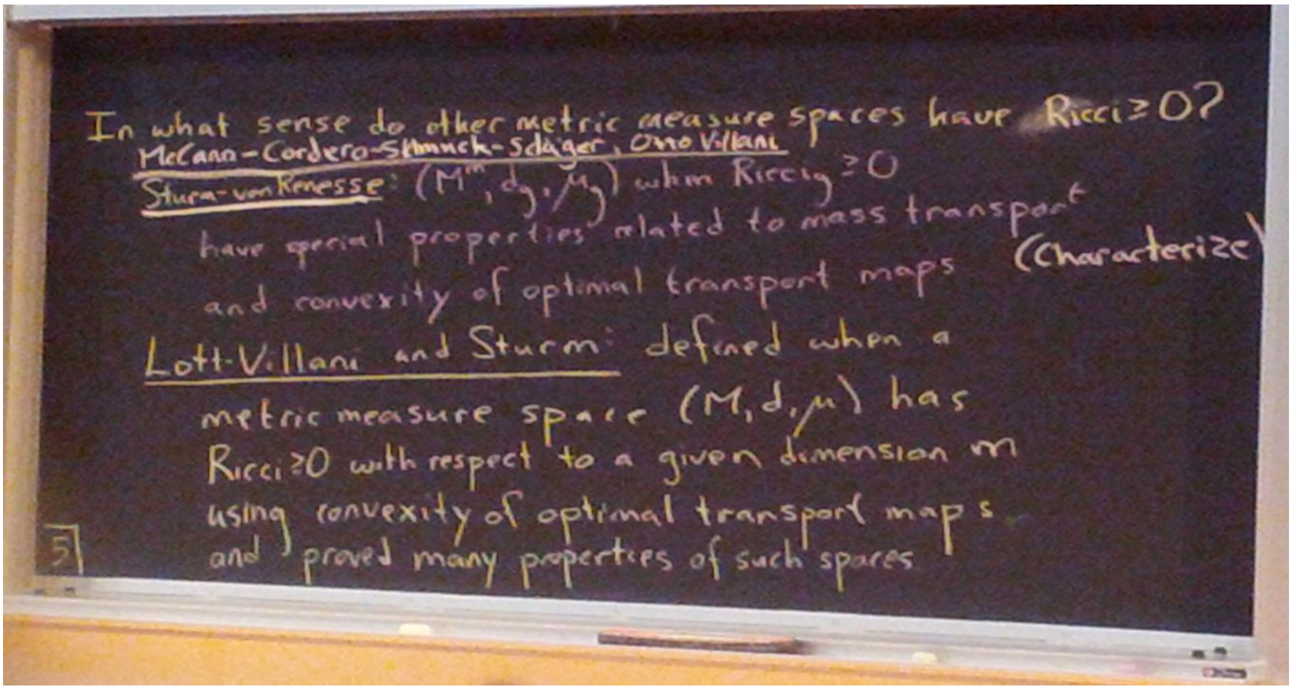
Example:

$M_1^2 \qquad M_2^2 \xrightarrow{GH} M'_\infty$

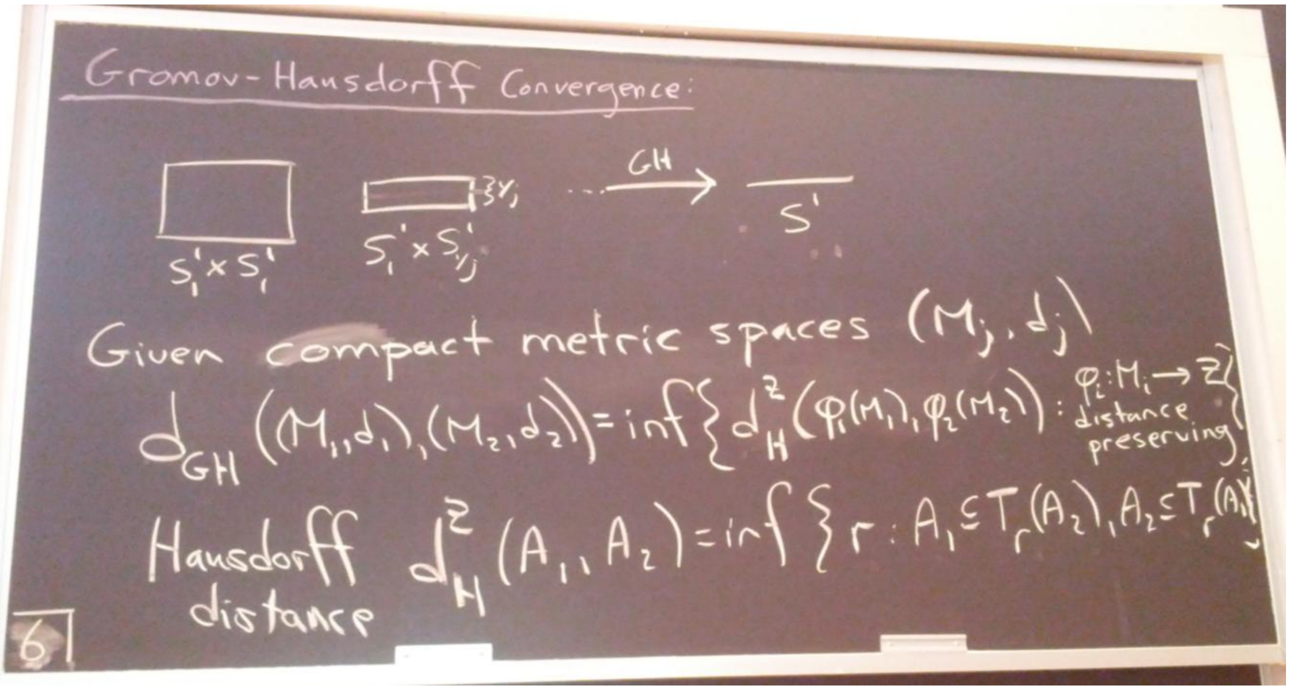
$\mu_1 = \mu_{g_1} \qquad \mu_2 = \mu_{g_2} \xrightarrow{Fukaya} \mu_\infty = x\,dx$

$\boxed{4}$

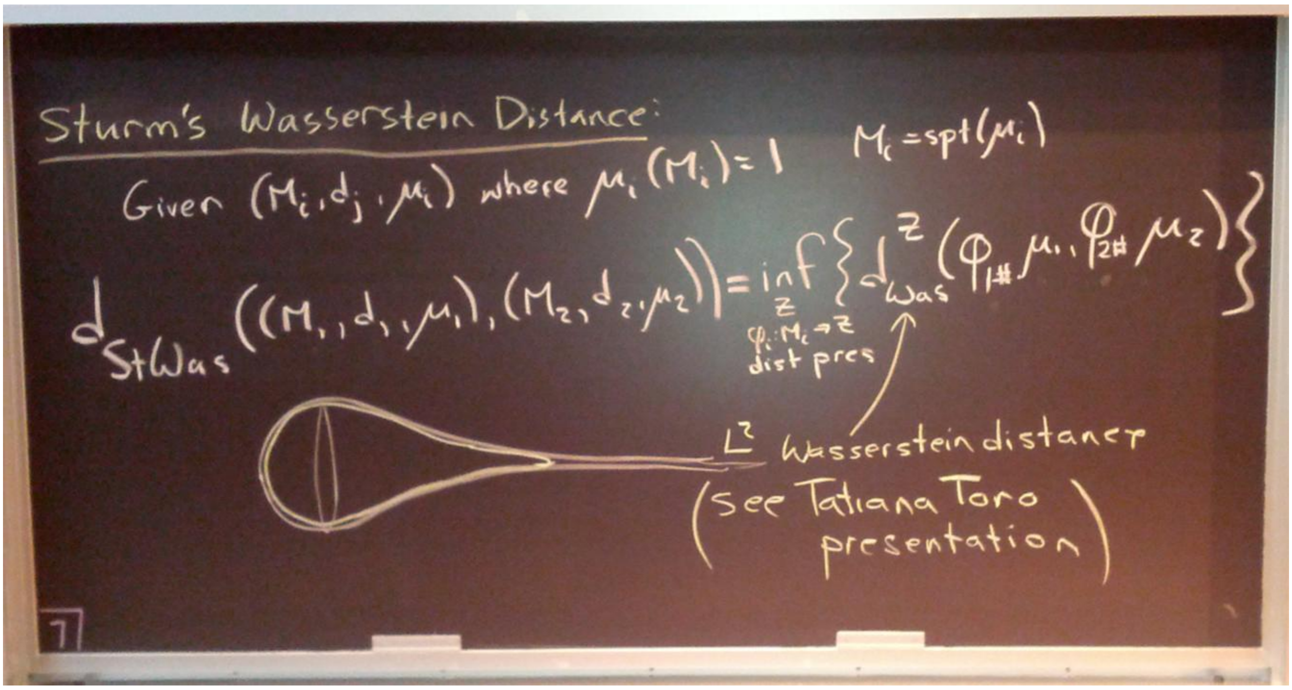
4


In what sense do other metric measure spaces have Ricci ≥ 0?
McCann-Cordero-Schmuck-Schläger, Otto Villani
Sturm-von Renesse: (M^m, d_g, μ_g) when $Ricci_g \geq 0$
have special properties related to mass transport
and convexity of optimal transport maps
(Characterize)
Lott-Villani and Sturm: defined when a
metric measure space (M, d, μ) has
Ricci ≥ 0 with respect to a given dimension m
using convexity of optimal transport maps
and proved many properties of such spaces
5

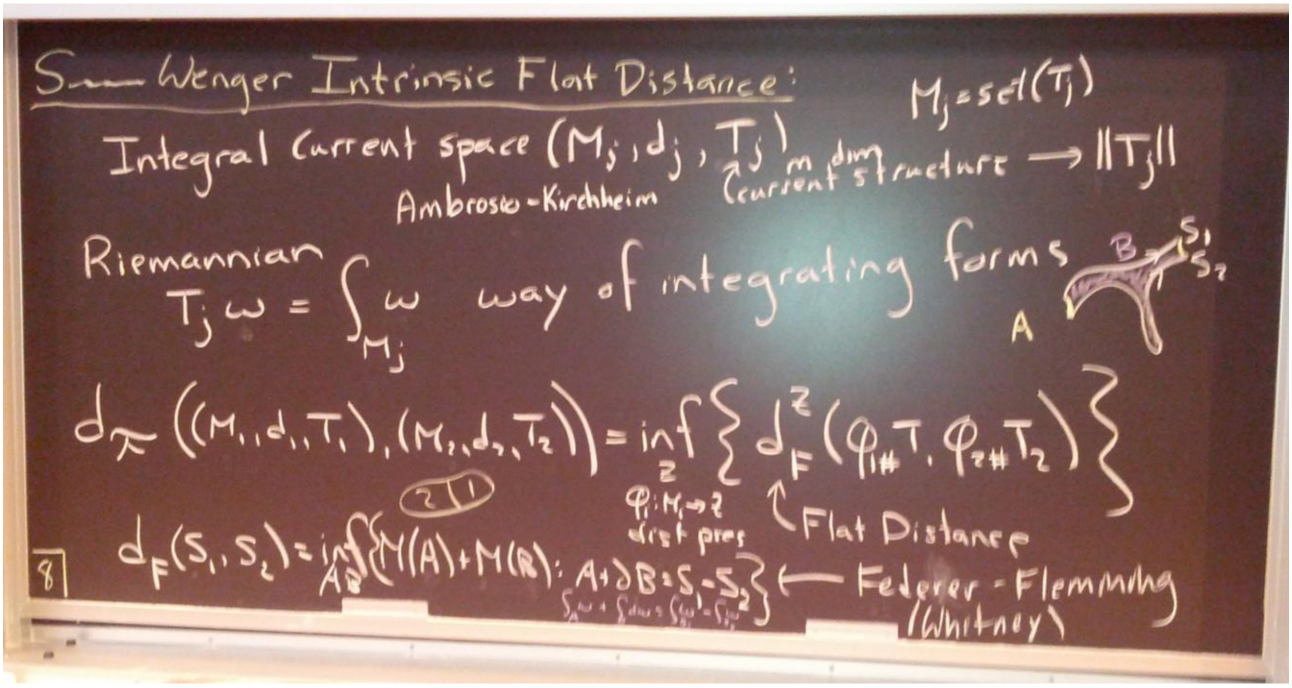
5


Gromov-Hausdorff Convergence:
$S^1_1 \times S^1_1$
$S^1_1 \times S^1_{1/j}$
$1/j$
GH
S^1
Given compact metric spaces (M_j, d_j)
$d_{GH}((M_1,d_1),(M_2,d_2)) = \inf\{ d_H^Z(\varphi_1(M_1), \varphi_2(M_2)) :$
$\varphi_i : M_i \to Z$ distance preserving $\}$
Hausdorff distance $d_H^Z(A_1, A_2) = \inf\{ r : A_1 \subseteq T_r(A_2), A_2 \subseteq T_r(A_1)\}$
6

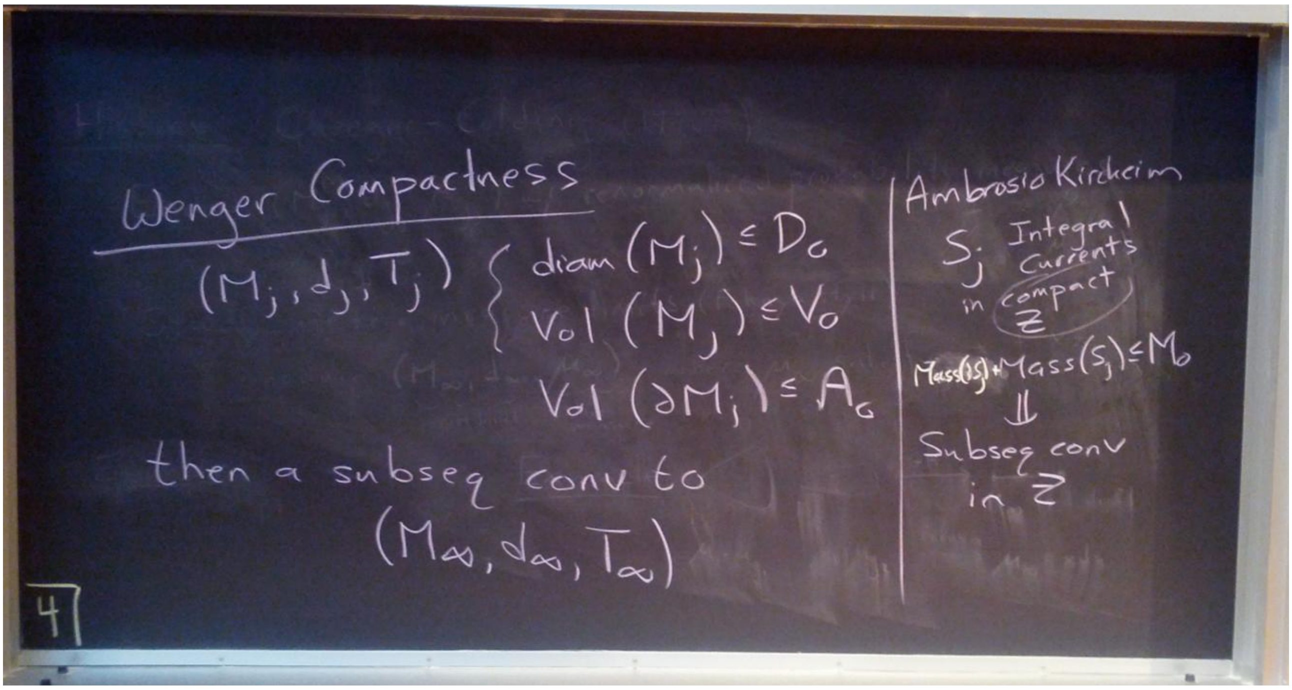
6


Sturm's Wasserstein Distance:
M_i = spt(μ_i)
Given (M_i, d_j, μ_i) where μ_i(M_i) = 1
d_StWas((M_1, d_1, μ_1), (M_2, d_2, μ_2)) = inf_Z {d^Z_Was(φ_1# μ_1, φ_2# μ_2)}
φ_i: M_i → Z dist pres
L^2 Wasserstein distance
(see Tatiana Toro presentation)
7

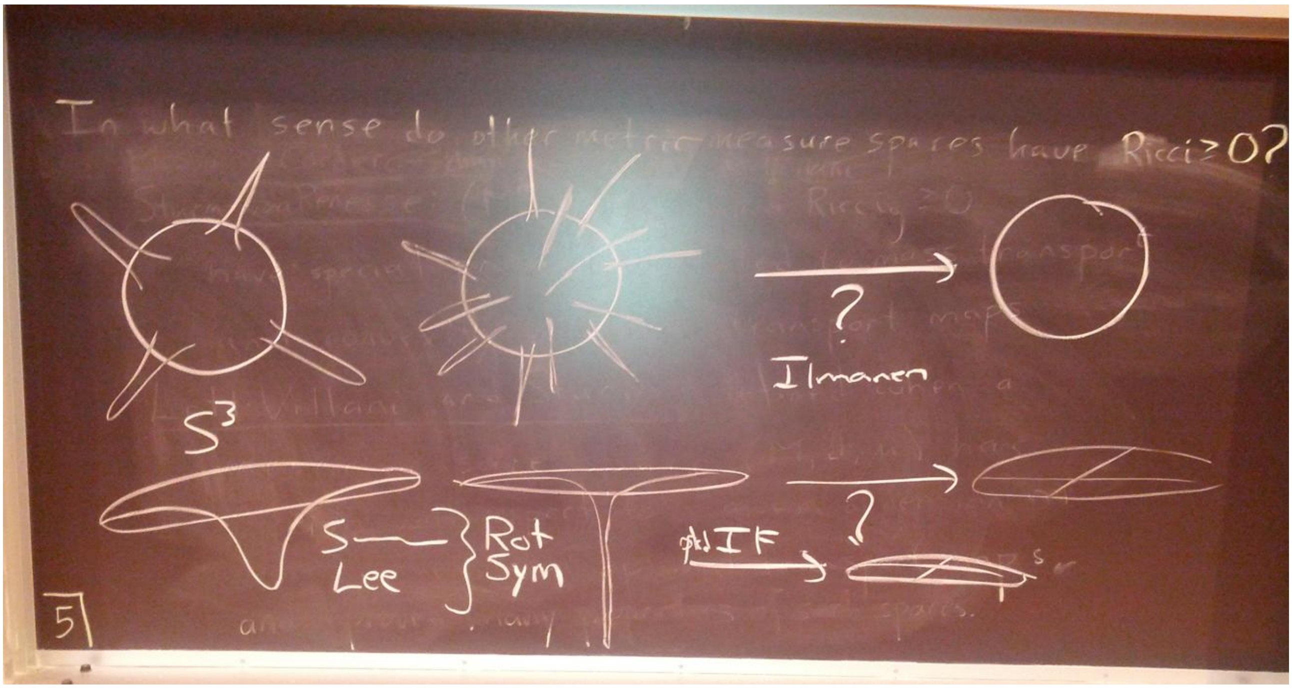
7


S— Wenger Intrinsic Flat Distance:
M_j = set(T_j)
Integral Current space (M_j, d_j, T_j) m dim current structure → ||T_j||
Ambrosio-Kirchheim
Riemannian
T_j ω = ∫_{M_j} ω way of integrating forms
B S_1 S_2
A
d_F((M_1, d_1, T_1), (M_2, d_2, T_2)) = inf_Z {d^Z_F(φ_1# T_1, φ_2# T_2)}
φ_i: M_i → Z dist pres
Flat Distance
2 1
d_F(S_1, S_2) = inf_{A,B} {M(A) + M(B): A + ∂B = S_1 − S_2} ← Federer-Flemming (Whitney)
8

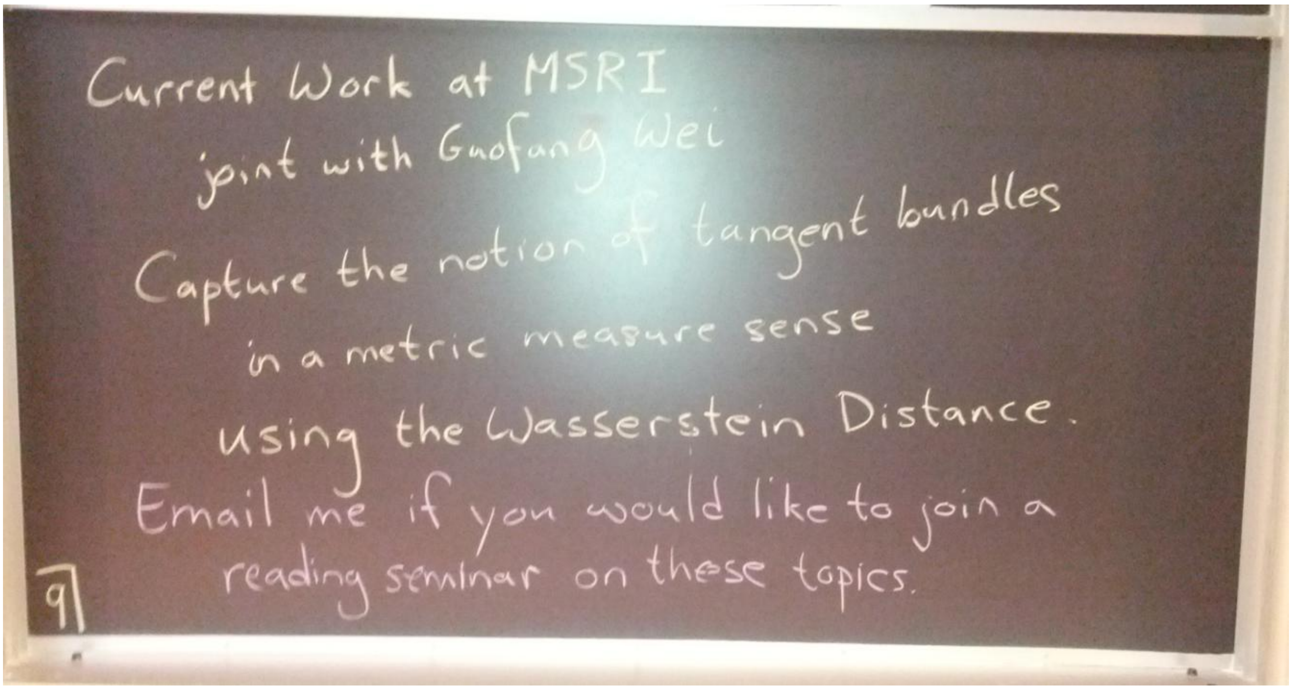
8


Wenger Compactness
(Mj, dj, Tj)
diam (Mj) ≤ D0
Vol (Mj) ≤ V0
Vol (∂Mj) ≤ A0
then a subseq conv to
(M∞, d∞, T∞)
Ambrosio Kircheim
Sj Integral currents
in compact Z
Mass(∂Sj)+Mass(Sj) ≤ M0
⇓
Subseq conv
in Z
4

9


In what sense do other metric measure spaces have Ricci ≥ 0?
?
Ilmanen
S3
S
Lee
Rot
Sym
IF
?
5

10


Current Work at MSRI
joint with Guofang Wei
Capture the notion of tangent bundles
in a metric measure sense
using the Wasserstein Distance.
Email me if you would like to join a
reading seminar on these topics.
9

(END)