Steady state competitive equilibrium coupling academic to labor market

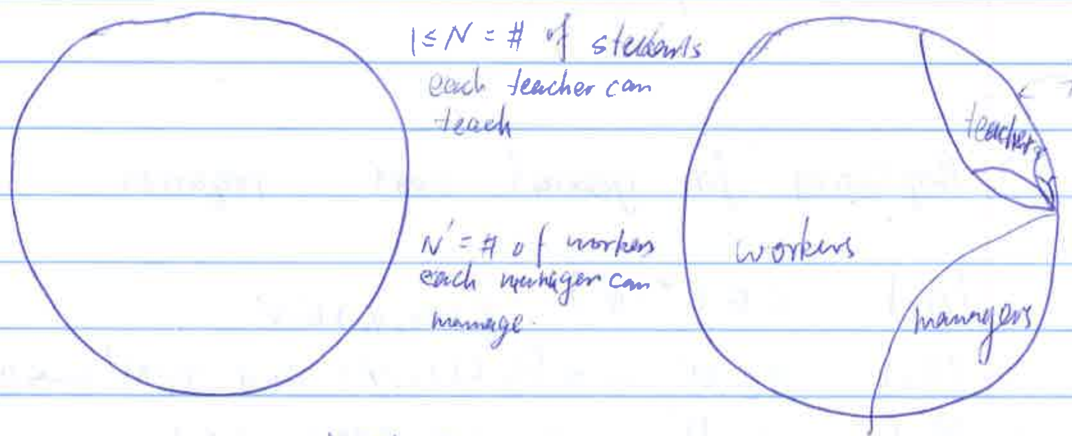
$$\max_{\substack{0\le\gamma\le\mu^+ \\ \quad\ \ \mu^-}} \int_{\text{on } M^+\times M^-} s(x,y)\,d\gamma(x,y) = \inf_{s(x,y)\le u(x)+v(y)} \int u\,d\mu^+ + v\,d\mu^-$$

② supply = demand

① stability

= γ-a.e. for optimizer

③ budget constraint



$\varepsilon^1 = \pi^1 \# \varepsilon \qquad \pi^1(k,a) = k$

$\varepsilon^2 = \pi^2 \# \varepsilon \qquad \pi^2(k,a) = a$

Model:

skill: $K = [0, \bar{k}[$, $\qquad \mu \in \mathcal{P}(K)$

$= A \qquad$ ↑ prior distribution of student skills

$b_\theta(k,a) = (1-\theta)k + \theta a \qquad$ skill ocupied when student k study with a

$Ce^a = w(a) =$ intrinsic satisfactiony generated by having $a \in A$

$e^{b_\theta(k,a)} = S(k,a) =$ surplus generated when worker $k \in K$ work with manager $a \in A$

steady state competitive equilibrium

- each individual wants to maxmize net utility
- students choose how much to invest in education
- adults choose whether to teach, manage or work, and who to match with in each case
- no individual has market power.
- adults skill distribution $\alpha \geq 0$, unknown, but self-reproducing

i.e need to find $\varepsilon \in P(K \times A)$ who studys with which teacher

$\lambda \in P(A \times A)$ who works with which manager

+ payoffs $v(a)$ = wages of adult $a \in A$ $\quad u, v \in C(A)$

$u(k)$ = net lifetime utility of student $k \in K$

Competitive Stable
(1a). $u(k) + \frac{v(a)}{N} \geq V(b_0(k,a)) + e^{b_0(k,a)} \quad \forall k, a \in A$
(1b). $v(k) + \frac{v(a)}{N'} \geq e^{b_0'(k,a)}$

steady state
2a) $\varepsilon^1 = k$
2b) $\alpha = b_{0\#}\varepsilon = \frac{\varepsilon^2}{N} + \lambda^1 + \frac{\lambda^2}{N'}$

equilibrium
3a) (1a) "=" ε - a.e
3b) (1b) "=" λ - a.e.

Variational Approach

$$\inf_{u,v \text{ satisfying } ①} \int_K u(k)\, d\bar{k}(k) = \int_{K\times A} u(k)\, d\varepsilon(k,a)$$

①+②

$$\geq \sup_{\varepsilon,\lambda \text{ satisfying } ②} \int c e^{b\theta(k,a)} d\varepsilon(k,a) + \int e^{b\theta'(k,a)} d\lambda(k,a)$$

Thm1
Infact, "=" holds and maximal is obtained

Q: [Is the min. attained?]

WLOG have a bound on $u \in L^1(dk)$

(1a) with $k=a$ $\Rightarrow$ $u(a) \geq v(a)\left(1-\frac{1}{N}\right) + ce^{a}$

$\Rightarrow$ $v \in L^1(dk)$ also bounded

would like something better, must accommodate possible singular behaviour at $\bar{k}$

- duality result assumes k to be doubling at $\bar{k}$.

and $c \geq 0$, $\theta, \theta', N, N' > 0$ with $\max\{\theta, \theta'\} < 1 < N$

Thm2 (singularity result)

Under various hypothesis a singularity occurs near $\bar{k}$, most likely:

$N\theta > 1 \Rightarrow \frac{dv}{dk}(\bar{k}-\Delta k) = \left(\frac{\text{const}}{\Delta k}\right)^{1+\frac{\log\theta}{\log N}}$ ← power

$N\theta < 1 \Rightarrow \frac{dv}{dk}(\bar{k}-\Delta k) \longrightarrow \text{const} < \infty$ as $\Delta k \to 0$

Thm 3

Under the same hypothesis as Thm 1

$\exists \ (u,v) \in C(A^\bullet)^2$ and (ε, λ) satisfying (1-3)(a,b)

$u(k) = \sup_{a \in A} C e^{b\theta(k,a)} + v((1-\theta)k + \theta a) - \frac{v(a)}{N}$

$V = \max\{V_w, V_m, V_t\}$ where $V_t(a) = N \sup_{k \in K} \{C e^{b\theta(k,a)} + V((1-\theta)k + \theta a) - u(k)\}$

$V_w(a) = \sup_{a' \in A} e^{b\theta'(a,a')} - \frac{V(a')}{N'}$

$V_m(a) = N' \sup_{a' \in A} e^{b\theta'(a',a)} - V(a)$

are both convex non-decreasing.

In fact, $\min\{u', u'', v', v''\} > 0$ if $c > 0$, or $N\theta^2 > 1$ (*)

With (ε, λ) have non-decreasing spt.

If (*) holds, any equilibrium $(\hat{\varepsilon}, \hat{\lambda})$ have non-decreasing spt.

If also k has no atoms, then $(\hat{\varepsilon}, \hat{\lambda}) = (\varepsilon, \lambda)$ are unique

If also $c > 0$ and $N\theta > 1 \Rightarrow u' = \hat{u}'$ k-a.e.

$v' = \hat{v}'$ $\alpha := b_{\theta\#}\varepsilon$ -a.e.

If also $k \geq \hat{k} << \mathcal{H}'$, spt $\hat{k} = \bar{k}$, then $u = \tilde{u}$ k-a.e.

Prop:

$CN\theta \geq N' p^{\bar{k}} \Rightarrow \underline{a_t} > \max\{\bar{a}_w, \bar{a}_m\}$

$N'\theta' \geq e^{\bar{k}} \Rightarrow \underline{a_m} > \overline{a_w}$

$\left.\begin{array}{l} N\theta \geq e^{k} \\ N'\theta' \geq e^{\bar{k}} \end{array}\right\} \Rightarrow$ least skill teacher of manager $\underline{a_t} \geq \bar{a}_m$

$V_t \geq 1 \Rightarrow$ education always improves skills