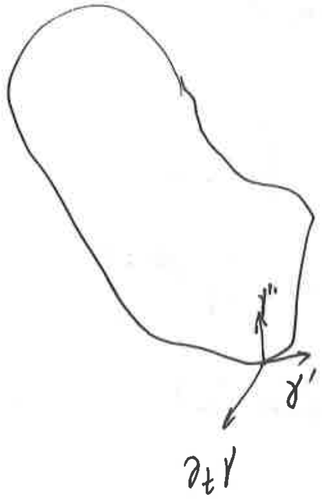
1. Binormal equation in $\mathbb{R}^3$

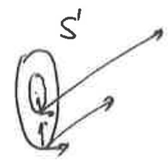


$\gamma \subseteq \mathbb{R}^3$ arc-length para. $\qquad \gamma' = \frac{\partial \gamma}{\partial \theta}$.

$\gamma' \perp \gamma''$

Binormal eqn.

$$\partial_t \gamma = \gamma' \times \gamma'' = k(\theta, t) \underset{\substack{\Downarrow \\ t \times \vec{n}}}{\vec{B}}$$

Ex. 

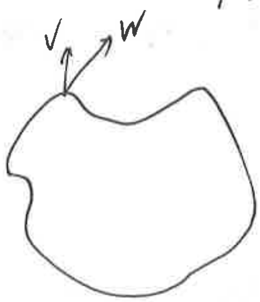
| LIA, vertex bilavant
Da Rios 1906 |

Properties:

1) Hamitonian eqn:

Ham. fn: $H(\gamma) = \text{length}(\gamma)$

Marsden-Weinstein. on oriented closed curves symm. struc.



$$\omega_\gamma^{MW}(V, W) = \cancel{V \times W} \int_\gamma i_W i_V \mu. \quad (\mu \text{ - volumn element})$$

$$= \int_0^L \mu(V, W, \gamma') d\theta$$

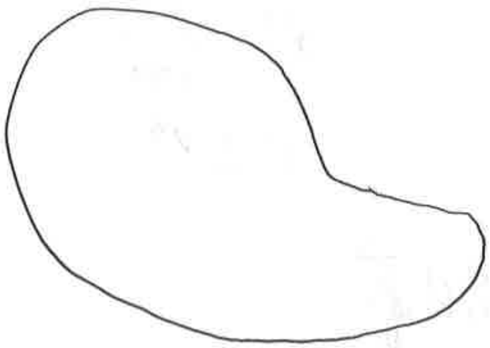
2). Integrable: $\exists$ Hasimoto transform:

binormal $\longrightarrow$ NLS

$\forall t$, $\psi(\theta) = \underset{\uparrow \text{curv.}}{k(\theta)} e^{i \int \tau \, d\theta}$. ($\tau$ - torsion.) $\qquad i\psi_t + \psi'' + \frac{1}{2}|\psi|^2 \psi = 0$

II. Generelization: binormal mean curvature flow
(skew)

$\mathbb{R}^n \supset P^{n-2}$. ~~close~~ closed manifold, codim 2.



Define Hami. fn

$H(P) = \text{volume}(P^{n-2})$.

(Marsden - Weinstein)

$$\omega_P^{mw}(V,W) = \int_P i_W i_V \mu.$$

Thm (Shashikanth, $n=4$, & Kh).
Haller - Vizman 2003.

Hami. eqn for ω_P & $H(P)$, is the skew-mean-curv flow:

$q_t = J(\overrightarrow{MC}_p(q))$; here $q \in P \subseteq \mathbb{R}^n$; J almost compl. struc. in $N_q P$. dim 2.

Def. i) For $q \in P^\ell \subseteq \mathbb{R}^n$, mean curvature vector $\overrightarrow{MC}(q) \in N_q P$ is trace of $\mathrm{II}_q : T_q P^{\otimes 2} \to N_q P$

ii (equiv.). $\overrightarrow{MC}(q) =$ average geod curv. of P in direction $\vec{u} \in S^{\ell-1} \subseteq T_q P$.

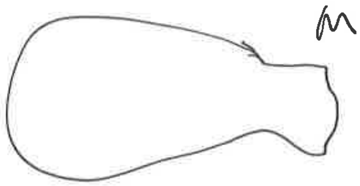
Ex $n=3$, $\vec{MC} = k\cdot\vec{n}$.

$$J(\vec{MC}) = k\cdot\vec{b} \quad (\text{here } \vec{b} = \vec{t}\times\vec{n}).$$

Pf idea. $\operatorname{grad} H = \frac{\delta H}{\delta P} = \vec{MC}$ (fastest volume change), $\operatorname{sgrad} H = J(\vec{MC})$ since W^{MW}-averaging of r/s in each $N_q P$ ☒

IV. Vorticity Euler eqn.

Euler eqn in $M\subseteq\mathbb{R}^n$: $\begin{cases}\partial_t v + (v\cdot\nabla)v = -\nabla p,\\ \operatorname{div} v = 0 \qquad v \parallel \partial M.\end{cases}$



M

vorticity:

$$\begin{array}{ccccc} v & \longrightarrow & v^\flat & \longrightarrow d v^\flat = & \xi \\ | & & \cap & \cap & \uparrow \\ \text{vel.} & & \Omega^1 & \Omega^2 & \operatorname{curl} v \end{array}$$

$$\partial_t \xi = -L_v \xi, \quad \text{for } \xi = \operatorname{curl} v.$$

Rm. $\mathfrak{g} = \operatorname{Vect}_\mu(M)$ $\quad \mathfrak{g}^* = \Omega^1/d\Omega^0 \cong d\Omega^1 \ni \xi$

the Euler eqn is Hamilt. on $\mathfrak{g}^*$, w.r.t Lie-Poisson str $\mathfrak{g}^*$, $H(\gamma) = \frac{1}{2}\int v^2 \gamma$.

Classically ξ - smooth $\quad$ Supp ξ = 0.

ξ (2-currents $\otimes$ codim = 1 = - vtx sheet)

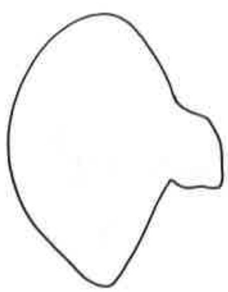
codim = 2, $\quad$ vtx $\quad$ filaments (___ ,

Ex. $\quad n=2.$

N point vertices.

N Vertex filaments mesh ...

$n=3.$ $\quad \xi = \delta v$



$v = \mathrm{curl}^{-1}\xi.$

$P^{n-2} = R.$

$\xi_P = \xi.$

~~We can~~ P.

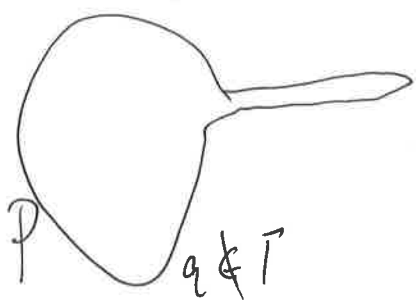
Note: $P \subset T^* = \mathrm{Vect}^*$

~~Parity~~ Parity $\langle P, v\rangle = \text{Flux}.\ v|_{\partial^{-1}P}.$

<u>Prop</u>: $W_p^{MW} = W_\xi^{KK}$ for $\xi = \delta_p$.

<u>Thm</u> (Sh, Kh).

The field potential. $V(q) = \int_\Gamma J\left(\mathrm{Proj}_{N_p}\left(\nabla_p G(p,q)\right)\right)\mu_{p}(p)$

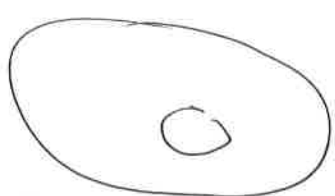


Satisfies. $dV^b = \delta_p = \xi$ (in the dist. sense)

(generlr. Boil-Savait formulas)

where $\Delta_p G(p,q) = \delta_p(q)$. Green's fn of Δ in $\mathbb{R}^n$.

<u>Note</u>: $V(q) = \begin{cases} < \infty & q \notin P \\ = \infty & q \in P \end{cases}$



$P_\varepsilon = P \setminus V_\varepsilon$

$V_\varepsilon(q) = \int_{P_\varepsilon} J(\quad)\mu.$

Similarly. $E = \frac{1}{2}\int V^2\mu \rightsquigarrow E_\varepsilon = \int (V, V_\varepsilon)\mu.$

Thm (Sh, Kh).

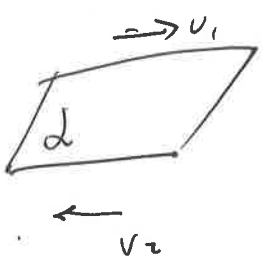
1). $q \in P \Rightarrow \lim_{\varepsilon \to 0} \frac{v_\varepsilon(q)}{\ln \varepsilon} = C_n \cdot J(\vec{MC}(q))$

2). $\lim_{\varepsilon \to 0} \frac{E_\varepsilon(v)}{\ln \varepsilon} = \tilde{C}_n \cdot \mathrm{volume}(P)$

$t \to (\ln \varepsilon)\, t \Rightarrow$ evolution skew-binormal eqn

LIA

V. Vortex Sheets.



$P^{n-1} \subseteq \mathbb{R}^n$

2-current

$\xi = \alpha \wedge \delta_P$

closed 1-form on P.

$\alpha = df$.

Prop: $\exists$ s/s on $\{\xi\}$.

simplistic structure.

Q: What is H? LIA?