

Teaching Mathematical Practices

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Critical Issues in Mathematics Education Workshop

Teacher Education in View of the Common Core

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

"Practice" in School Mathematics

1. School mathematics has always been about "practice," but it has been more about *drill and practice*, not *the practice of mathematics*.

2. The Common Core State Standards offer the possibility of re-orienting school mathematics around a more robust conception of **mathematical competence**.

Overview

1. Review of CCSSM, and Mathematical Practices

2. Instructional example, I: Making sense of problems [MP 1, 3, 6]

3. Instructional example, II: Reasoning, language, generalizing [MP 2, 3, 4, 6, 7, 8]

Common Core State Standards for Mathematics (CCSSM)

What makes them distinctive (uncommon)?

1. They are common, i.e. shared by (almost) all states. They are derived from the *intersection* of the 50 state standards, unlike commercial textbooks, that tend to be the union.
2. This presents an unprecedented opportunity for national curricular coherence, thus removing a major obstacle to effective teacher education.
3. They prominently feature Mathematical Practices and make them a central part of the curriculum, not just paying rhetorical homage. If implementation of CCSSM does not robustly include Mathematical Practices, little significant change will be accomplished.

Threats to Implementing Mathematical Practice in School

1. We know too little about what these looks like in action when engaged by learners, especially at the elementary grades where they must begin to be successful.
2. Consequently, we also lack knowledge of what it takes to teach mathematical practices in ways that have integrity and are fundamentally connected to core content.
3. Some people may think of the practices as "extras," to be taught only after "basics" are mastered. In other words, people won't see the practices as "basic."
4. Some people may not think that all students are capable of engaging in mathematical practices.

Challenges

1. Students must learn mathematical practices, interwoven with content, starting in the early grades. They are fundamental, not an add-on, to follow the learning of basic content and skills.
2. Many teachers do not now know how to enable this, nor do they have appropriate resources and support to do so.
3. A first step is to make the *meaning* of the mathematical practices known, in concrete, vivid, and instructionally germane ways.
4. Only then can the work of curricular and instructional design take hold.
5. And teachers must be given access to the knowledge and skills needed to make mathematical practices an integral part of their instruction

Resources

1. A history of efforts to incorporate some version of mathematical practices: Process standards of NCTM; Strands of mathematical proficiency in *Adding It Up*.

2. Extensive research literature on certain practices – problem solving; reasoning and proving; modeling.

3. Mathematicians' reflections on practices of the discipline.

4. Rare, but edifying examples of instruction in which mathematical practices are seen to be taught and learned. These examples give a proof of concept, and much more.

5. English language arts (at the elementary level), particularly reading and writing, have long been dominated by a focus on practices. This presents a domain of school learning in which teachers have successfully made practices central to instruction.

Practice(s)

- *How can I get to Carnegie Hall?*
- *Practice, practice, practice!*

Four meanings (M. Lampert):

1. Practice (v.) as in rehearsal, "drill and practice." Repeatedly doing something to increase the quality of performance, knowledge, and skill.
2. The practice (n.) – of law, medicine, mathematics, teaching, etc.
3. The practices (n.) (not the plural of practice) — of law, or medicine, or mathematics, or teaching, or etc. The fundamental things practitioners DO – activities, skills, methods, dispositions, of the practice. These include learning by practice.
4. Practice (n.) as in "theory vs. practice," or "theoretical vs. practical"

Mathematical Practices, according to CCSSM:

How do we make them also practices of teaching and learning?

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

How do we make these come alive for teachers and learners?

Strands of mathematical proficiency in *Adding It Up*

A1.	Adaptive reasoning	(reasoning and proving)
A2.	Strategic competence	(problem solving skills)
A3.	Conceptual understanding	(comprehension of mathematical concepts, operations, and relations)
A4.	Procedural fluency	(skill in carrying out procedures flexibly, accurately, efficiently and appropriately. "basic skills")
A5.	Productive disposition	(habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

NCTM Process Standards

- N1. Problem solving
- N2. Reasoning and proof
- N3. Communication
- N4. Representation
- N5. Connections.

HB Personal Draft List

H1. Questioning	(asking "natural questions")
H2. Exploring	("eyes-open" exploring, experimenting)
H3. Representing	(constructing, using, and comparing representations)
H4. Structure seeking	(seeing patterns, useful ways of organizing, representing, interpreting situations)
H5. Consulting	(expert colleagues, the literature, Google, etc.)
H6. Connecting	(seeing connections, analogies, similarities with other situations)
H7. Proof seeking	(trying to justify or verify a claim/conjecture/solution)
H8. Being opportunistic	(being alert to promising 'signals' in the mathematical work)
H9. Proving	(producing a clear and rigorously convincing presentation of a justification of a mathematical claim)
H10. Analyzing/evaluating proofs	(critically evaluating proposed proofs; analyzing them as valuable resources)
H11. Exercising judgment & taste	(developing a sense of the significance, lucidity, coherence, elegance of a piece of mathematics, or a mathematical idea)

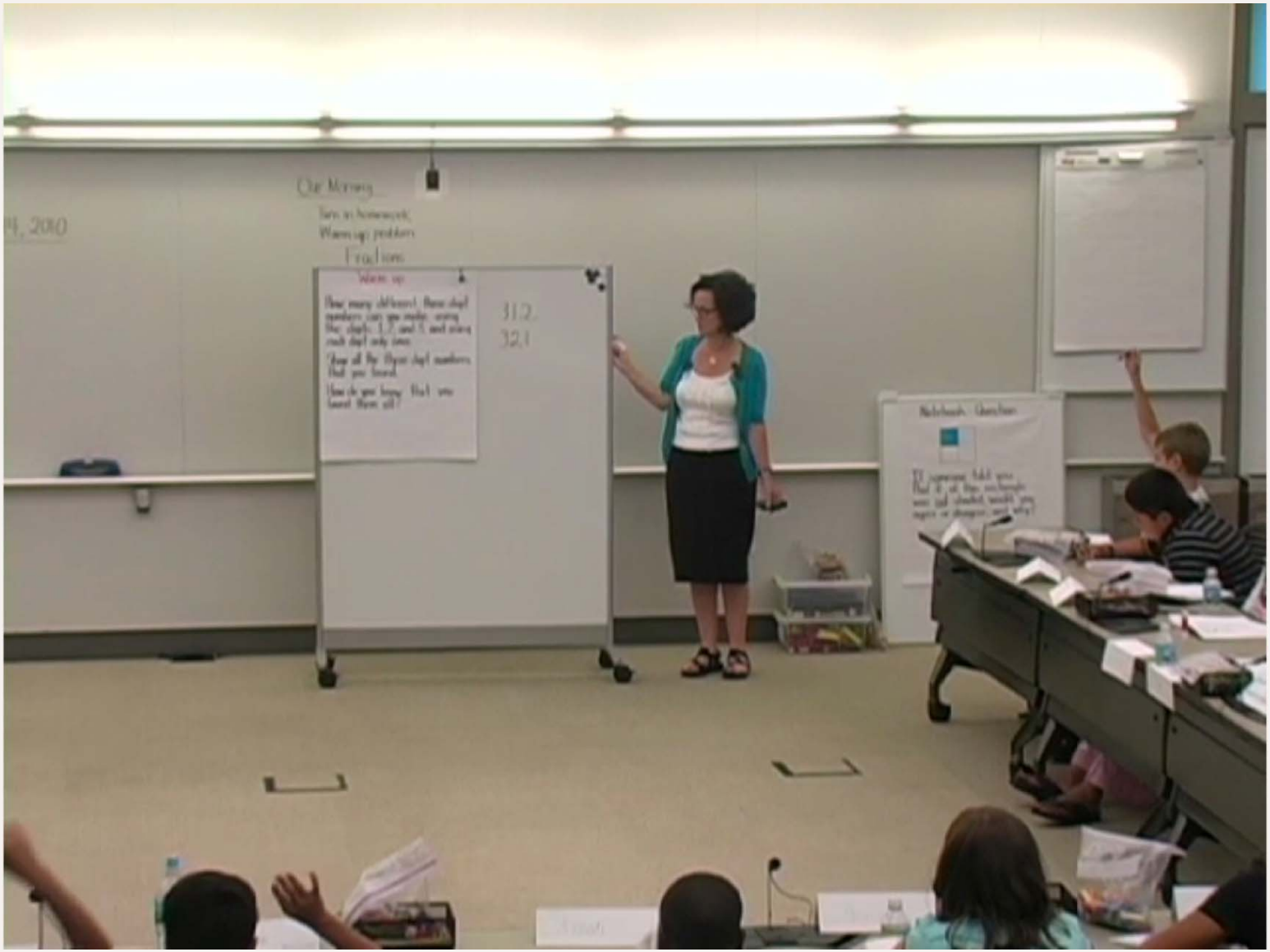
What is the work of teaching these mathematical practices?

A first look

Instructional Example, I: Making sense of problems

What mathematical practices do you see being taught/learned?

- One week summer lab class.
- Mathematically struggling students passing from 4th to 5th grade.
- Warm-up problem:
  - *How many three-digit numbers can you make using the digits 1, 2, and 3, and using each digit only once.*
  - *Show all the three-digit numbers that you found.*
  - *How do you know that you found them all?*



Mathematical Practices in Use

MP1. Make sense of problems and persevere in solving them.

Explaining the meaning of a problem and looking for entry points to its solution.

Analyze givens, constraints, relationships, and goals.

Check answers to problems and continually ask, "Does this make sense?"

Seek to understand the approaches of others.

MP3. Construct viable arguments and critique the reasoning of others.

Understand and use stated assumptions, definitions in constructing arguments.

Analyze situations and recognize and use counterexamples.

Justify conclusions, communicate them to others, and

Respond to the arguments of others.

MP6. Attend to precision.

Seek to communicate precisely to others.

Instructional Example, II

Teaching and learning of a range of mathematical practices:

– reasoning and proving
– precision with mathematical language
– discerning and formulating mathematical patterns

The emergence of "Sean Numbers"

- Diverse classroom of third graders
- January, middle of the school year
- They have been exploring properties of even and odd numbers
- One student, Sean, announces something that he noticed about the number 6: "It can be odd as well as even."
- We shall follow the mathematical adventure that this precipitates.

There are four video clips.
Notice in each one the mathematical practices in play, by both students and teacher.

1. Sean's idea about the number 6

2. Other students' responses to Sean

Mathematical practices of class

3. Mei's response to Sean

Mathematical practices of Mei and Sean

4. The emergence of "Sean Numbers"

Mathematical practices of class

What is the work,
and what are the demands,
of the teaching
of mathematical practices?

Teaching Mathematical Practices

1. Modeling, with narration
2. Structured guidance, through questioning, prompting
3. Explicit frameworks (e.g., providing explanations with reference to all elements required for a complete solution)
4. Mathematical language for practice (e.g., *proving, conjecture, definition, conditions, counterexample, representation*) to support explicit thinking
5. Posting mathematical resources (e.g., already-established ideas, terms, common knowledge)

Thank You!

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