The Mathematical Education of Teachers II

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MSRI Critical Issues in Education

The Mathematical Education of Teachers II

The draft METII document can be downloaded from the website of the Conference Board of the Mathematical Sciences (CBMS) at *www.cbmsweb.org*

You can submit comments until April 28.

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Why MET2?

- The Common Core State Standards for Mathematics;
- Offer recommendations for professional development;
- Discuss needs of math specialists, early childhood teachers, special education teachers.

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Chapter 3: Recommendations

Recommendations on Mathematics for Teachers

1. Develop a good understanding of the math they will teach;
2. Need for coursework, with time to engage in reasoning, explaining, and making sense of math;
3. Need for ongoing professional development;
4. Need to develop habits of mind of a mathematical thinker; need for flexible and interactive styles of teaching.

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Chapter 3: Recommendations

Recommendations on the Roles for Mathematicians in Teacher Education

5. Teacher education is an important part of mathematics departments' missions

6. The need to improve mathematics teaching at all levels and to professionalize mathematics teaching.

   Please join the Mathematics Teaching Community at *https://mathematicsteachingcommunity.math.uga.edu*

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Ch 4: Elementary Teachers

Studying the math teachers will teach:

"...this report recommends that before beginning to teach, an elementary teacher should study in depth, and from a teacher's perspective, the vast majority of K–5 mathematics, its connections to prekindergarten mathematics, and its connections to grades 6–8 mathematics. By itself, this expectation is not sufficient to guarantee high quality teaching. However, there is no substitute: a strong understanding of the mathematics a teacher will teach is necessary for good teaching. "

Ch 4: Elementary Teachers

Engaging in mathematical practice:

"Several points about the CCSS Standards for Mathematical Practice bear emphasizing. . . . *engaging in mathematical practices takes time and opportunity*, so that coursework and professional development for teachers must plan with that in mind. Time and opportunity to think about, discuss, and explain mathematical ideas are essential for learning to treat mathematics as a sense-making enterprise."

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Operations and Algebraic Thinking (K–5).

- The different types of problems solved by addition, subtraction, multiplication, and division, including “algebraic” problem types (e.g., Start Unknown Add To).
- Learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms).
- Recognizing the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning “the same amount as” rather than a “calculate the answer” symbol (MP6).

Operations and Algebraic Thinking (K–5).

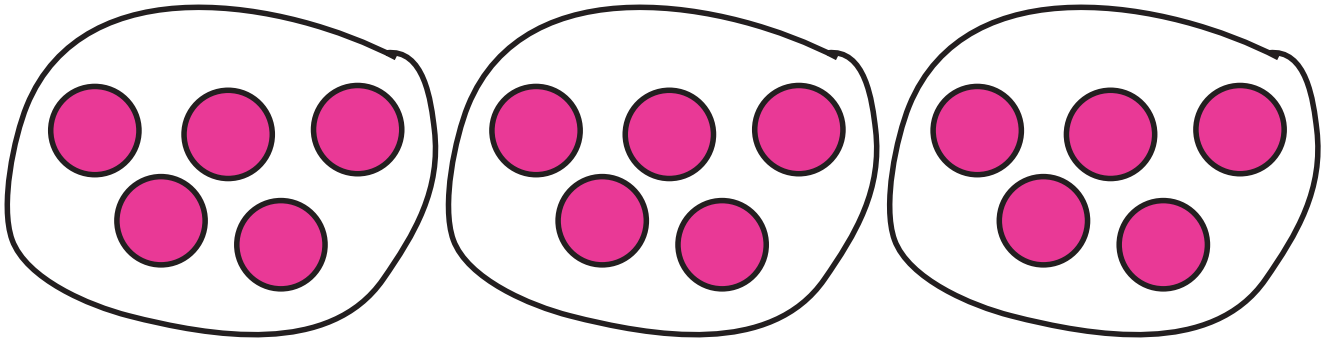
Sample activities for teachers:

1. Use arrays to explain the commutativity of multiplication and recognize that commutativity is not obvious. MP 3, 5.

The commutative property of multiplication

Why should multiplication be commutative? This is not obvious!

3×5



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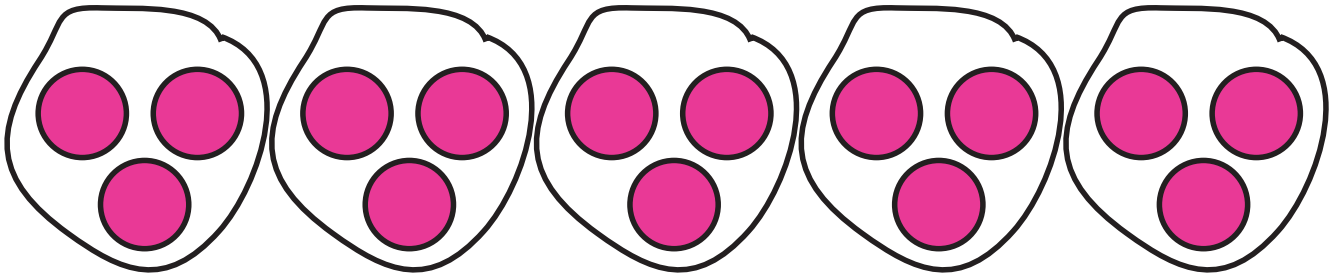

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The commutative property of multiplication

So why is $A \times B$ equal to $B \times A$???

The temptation is just to check a bunch of examples, but is there some underlying reason why it always works?

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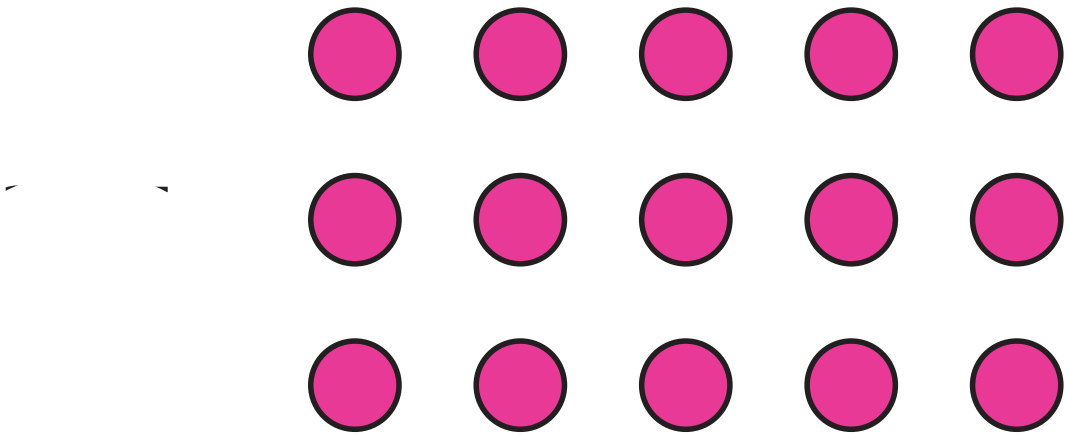
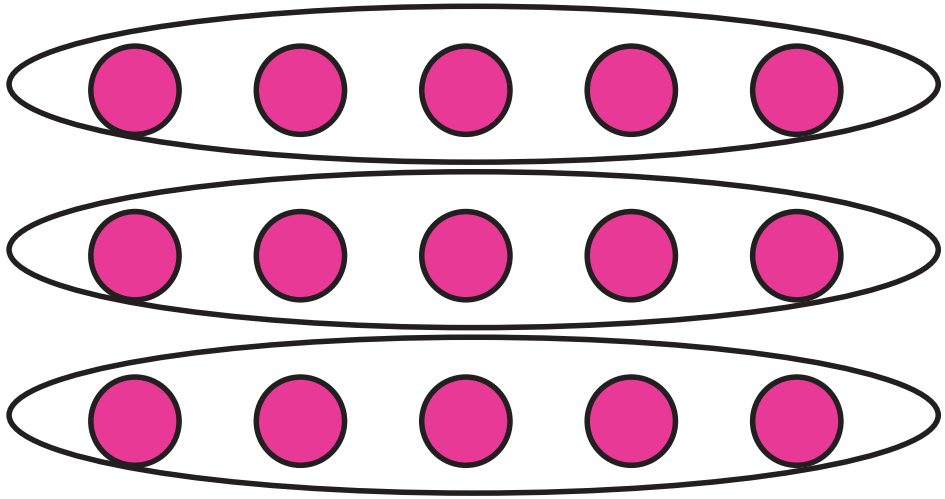


The commutative property of multiplication

3×5



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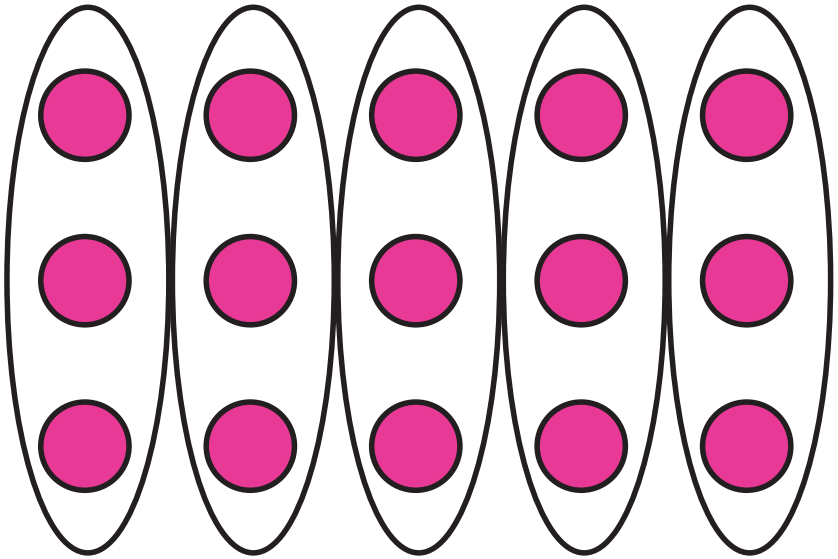

5×3

4. Explain how to solve equations such as $283 + 19 = x + 18$ by "thinking relationally" rather than by applying standard algebraic methods. MP 3, 7, 8.

Progressions for the CCSS

The discussion refers to the Progressions for the CCSS

Go to:
ime.math.arizona.edu/progressions/

Draft progressions exist or are planned for each domain within the K–8 CCSSM and for each conceptual category within the high school CCSSM.

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Math to study in professional development:

"The mathematics of elementary school is full of deep and interesting ideas, which can be studied repeatedly, with increasing depth and attention to detail and nuance. Therefore, whereas prospective teachers will undertake an initial study of elementary mathematics from a teacher's perspective in their preparation program, practicing teachers will benefit from delving more deeply into the very same topics."

Math courses for preparing teachers:

"It bears emphasizing that familiar mathematics courses such as college algebra, mathematical modeling, and liberal arts mathematics, *cannot substitute for the study of mathematics for elementary teachers*, although they might make reasonable additions. Also, it is unlikely that knowledge of elementary mathematics needed for teaching can be acquired through experience in other professions, even mathematically demanding ones."

Ch 4: Elementary Teachers

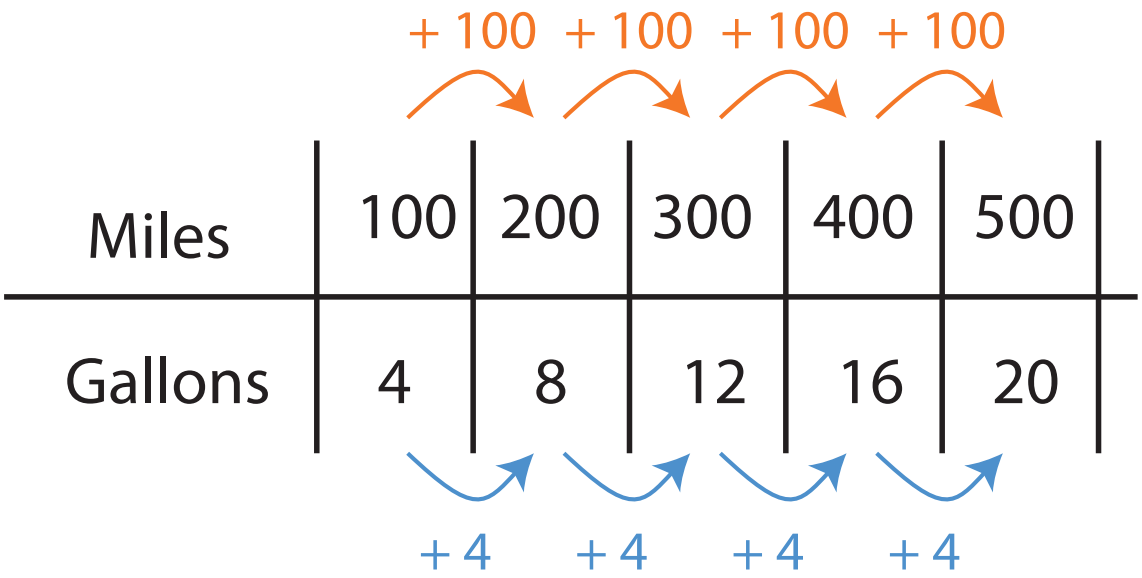
Short sections on:

- Challenges in preparing elementary teachers
- Preparation and professional development for elementary mathematics specialists
- Preparation and professional development for early childhood teachers
- Preparation and professional development for special education teachers

"Preservice and practicing middle grades teachers need to be aware of representations, be they drawings, tape diagrams, number lines, or physical models, used in the earlier grades and how those representations may lend themselves to establishing and extending mathematical ideas into the middle grades."

Ratio and Proportional Relationships (6–7).
Sample activities for teachers:

2. Compare and contrast different ways to find values in proportional relationships and in inversely proportional relationships. For example, explain why linear interpolation can be used with proportional relationships but not with inversely proportional relationships. MP 3, 4, 7.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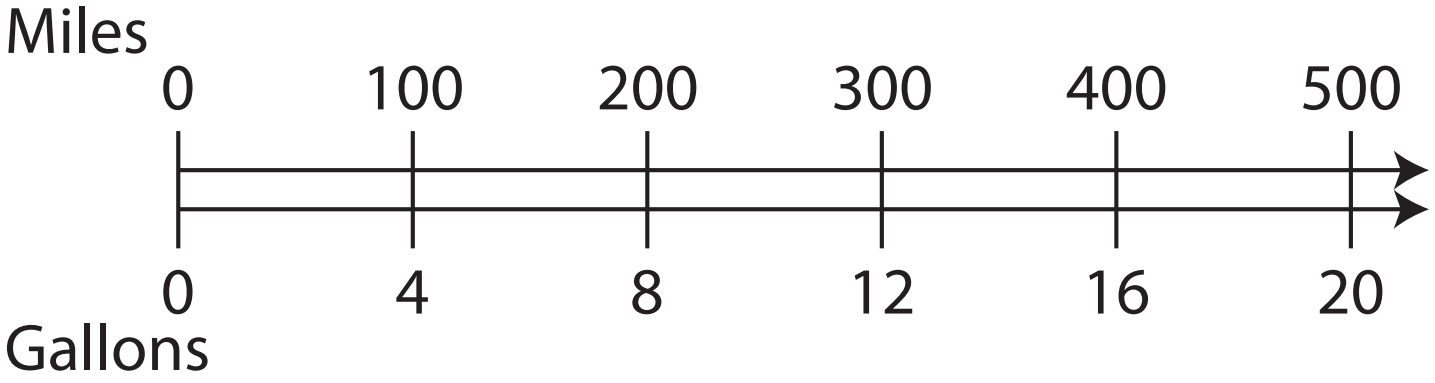
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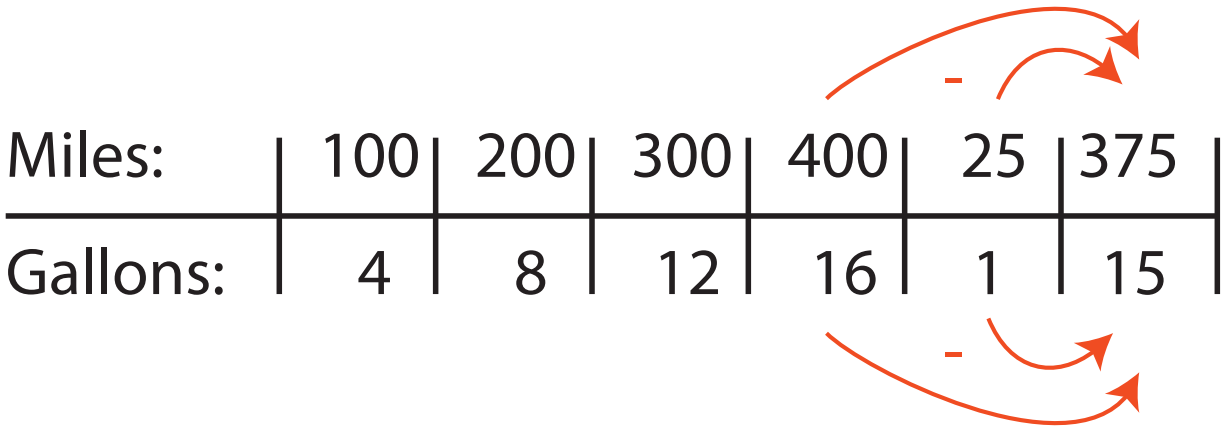
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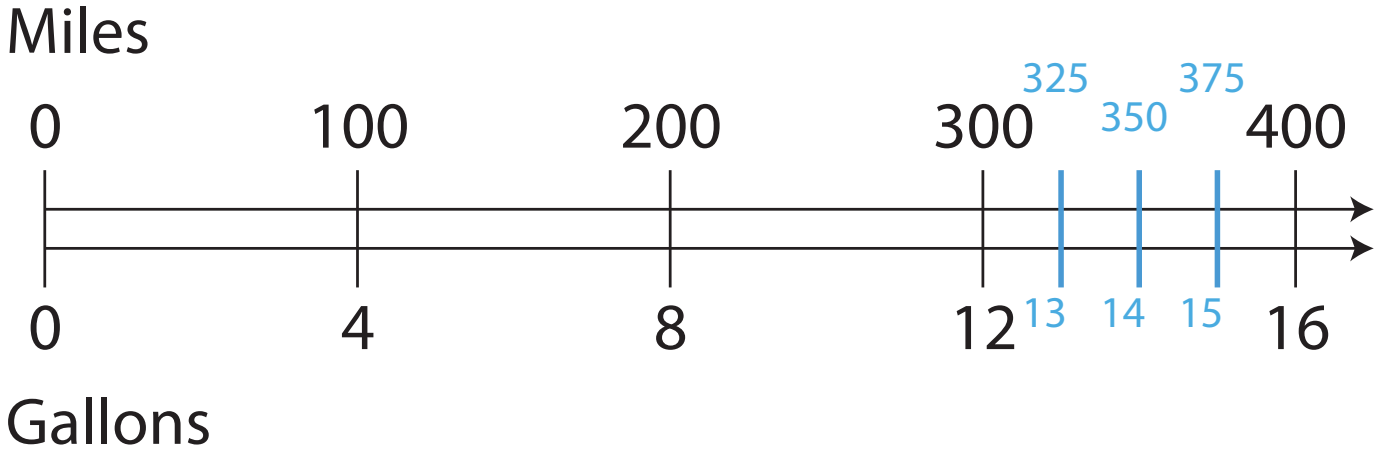
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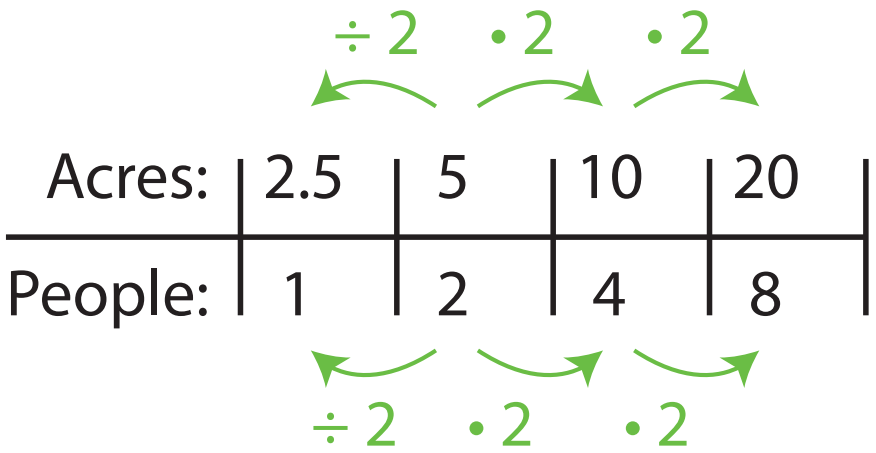
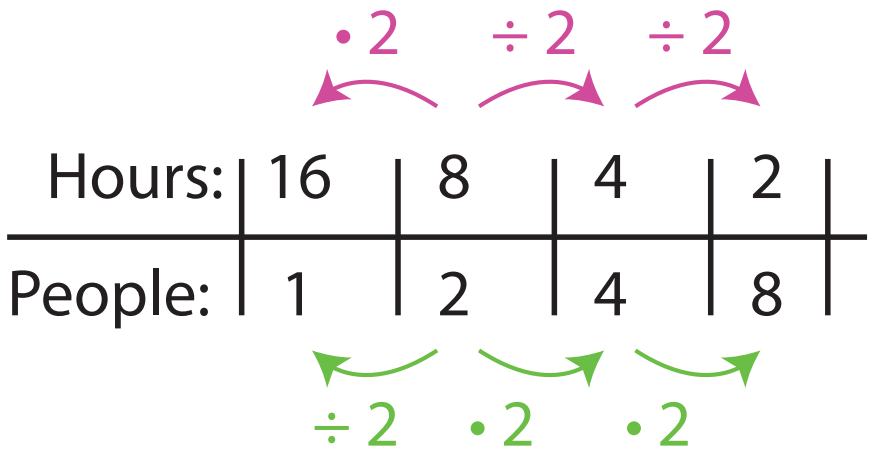
Suppose that 2 people take 8 hours to mow 5 acres of grass. (Assume all the people work at the same steady pace.)

Proportional relationship



***Inversely* proportional relationship**



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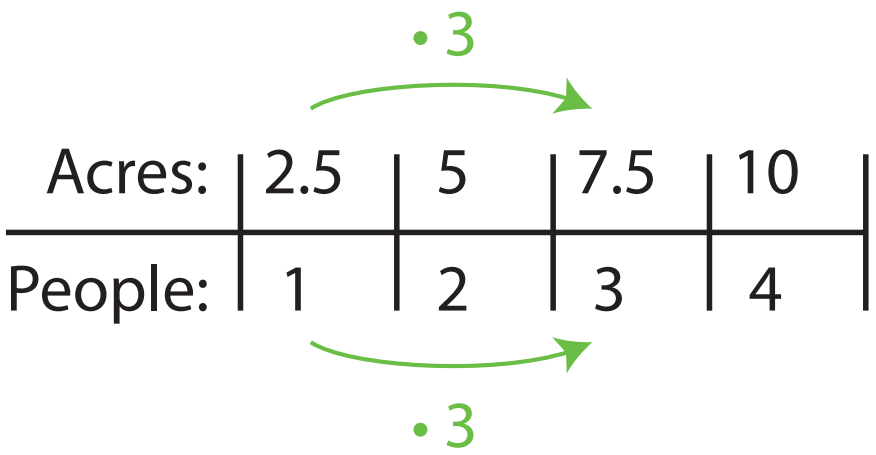
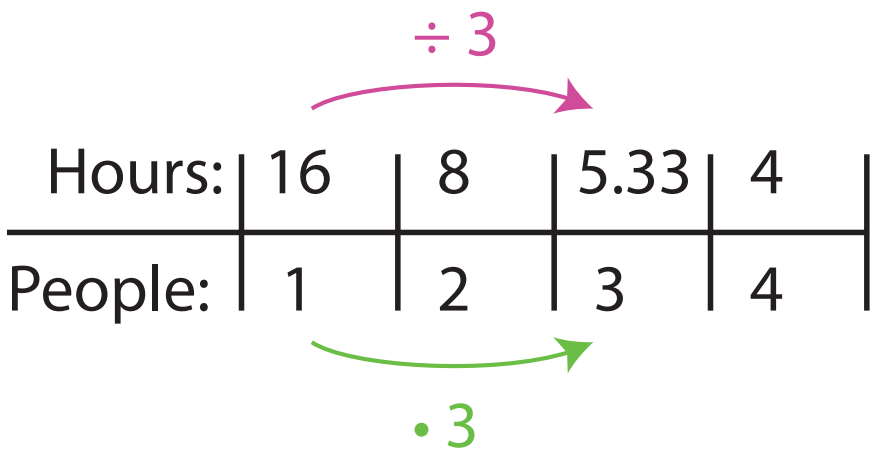
Proportional relationship



acres ÷ # people = 2.5

***Inversely* proportional relationship**



hours • # people = 16

Ch 6: High School Teachers

1. essential topics and experiences for preparation;
2. desirable topics for preparation, essential in early-career education
3. professional development topics and experiences.

Ch 6: High School Teachers

A double discontinuity:

Felix Klein:
“The young university student [was] confronted with problems that did not suggest . . . the things with which he had been concerned at school. When, after finishing his course of study, he became a teacher . . . he was scarcely able to discern any connection between his task and his university mathematics.”

Ch 6: High School Teachers

Not just forward looking, but also connecting back:

“the mathematical topics in courses for prospective high school teachers and in professional development for practicing teachers should be tailored to the work of teaching, examining connections between middle grades and high school mathematics as well as those between high school and college.”

Ch 6: High School Teachers

The need for opportunities to engage in mathematical practices and develop mathematical habits of mind:

“teachers need opportunities for the full range of mathematical experience themselves: struggling with hard problems, discovering their own solutions, reasoning mathematically, modeling with mathematics, and developing mathematical habits of mind.”

Ch 6: High School Teachers

"A comparison of arithmetic in $\mathbb{Z}$ and $\mathbb{Z}/n\mathbb{Z}$ helps teachers understand the importance of the lack of zero divisors when teaching the "factor to solve" techniques for quadratic and higher-degree equations."

Problem for prospective teachers:
Add words to turn these equations into a coherent logical argument:

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0, \quad x - 2 = 0$$

$$x = 3, \quad x = 2$$

Does the argument work over $\mathbb{Z}/6\mathbb{Z}$?

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