# Developing Materials that Support Teachers' Capacity to Teach Mathematics in the Common Core

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# Acknowledgements


## The Dev-TE@M Project

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- Meghan Shaughnessy
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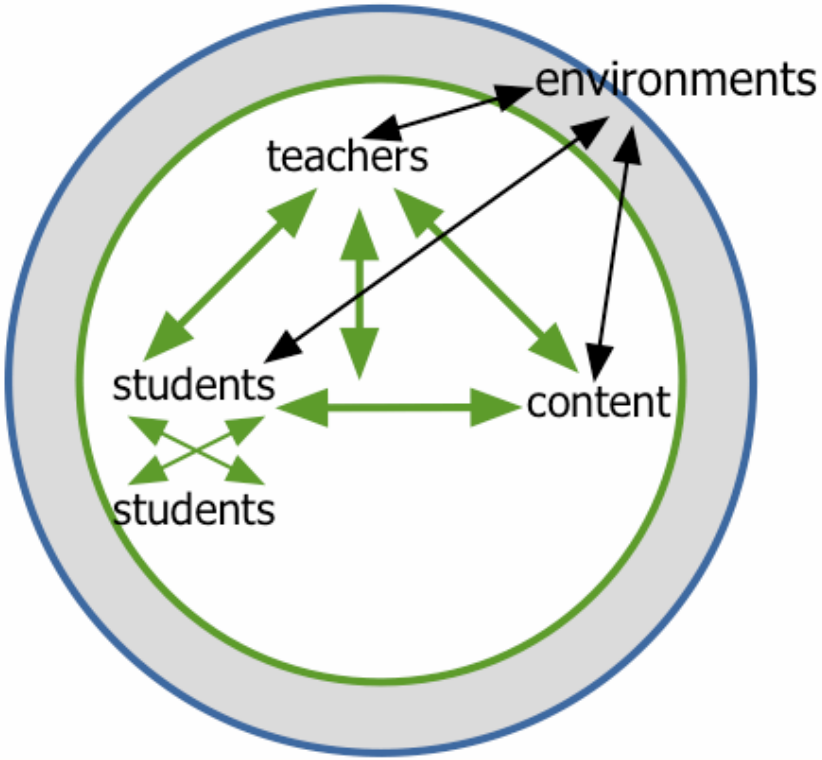
# Overview

- Project overview
- Approaches to supporting the development of teachers'
  - Command of the mathematics of the CCSS
  - Ability to effectively work on this mathematics *in practice*

# Project Rationale: Developing Professional Knowledge




Dev-TE@M materials integrate attention to the following core elements

1. mathematical knowledge for teaching
2. students' thinking about mathematics
3. essential teaching practices that support student learning; and
4. routines for learning in and from practice

# Materials Features

## Quick facts:

- Ten 2-hour sessions
- Professional development for *practicing* elementary mathematics teachers
- Integrated content
- Practice-based design
- Accessible via the web with multimedia components
- Facilitated sessions conducted in real-time

# Representing and Comparing Fractions

## Module Content

- **Mathematics:** *Representation, definition, equivalence, and comparison of fractions*
- **Student Thinking:** *Students' ideas and approaches to comparing and representing fractions*
- **Teaching Practice:** *Practices of using representations in classroom teaching*
- **Learning from Practice:** *Professional practices for analyzing images of blackboard and other public recording spaces to improve teaching and learning*

## Common Core

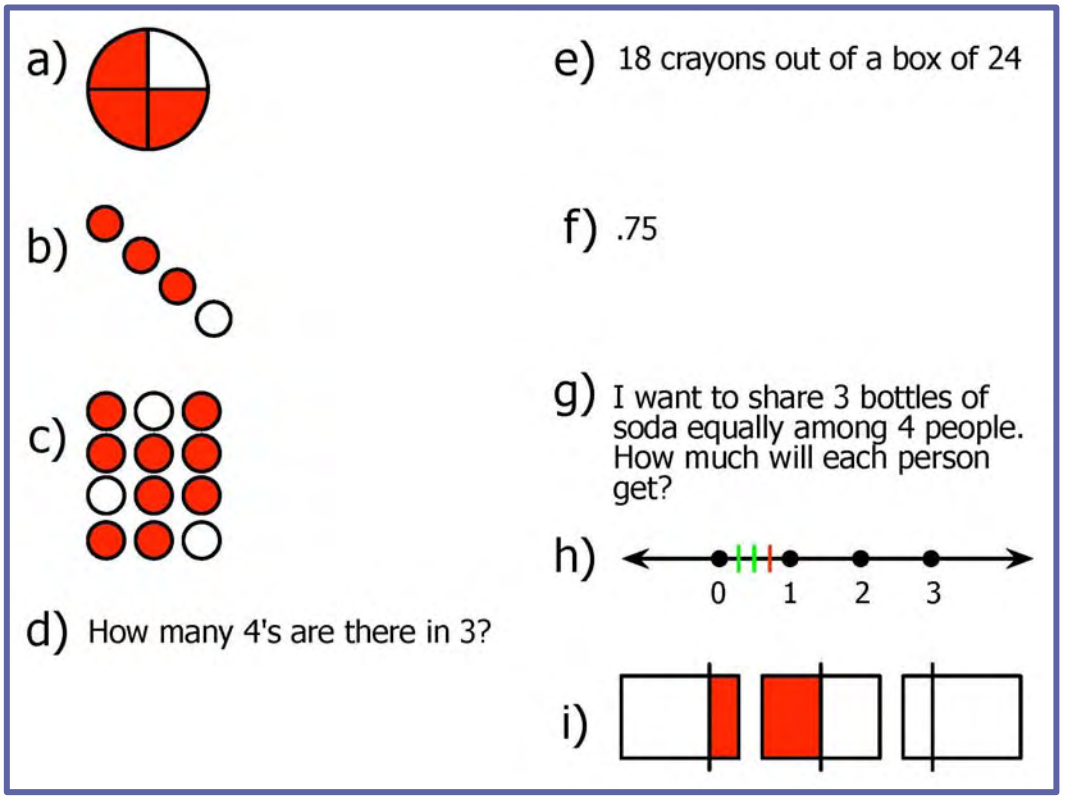
**Grade 3**:
Develop an understanding of fractions as numbers

**Math Practice #2:**
Construct viable arguments and critique the reasoning of others

# Designing materials to support elementary teachers in

# deepening their understanding of the mathematics of the CCSS

# Mathematics Example 1: Representations of 3/4

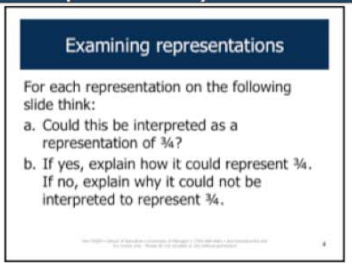
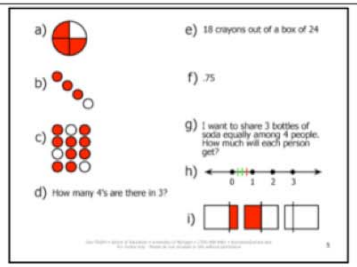



- Begin work on constructing viable arguments and critiquing the reasoning of others
- Set up working on the definition of a fraction
- Begin work on the number line

# Connections with CCSSM

***Part 3: Analyzing representations of ¾ (~20 minutes)***

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will be able to interpret and explain multiple representations of a fraction. | 1. Introduce Part 3 and watch the video with slides.<br>2. Participants record their thinking about each representation. | • Video (01:49): Task introduction<br>• Handout: Representations of ¾ |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 3 by having participants watch the *video* in which Dr. Ball introduces the task that participants will be working on in this segment – looking at potential representations of ¾ and deciding:<br>• Could this be interpreted as a representation of ¾?<br>• If yes, explain how it could represent ¾. If no, explain why it could not be interpreted to represent ¾.<br>  | *Participants may move to talking about teaching practices when answering the first question. Encourage participants to focus on mathematics in this part of the session*<br><br>*CCSSM Link: In working on this activity, participants are engaged in several of the CCSSM mathematical practices including: making sense of problems and persevering in solving them (#1), reasoning abstractly and quantitatively (#2), constructing viable arguments and critiquing the reasoning of others (#3), and precisely communicating their thinking to others (#6). While the major focus on fractions starts in Grade 3, in the earlier grades, students work on foundational ideas such as partitioning shapes into equal shares and naming a share using fraction language (1.G.3, 2.G.3, 3.G.2).*<br><br>*See Session 2, Part 4 for a document explaining how each representation can be interpreted as a representation of ¾. You may want to consider distributing this document to participants at the end of the session.*<br><br>*See Session 3, Part 6 for the "Working Definition of a Fraction." While the participants will not see this working definition until Session 3, the work that they are doing in Session 2 is designed to begin to elicit aspects of the definition. Reviewing this working definition now may be useful for you in preparing for Session 2.* |
| 2. Distribute the *Handout: Representations of ¾.* Have participants work independently on deciding whether each figure could be interpreted as a representation of ¾. Then provide time for participants to discuss their thinking with a partner.<br>  | *Be sure to point out that you are not asking participants to consider whether they would use the representation with their students. Asking about a representation's difficulty for students and how you could use it in the classroom are important questions for teaching, but in order to answer such questions, teachers first need to be able to analyze a representation and explain how it does or does not represent the intended idea. It is that careful analysis and explanation that this task is meant to address; it is not about whether and how to use the representations with students.*<br><br>*Allowing initial time to independently consider the representations will support participant interactions later on. In the next segment, participants will have the opportunity to discuss particular points about several of the representations.*<br><br>*Participants may comment that knowing multiple representations is important for teaching because different students understand different representations. Try to help participants see that there is a larger issue – different representations make different mathematics visible.* |

*CCSSM Link:* In working on this activity, participants are engaged in several of the CCSSM mathematical practices including: making sense of problems and persevering in solving them (#1), reasoning abstractly and quantitatively (#2), constructing viable arguments and critiquing the reasoning of others (#3), and precisely communicating their thinking to others (#6). While the major focus on fractions starts in Grade 3, in the earlier grades, students work on foundational ideas such as partitioning shapes into equal shares and naming a share using fraction language (1.G.3, 2.G.3, 3.G.2).

# Mathematics Example 2: Definition of a Fraction

- Identify the whole
- Make *d* equal parts
- Write $^1/_d$ to show one of the equal parts
- If you have *d* of $^1/_d$, then you have the whole
- If you have *n* of $^1/_d$, then you have $^n/_d$

- *n* and *d* are whole numbers
- $d \neq 0$

# Video

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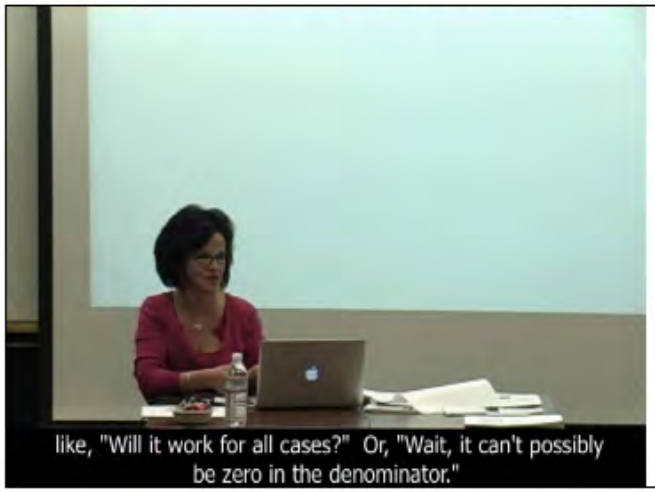
## Session 3-6: A working definition of a fraction




### A working definition of a fraction

- Identify the whole
- Make $d$ equal parts
- Write $^1/_d$ to show one of the equal parts
- If you have $d$ of $^1/_d$, then you have the whole
- If you have $n$ of $^1/_d$, then you have $^n/_d$

- $n$ and $d$ are whole numbers
- $d \neq 0$

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**Overview**

This part further develops a working definition of a fraction and then provides practice applying it to specific examples.

**Key points**

- A working definition of a fraction (see slide).
- Definitions can support the explanation and representation of mathematical ideas.

**Resources**

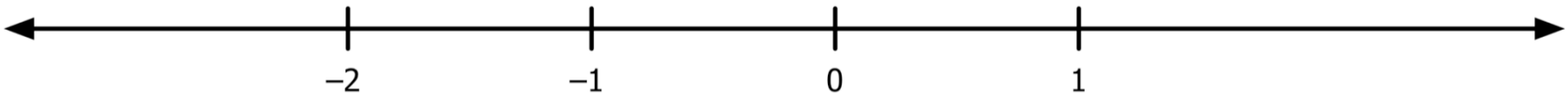
Handout: A working definition of a fraction

# Video

# Mathematics Example 3: Properties and Conventions of the Number Line

Initial exploring by locating fractions on the line:

Place 1 ½ on the line and explain how you figured it out to a partner



# Properties and Conventions

The horizontally displayed number line has the following key properties and conventions:

- **Locating Numbers**
- **Two infinite directions**
- **Positive and negative**
- **Unit interval**
- **Labeling Points**
- **Symmetry**

**Unit interval:** The interval from 0 to 1 is defined to be the unit interval and its length is the *unit distance*.

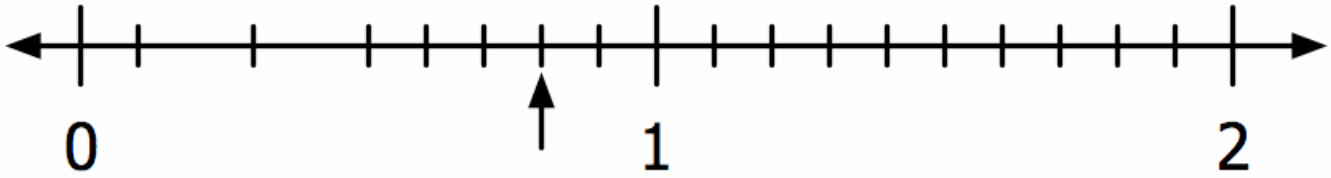
# Designing materials to support elementary teachers in

# TEACHING

## the mathematics of the CCSS

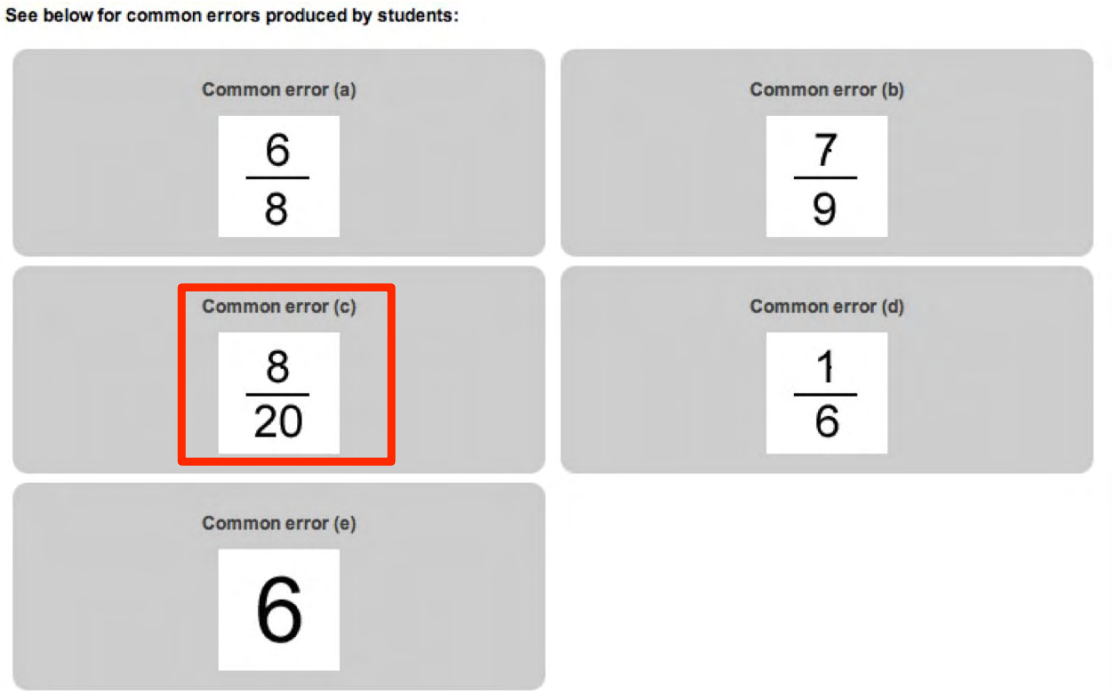
# Teaching Example 1:
# Analyzing Student Use of the Number Line

What value should be written where the arrow is pointing?




Anticipate responses that students would give.

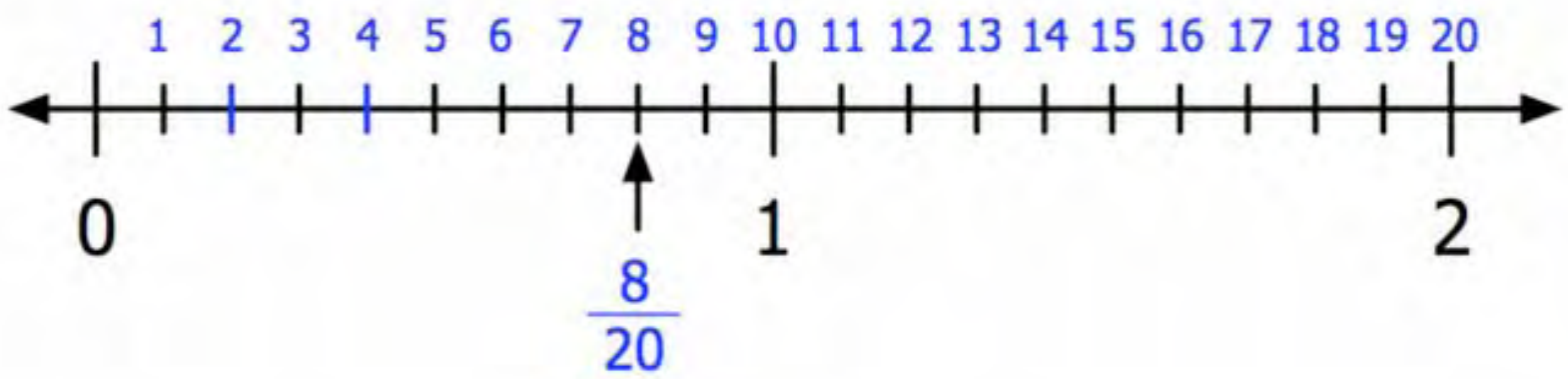
# Analyzing Common Responses




For each response:

- Explain how a student might have arrived at that answer.
- Consider how you might respond to a student who gave that answer.

## Representing the Point on the Number Line as "8/20"

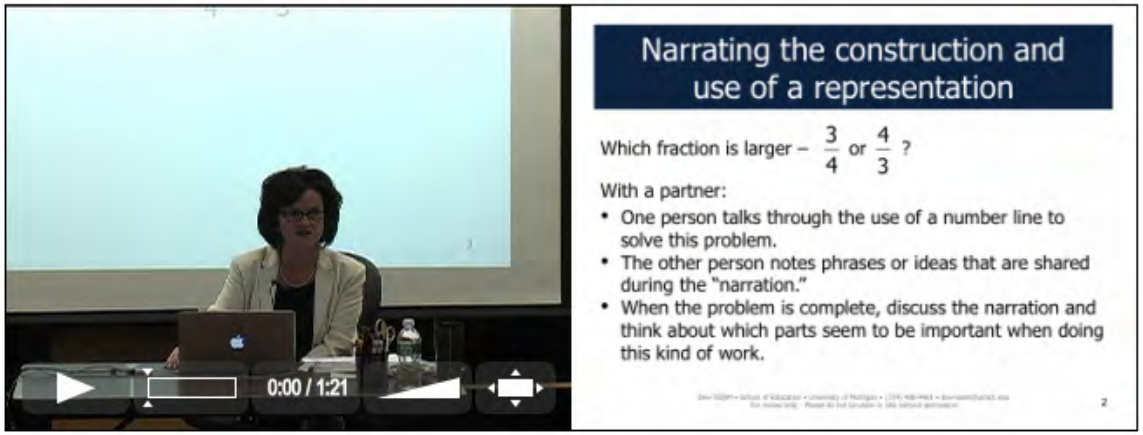



*It's ten from 0 to 1 so it has to be another ten from 1 to 2 so that would make twenty, then I counted from 0 to the arrow to get eight.*

# Teaching Example 2: Narrating the Construction and Use of a Representation

Dev-TEAM

**Session 5-2: The practice of narrating**




**Overview**

This part begins to articulate the practice of narrating by exploring what is involved in narrating the construction and use of a number line to compare fractions.

**Key points**

- When practicing narrating, use the actual language that would be used with students. That is, avoid talking about what "might" be done or "would" be said to students. Instead, talk as if talking to a class.
- Narrate *while* doing the construction. Do not wait until after the representation is constructed to explain what was done.

# Narrating the Construction and Use of a Representation

- Make clear the mathematical problem or context.
- Describe how a particular representation is useful for this problem.
- Construct the representation and use it to solve the task while describing and giving meaning to each step.
- Summarize what the representation has helped to do.

# Analyzing Others' Narrations

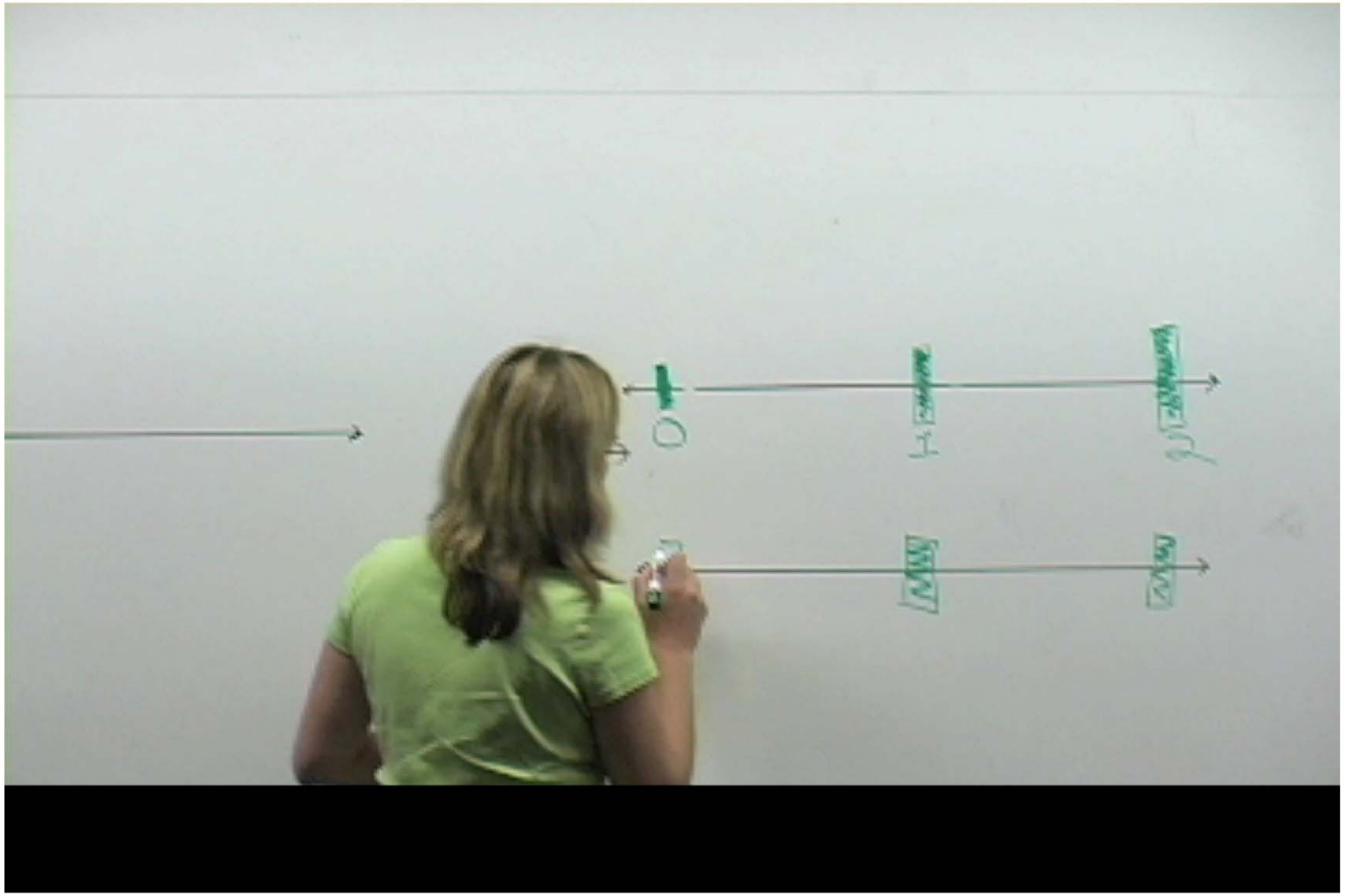
**Context:**

- A teacher in a professional development
- Comparing $^{3}/_{4}$ and $^{4}/_{3}$ using a number line

**Focus questions:**

- Notice the process of narration used and the key features of the number line that were made explicit.
- Is there anything that you would add to or change about the narration in the video clip to make it more clear and/or complete?

# Video



# Materials that Support Narrating the Construction and Use of a Representation

Participants:

- Engage in the rehearsal seen in the video
- Analyze peers' narrations
- Watch and analyze videos of teachers in the PD which:
  - Enrich or orient discussion
  - Enables participants to hear the ways in which "colleagues" articulate key ideas
  - Are easily revisited

# Closing Points

# Other Features of the Materials

- Elementary classroom videos
- Examples from curriculum materials
- Planning for and using public recording space
- Classroom Connection Activities (CCAs)

# Summary

Dev-TE@M modules integrate attention to:

1. mathematical knowledge for teaching
2. students' thinking about mathematics
3. essential teaching practices that support student learning
4. routines for learning in and from practice

**Provides opportunities for teachers to deepen their understanding of Common Core content AND support elementary teachers in TEACHING the mathematics in the Common Core**

# Thank you!

## For further information about our project:
## www.umich.edu/~devteam