

Local structure of groups and of their classifying spaces

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This will be a survey talk on the close relationship between the local structure of a finite group or compact Lie group and that of its classifying space. By the “ p -local structure” of a group G , for a prime p , is meant the structure of a Sylow p -subgroup $S \leq G$ (a maximal p -toral subgroup if G is compact Lie), together with all G -conjugacy relations between elements and subgroups of S . By the p -local structure of the classifying space BG is meant the structure (homotopy properties) of its p -completion $BG_p^\wedge$.

For example, by a conjecture of Martino and Priddy, now a theorem, two finite groups G and H have equivalent p -local structures if and only if $BG_p^\wedge \simeq BH_p^\wedge$. This was used, in joint work with Broto and Møller, to prove a general theorem about local equivalences between finite Lie groups — a result for which no purely algebraic proof is known.

As another example, these ideas have allowed us to extend the family of p -completed classifying spaces of (finite or compact Lie) groups to a much larger family of spaces which have many of the same very nice homotopy theoretic properties.