BINARY BLACK HOLES IN STRONG FIELD GRAVITY, GRAVITATIONAL WAVES AND ELECTROMAGNETIC SIGNATURES FROM THEIR ACCRETION DISKS

MANUELA CAMPANELLI

Advanced LIGO - small bh/bh mergers, neutron star mergers, relatively nearby. Under construction.

eLISA/NGO - supermassive black holes, proposed, might go through with ESA.

PTA - radio telescopes. currently up. super-super massive black holes (10^9 solar masses)

Still haven't found relativistic binary black holes: closest may be .1 parsecs?

Couldn't evolve long enough to look at physics, since mathematical formulation wasn't stable enough (well-posedness?). Had to reformulate for modern methods (SpEC, moving puncture approach)

Her group working on extreme binary black holes: high spins, mass ratios, large distances. graph on "cornering Extreme Black hole binaries is what cases they haven't really explored yet.

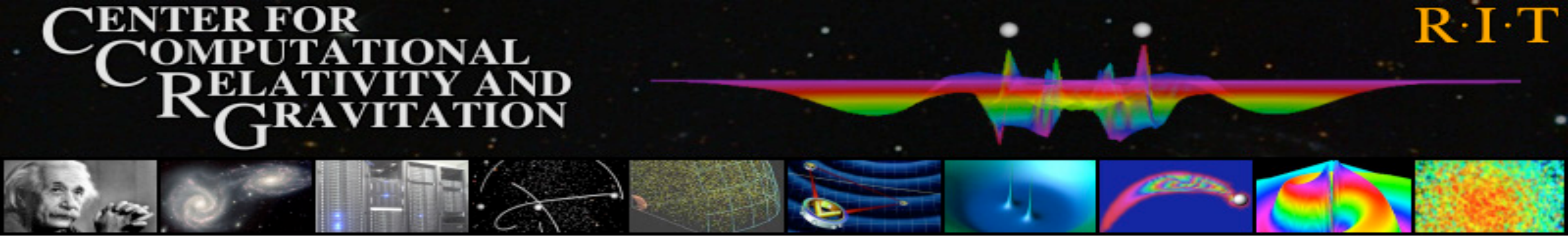
Use adaptive tools to resolve physics near the small object for high mass ratio.

Used numerical relativity to make sure it agrees with Post-Newtonian expansion, but expensive, so can't always do (months at a time). Very good agreement.

Escape velocity from Milky Way is like 1000 km/s, so no galaxy could hold a black hole that gets up to that max kick of 5000 km/s.

Could be close to Hubble time to get BHs to inspiral from parsecs.

Not much gas near BBH during far inspiral, so want to do accurate post-newtonian work to see how much gas is available during final inspiral. Important, so we can run full simulation of final inspiral, so we can find characteristic light signature.



Binary Black Hole Mergers:
Gravitational Radiation, Kicks, and Gas Dynamics

Manuela Campanelli
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Connections for Women: Mathematical General Relativity
Mathematical Sciences Research Institute (MSRI), Berkeley CA
September 03, 2013, 10:45 AM - 11:45 AM

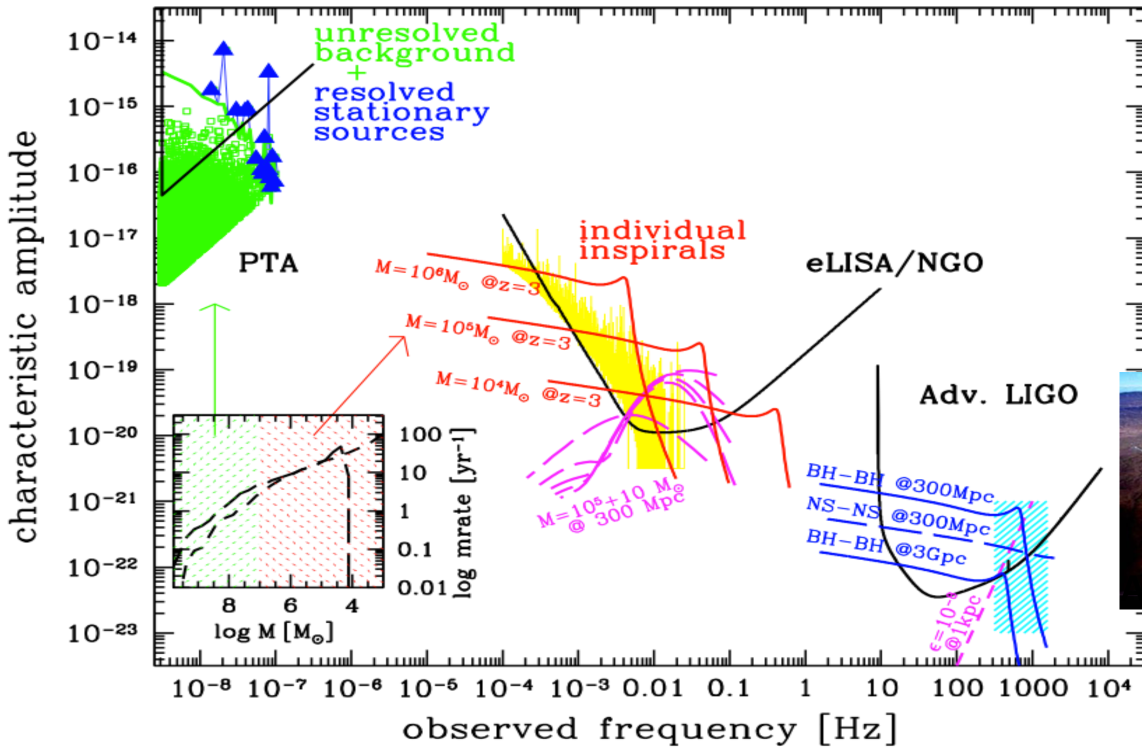
Collaborators @RIT:
Bowen, Lousto, Noble, Yunes, Zlochower, Zilhao
&
Krolik (@JHU), Yunes (@Montana), Mundim (@AEI, Germany),
Nakano (Yukawa Institute, Japan).

R·I·T

Gravitational Wave Astronomy

- BBH mergers are ideal source for a wide range of GW detectors.
  - Their peak GW luminosity outshines the entire observable universe (L_{GW}~10^{54} erg/s)
- GWs travel essentially undisturbed from the source to us

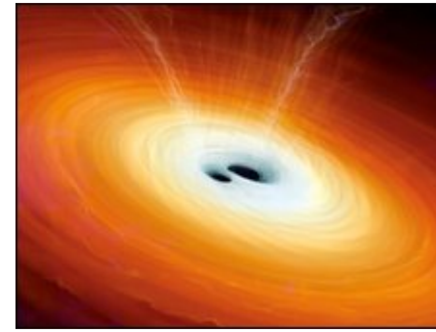
Now Observing!
Supermassive BBH
McWilliams++ 2012



2015 : Early Science
2019 : Full Sensitivity
0.4 - 1000 events/yr?
Only neutron star and small BH systems

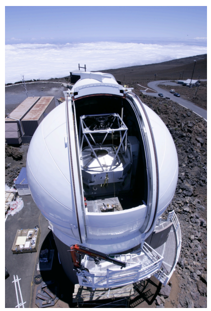
Proposed, 2022?
Guaranteed sources:
supermassive BBH and compact objects falling into a MBH ($10^6 M_\odot$)

Multi-Messenger Astronomy



- BBH Mergers could also be observable in EM spectrum, provided that enough gas is present during the merger stage.

- High-cadence all-sky survey astronomy data could differentiate EM signatures from BBH mergers from those of single AGNs



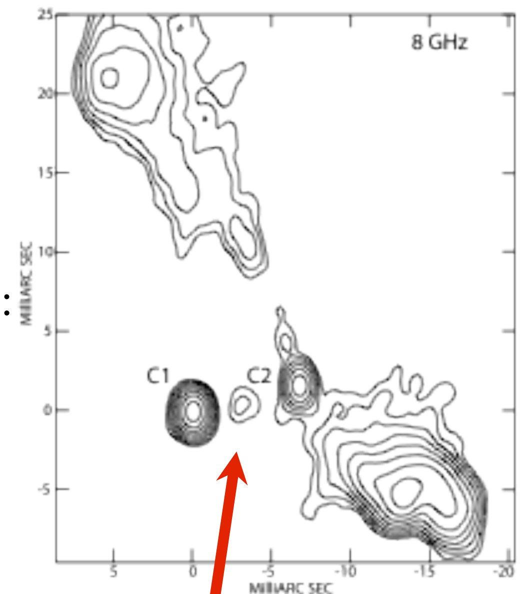
Pan-STARRS:
- 2010-??
- 4 skies per month

Large Synoptic Survey Telescope (LSST):
- 2021-2032
- 1 sky every 3 days

- Great potential for coordinated GW-EM astronomy:
  - GW Detection/Localization <---> EM Detection/Localization;
  - GW and light are connected theoretically but originate in wholly different mechanisms --> independently constrain models;
  - Cosmological Standard Sirens: distance vs redshift measurements [Schutz 1986, Holz & Hughes 2005]
  - Understanding of BH dynamics, merger scenarios, highly relativistic plasma, jet formation, etc
  - Either GW or EM observations of close supermassive BH binaries *would be the first of its kind*!

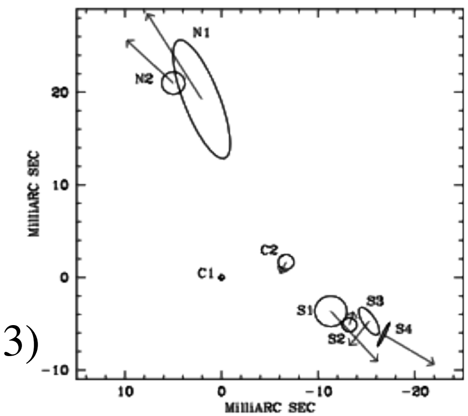
Evidence of BBH ...

SMBHs are observed at the centers of all galaxies with bulges (Gueltekin++09), but there are very few observations of close merging pairs



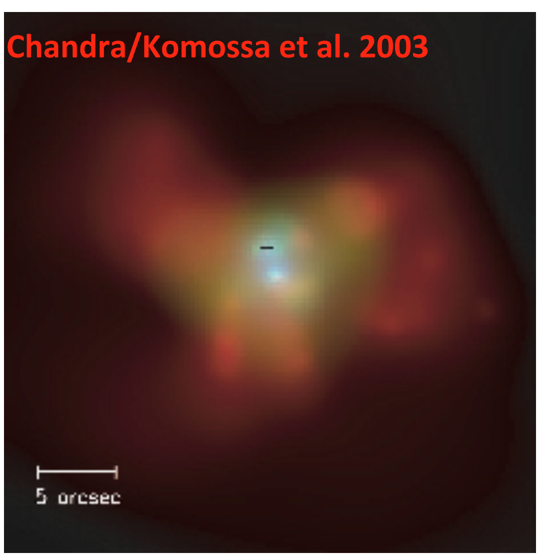
0402+379: (Xu et al. 1994, Maness et al. 2004, Rodriguez et al. 2006):

- Radio observation
- **Separation = 5 pc**



NGC 6240: (Komossa et al. 2003)

- Optical ID: (Fried & Schulz 1983)
- **Separation = 0.5 kpc**



Weakly Emitting Gravitational Waves

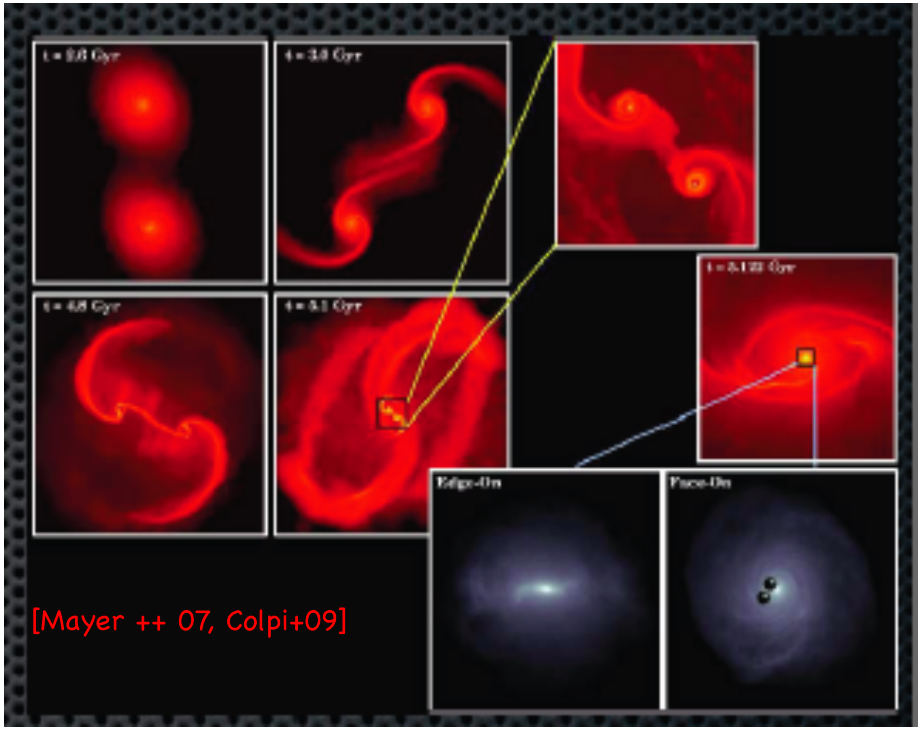
SDSS J153636.22+044127.0

(Lauer & Boroson 2009)

Separation = 0.1pc

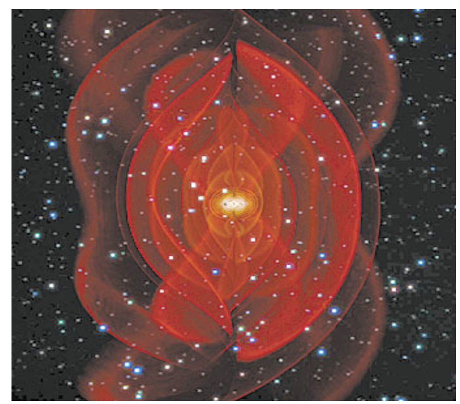
1pc = 1 parsec = 3.26 light-years
= 1.9 x 10^{13} miles

Supermassive Black-Hole Mergers



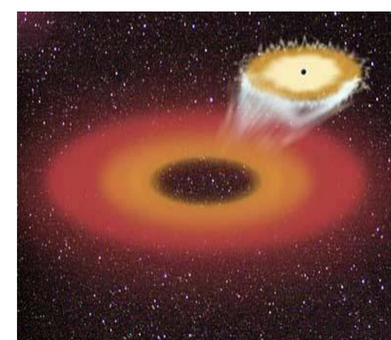
- Hierarchical build-up of galaxies from smaller structures (ΛCDM) ➔ galaxies merger ➔ BBH mergers

- Torques from gas, stellar dynamical friction, gravitational slingshot bring the pair to sub-pc scales …



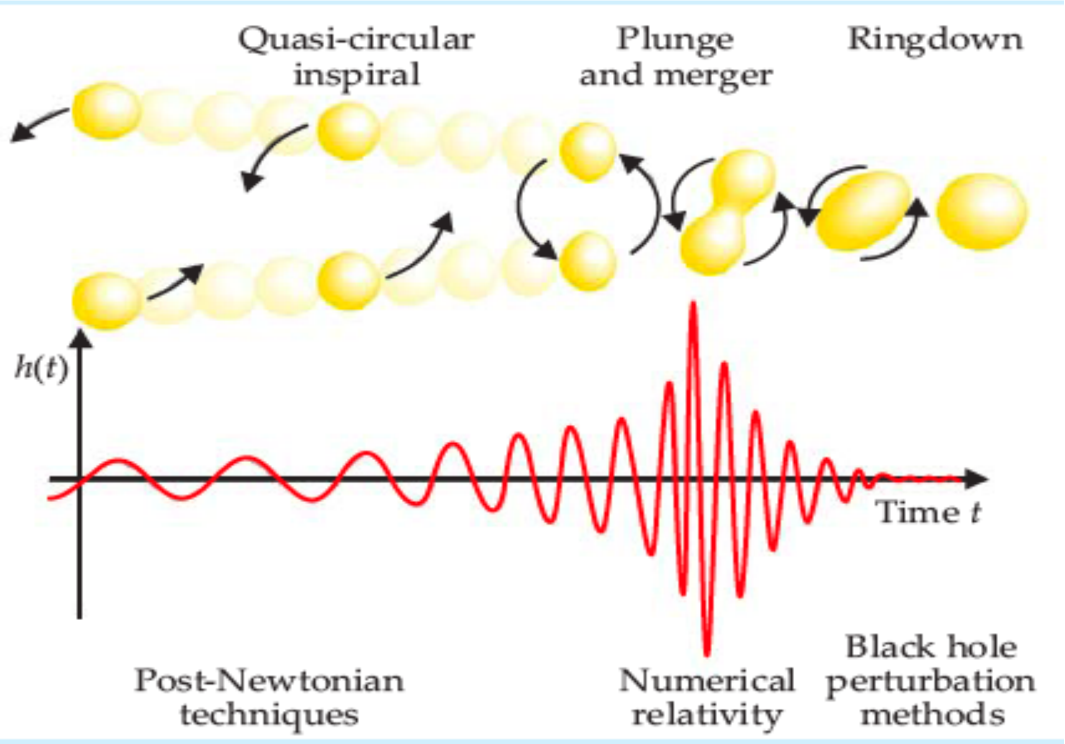
- Then, **GW** emission (3-10% of the total mass) drive the binary to the final merger

- The BH remnant will **recoil** from its host structure, depending on the BH spins and masses at merger.



Numerical Relativity

To model the final stages of BBH mergers, we need to evolve the GR Equations

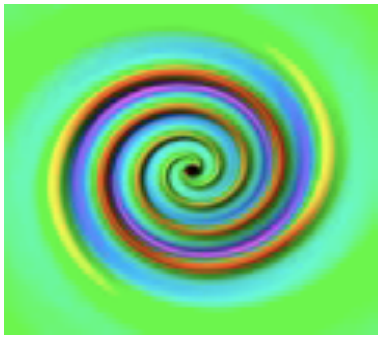


Artistic representation, Baumgarte & Shapiro (Physics Today 2011)

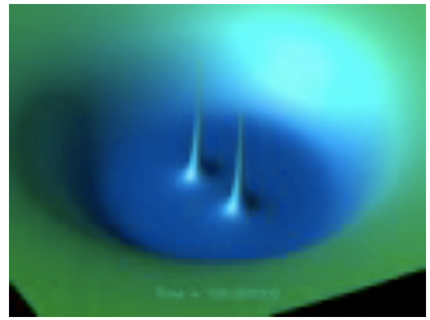
- Numerical Relativity can be used to calculate:
  - Gravitational Waves (Waveforms)
  - Astrophysics of the BH remnant, such as the final kick and final spin
  - Accretion Disks Dynamics (GR-MHD)

Modern Numerical Relativity

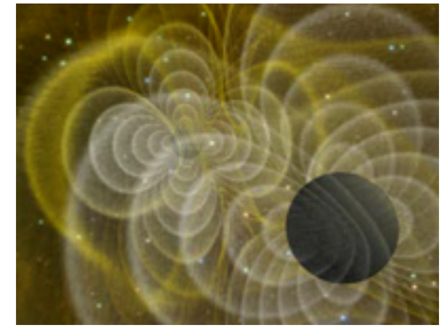
There has been an ongoing effort since the 60's to do this, but it is only in the last 8 years has it actually been possible to evolve multiple BBH spacetimes stably and accurately enough.



Pretorius, Phys Rev Lett 95 (2005)



Campanelli +, Phys Rev Lett 96 (2006)



Baker +, Phys Rev Lett 96, (2006)

GWs carry away a full 4% of their initial energy in roughly an orbital time, and leave behind a moderately spinning BH with a/M = 0.7

Spectral Einstein Code (SpEC):

Generalized Harmonic

Highly-accurate (converge exponetially with resolution), but less flexible (care needed to get BBH merger)

Moving Puncture Codes:

BSSN + Punctures, AMR

Less-accurate (polynomial convergence of FD methods), but more flexible and robust

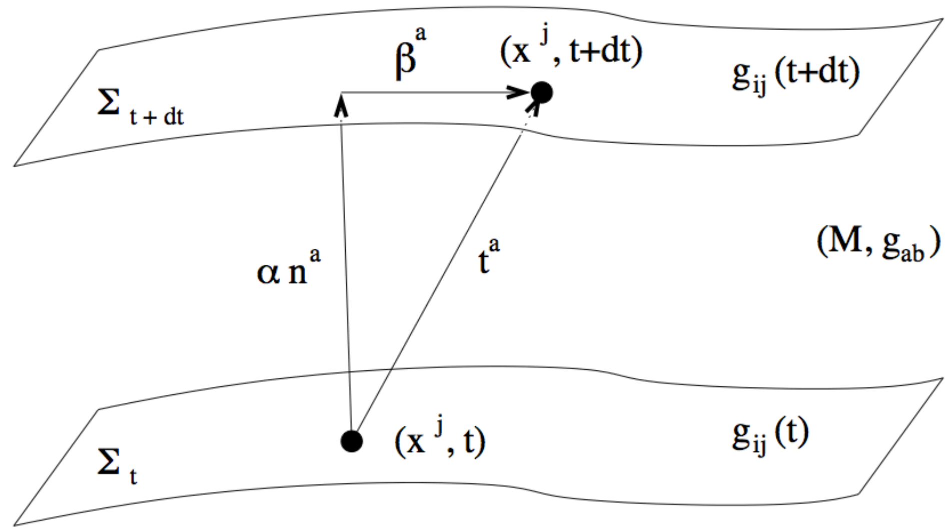
Open-Source Codes:

einstein toolkit

3+1 Numerical Relativity

(Arnowitt, Deser, Misner, 1962)

- Foliates 4-dimensional manifold into space-like hypersurfaces parameterized by time;
- Turn Einstein's equations into a Cauchy problem with hyperbolic PDEs and elliptic constraint equations;
- Equations are solved via finite differencing on nested meshes of ever decreasing grid spacing;
- In practice, we now work with a strongly hyperbolic 3+1 formulation (BSSN).



$$ds^2 = \left(-\alpha^2 + \beta^j\beta_j\right) dt^2 + 2\beta_j dx^j dt + g_{ij} dx^i dx^j$$

$$K_{ij} = -\frac{1}{2}\pounds_n g_{ij}$$

12 Coupled 1st-order hyperbolic PDEs:

$$\pounds_t g_{ij} = -2\alpha K_{ij} + \pounds_\beta g_{ij}$$

$$\pounds_t K^a{}_b = \pounds_\beta K^a{}_b - D^a D_b \alpha + \alpha\left\{{}^{(3)}R^a{}_b + K K^a{}_b + 8\pi\left[\frac{1}{2}\gamma^a{}_b\left(S^c{}_c - \varrho\right) - S^a{}_b\right]\right\}$$

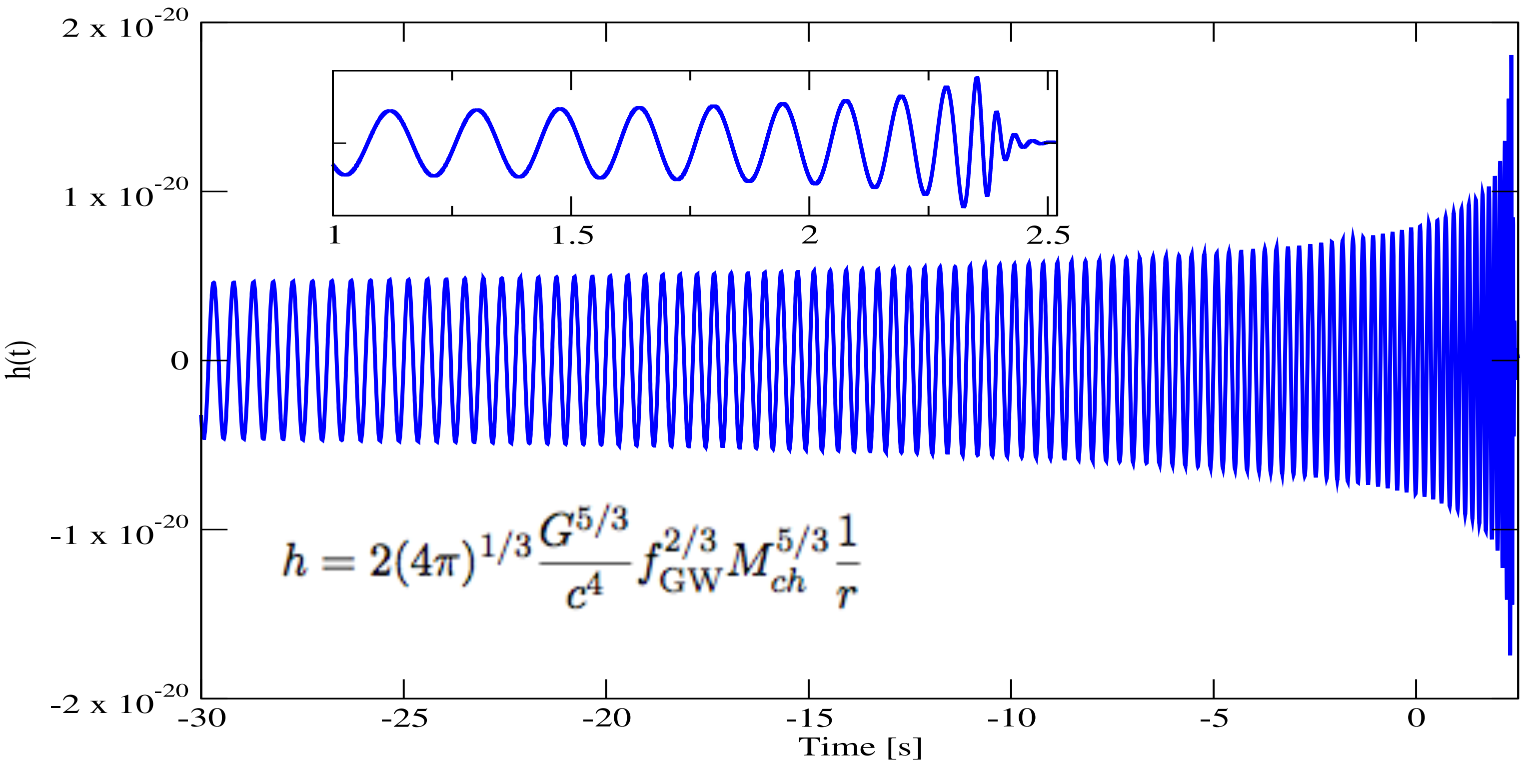
4 Elliptic Constraint Eqs:

$$D_b K^{ab} - D^a K = 8\pi j^a \qquad {}^{(3)}R + K^2 - K^a{}_b K^b{}_a = 16\pi\varrho$$

4 Gauge Conditions: $\alpha(x^\mu)\ ,\ \beta^i(x^\mu)$ Relating coords between neighboring slices

Gravitational Radiation Waveforms

Waveforms encode information about many parameters: BH masses & spins, orbital parameters, source distance, sky position, and are essential on assisting GW detectors, such as LIGO, to predict what to expect and for physical information extraction …

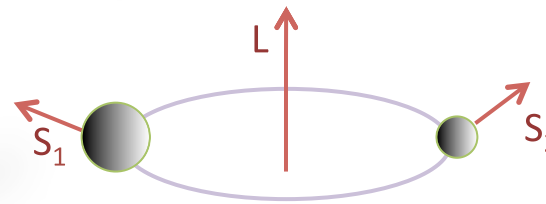


$$h = 2(4\pi)^{1/3}\frac{G^{5/3}}{c^4} f_{\rm GW}^{2/3} M_{ch}^{5/3}\frac{1}{r}$$

- GR is scale invariant, so waveform are independent of the total mass.

$$\lim_{r\to\infty}\left[r\,\psi_4^{\ell m}(r,t)\right] = \left[r\,\psi_4^{\ell m}(r,t) - \frac{(\ell-1)(\ell+2)}{2}\int_0^t dt\,\psi_4^{\ell m}(r,t)\right]_{r=R_{\rm obs}}$$

Spanning Through BBH Parameter Space

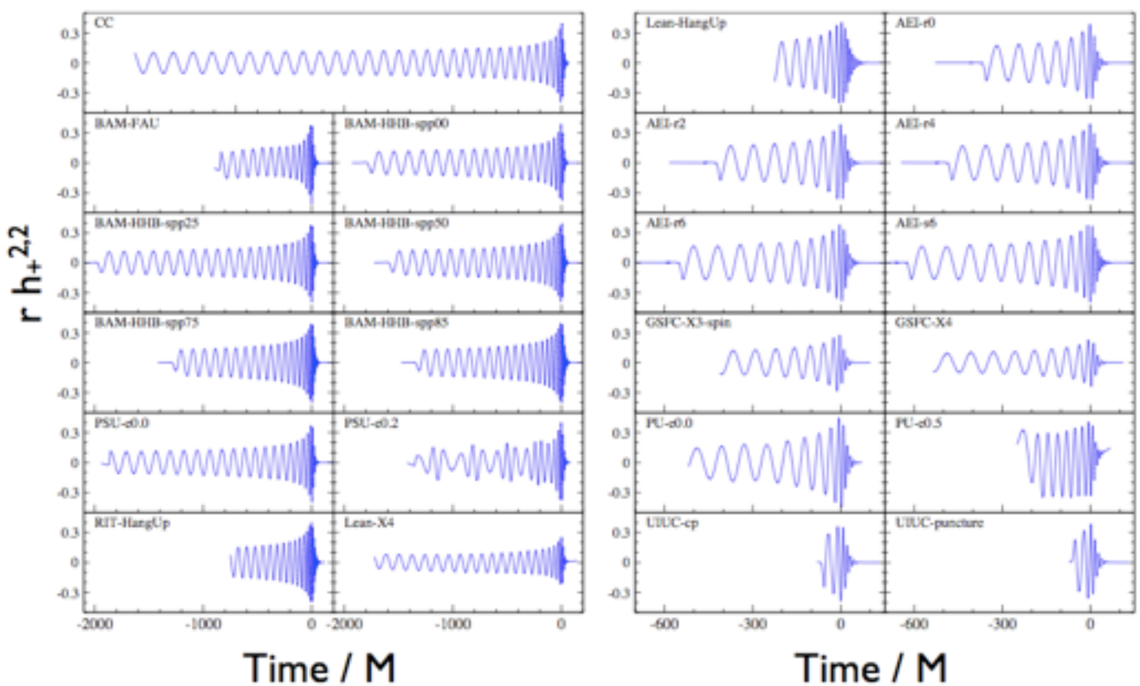


- BBH span over a large parameter space:

  mass ratio (1 parameter) : $q = m_1/m_2 \leq 1, \quad \nu = \eta = m_1 m_2/(m_1+m_2)^2$

  spin (6 parameters) : $\vec{S}_i = m_i^2 \vec{\alpha}_i, \quad |\vec{\alpha}_i| \leq 1$

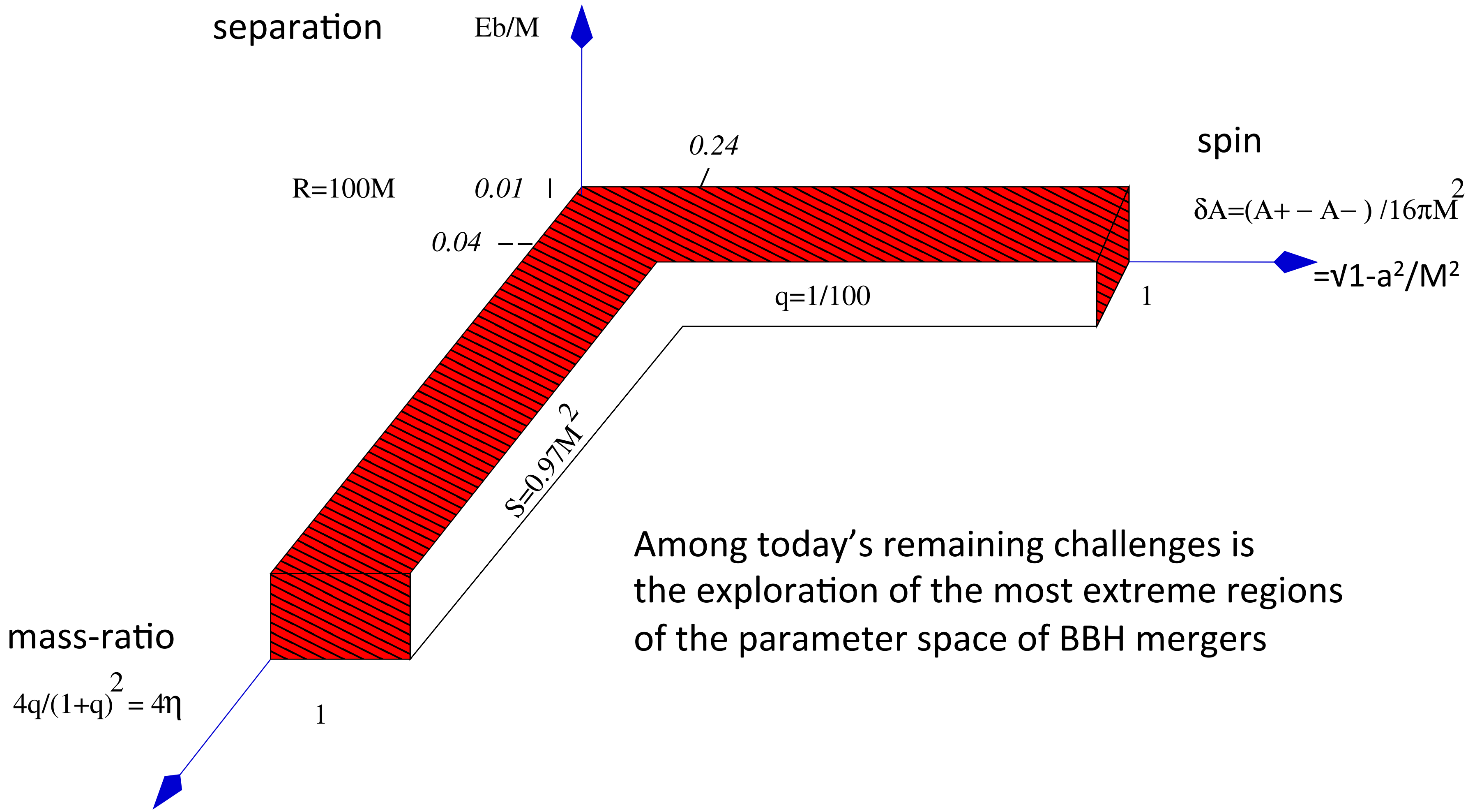
  eccentricity (1 parameters) : e



- **NINJA I**: BBH waveforms used to test of all data analysis algorithms [Aylott++ 2009, Cadonati++ 2009]
- **NINJA 2**: BBH analysis in real data in close collaboration with LSC/Virgo.
- **NRAR**: NR groups span BBH parameter space.

https://www.ninja-project.org/

Cornering Extreme Black Hole Binaries

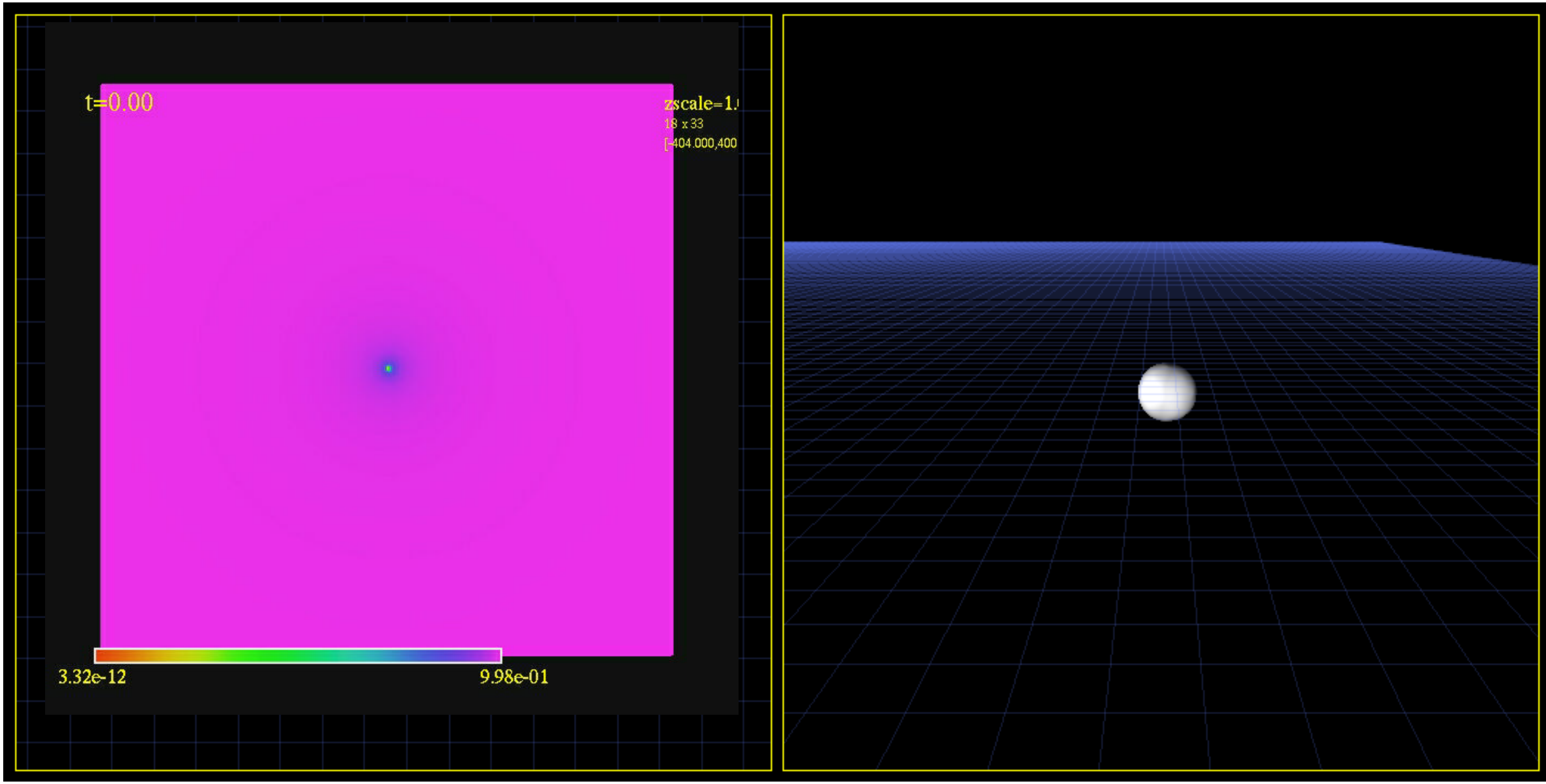


Among today's remaining challenges is the exploration of the most extreme regions of the parameter space of BBH mergers

Courtesy by Carlos Lousto, 2013

The Large mass-ratio Corner, q= 100:1

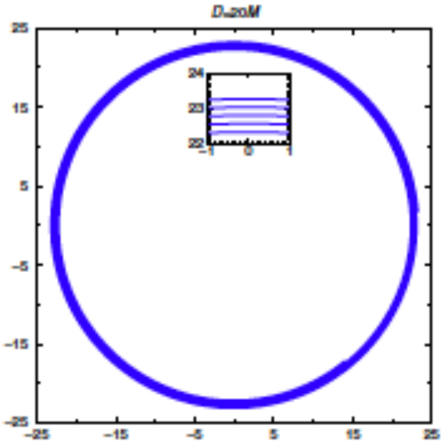
Lousto & Zlochower, Phys. Rev. Lett. 2011



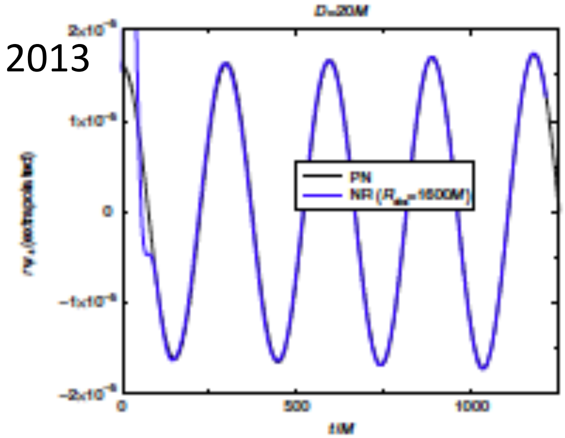
15 levels of refinements in AMR guided by BH perturbation theory, adapted gauge conditions.

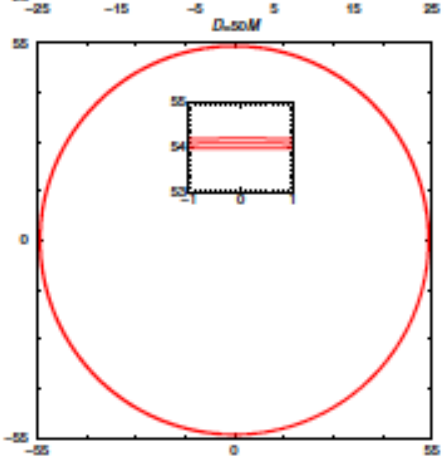
The Outer Limits of Black Hole Binaries

Lousto & Zlochower, Phys. Rev. D 88 024001, 2013

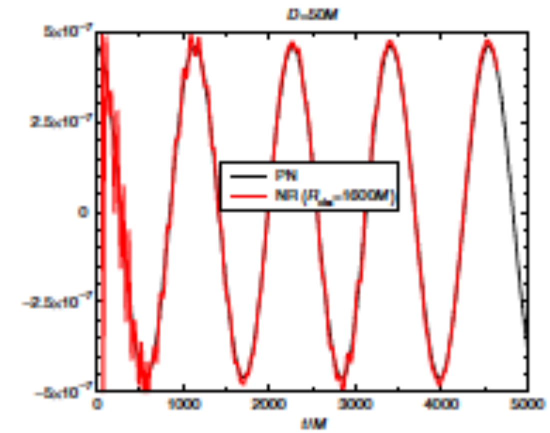


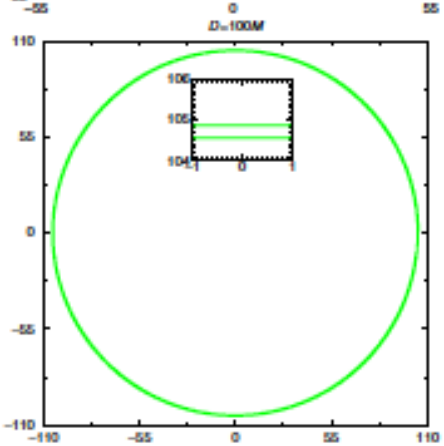
T_{NR}=615M T_{PN} =601M





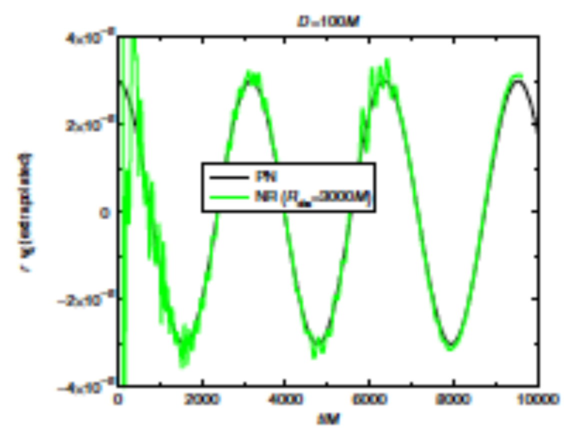
T_{NR}=2313M, T_{PN}=2283M





T_{NR}=6422M, T_{PN}=6370M

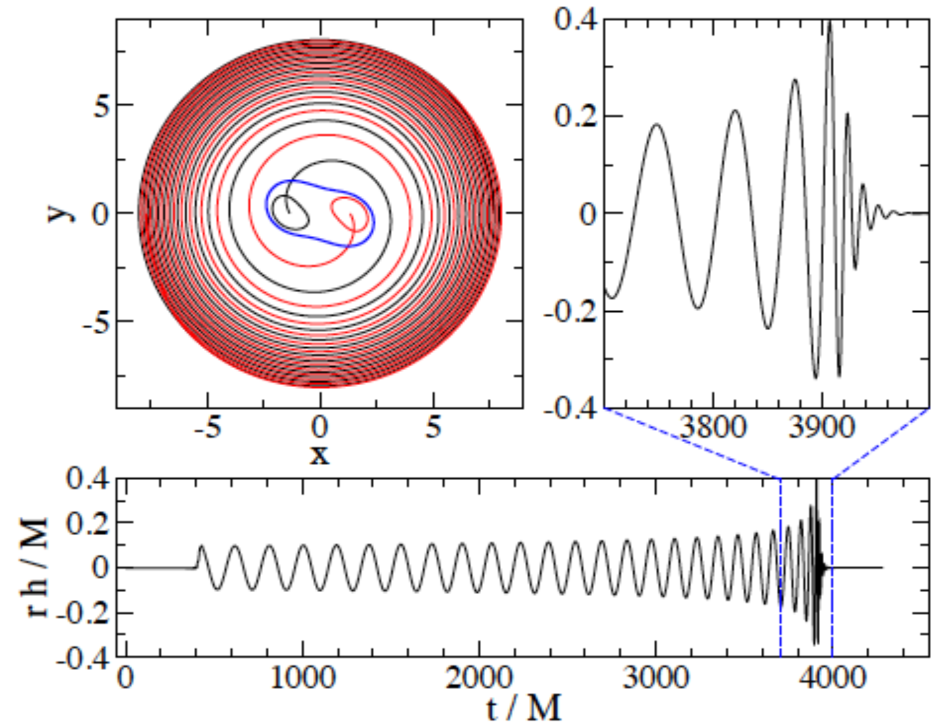
(Orbital Period)



The High Spin Corner

Lovelace+, Phys. Rev. D, 2011

Make a 12 orbits evolution of BBH with spins=0.97. Radiates over 10% of its mass in GW. The brightest source in the entire Universe!



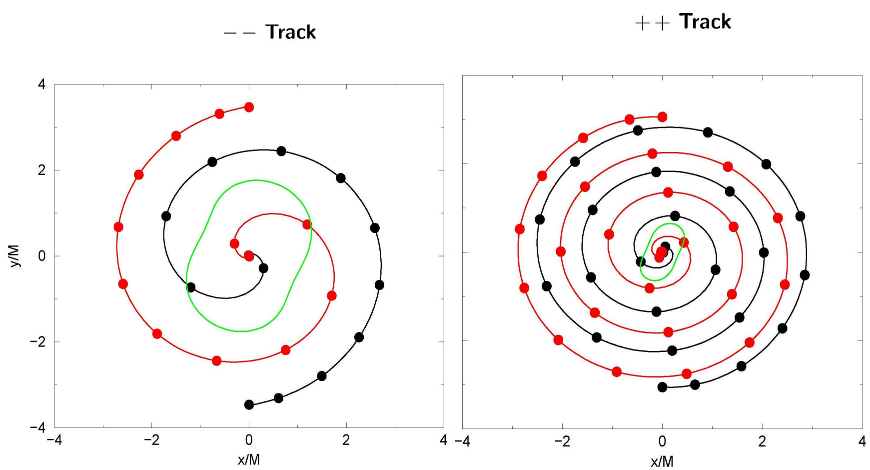


Figure 4: Puncture tracks for the − − configuration.

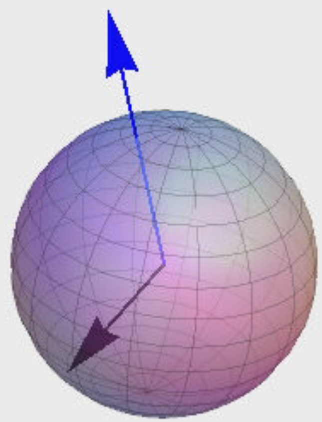
Figure 6: Puncture tracks for the + + configuration.

Campanelli+, Phys Rev D, 2006

Orbital-hangup effect: When spins are aligned with L, repulsive spin-orbit coupling delays the merger, maximizing the amplitude of gravitational radiation.

Spin Dynamics: Precession

Lousto & Zlochower, arXiv:1307.6237

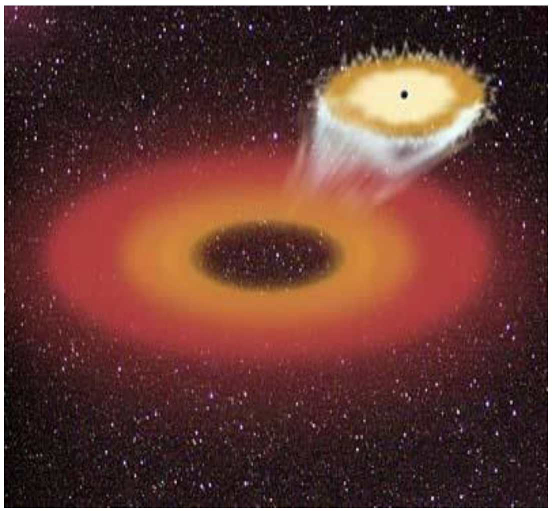


Two equal-mass, spinning BHs, with spins nearly counteraligned with L

J = L + S

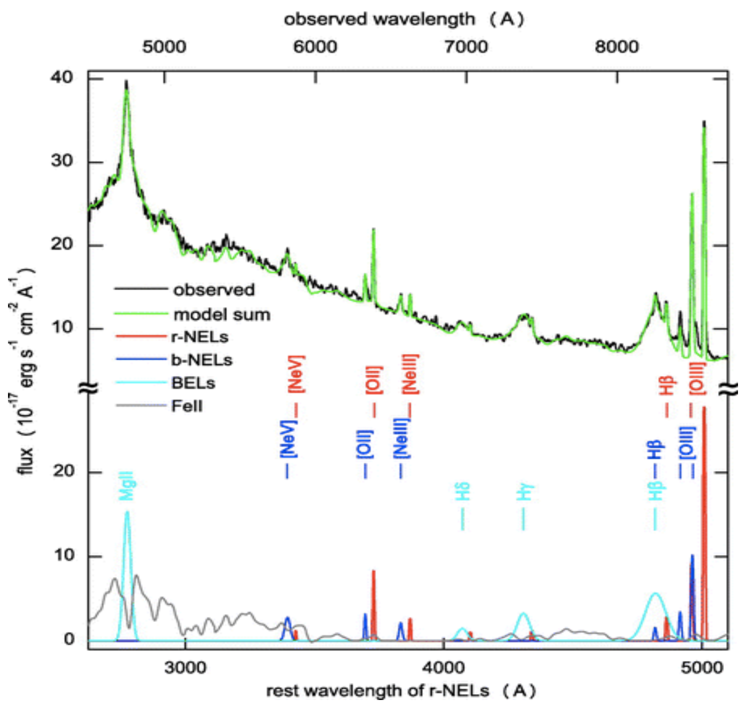
Gravitational Radiation Recoils

The asymmetric beaming of GW radiation (due to unequal masses and/or spins) at merger can cause the BH remnant to recoil, and if the recoil is large enough the the BH can “escape” from its host structure.



Consequences for growth of SMBHs in galaxies and IMBH formation in globular clusters

Possible observations: off-set galactic nuclei, displaced active galactic nuclei, population of galaxies without SMBHs, x-rays afterglows, feedback trails, double-peaked NRL emitters



Double-peaked NRL emitters [Komossa+08)

- edge of the disk of kicked BH
- ionized gas of the disk left behind

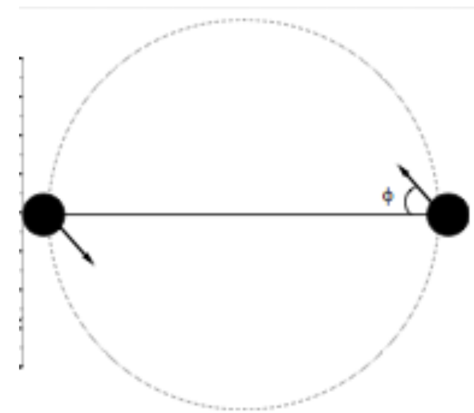
Spins Dynamics and Gravitational Radiation Recoils

- Ideal calculation for NR

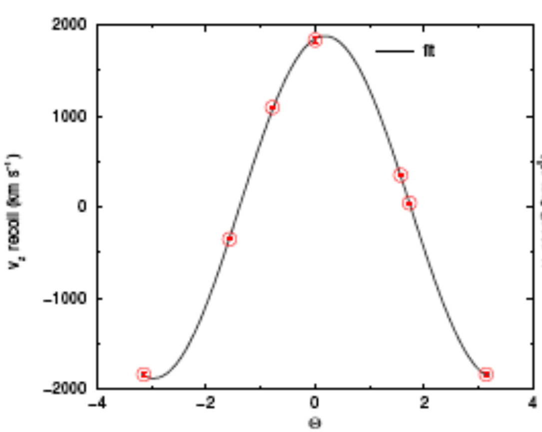
$$\frac{dP_i}{dt} = \lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \ell_i \left|\int_{-\infty}^{t} \Psi_4 d\tilde{t}\right|^2 d\Omega\right]$$

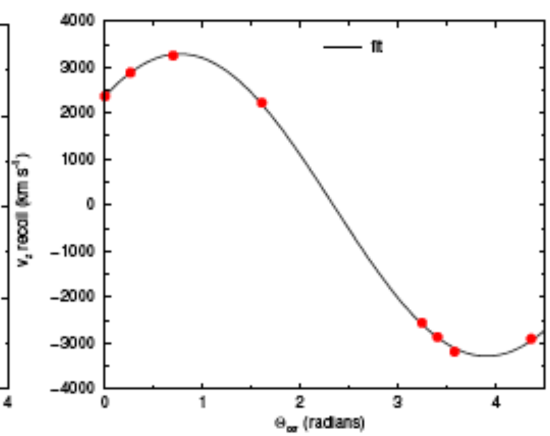
- While unequal-mass BBH lead to a maximum recoils < 200 km/s, it was found that spin-orbit coupling effects can lead to very large kick velocities, up to 4000 Km/s (superkicks).

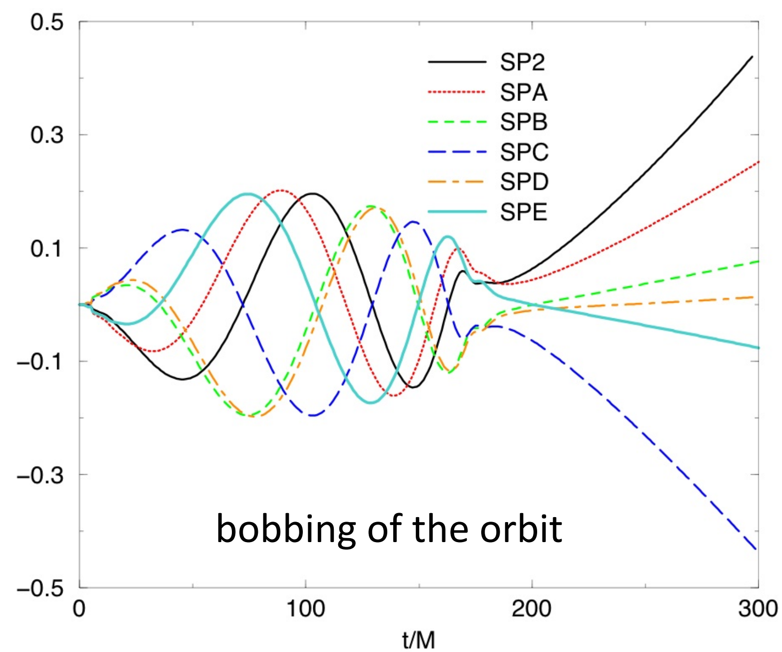
- Recoil velocity depends sinusoidally on the initial phase of the binary, and linearly (at leading order) on the spin magnitude.



Equal-mass BBH, in-plane, opposite BH spins [Campanelli+07a,b, Gonzalez+07]

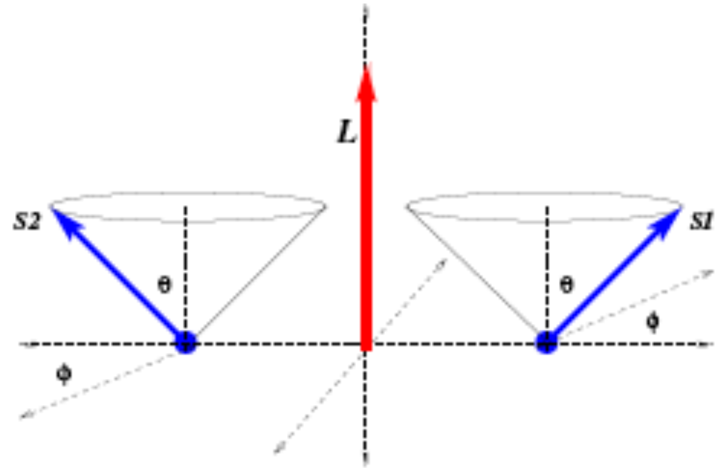




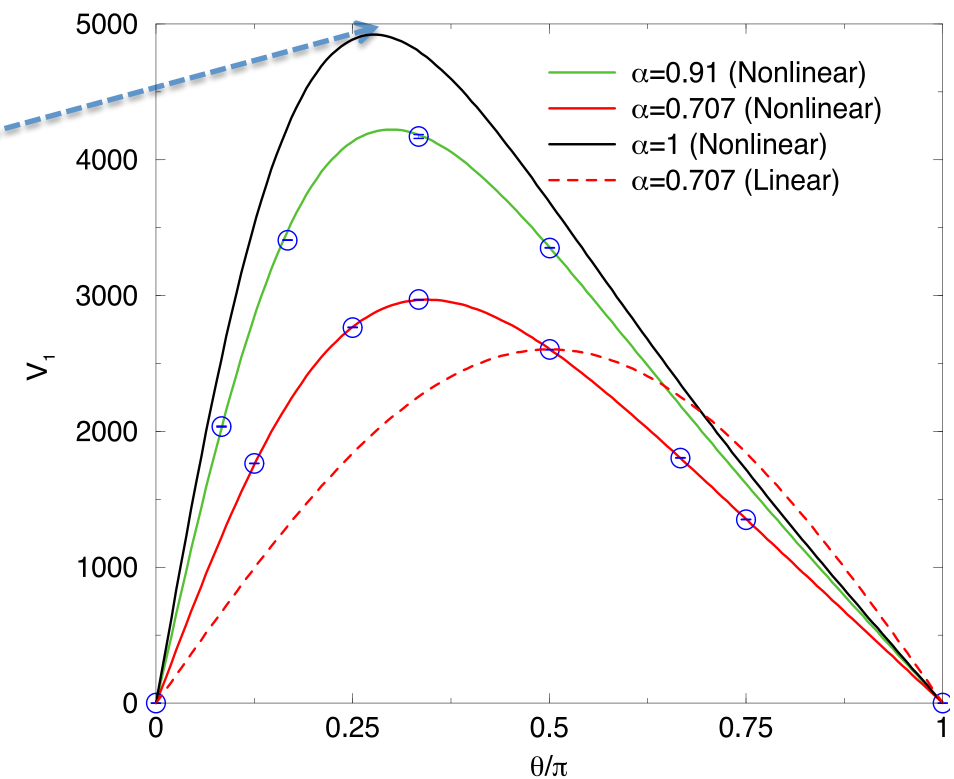


More on large GW Recoils

- When spins are aligned with L, repulsive spin-orbit coupling delays the merger (orbital-hangup effect), maximizing the amplitude of gravitational radiation (up to 10%) [Campanelli+ 06].
- Combined with the superkick effect (which maximizes the asymmetry of momentum radiated), this leads to very large recoils [Lousto & Zlochower, 11].



Peak occurs at 5000 km/s for **nearly aligned spins**



Three parameters family of initial configurations depending of φ, θ, and spin magnitude, a. Each dot in the plot are 6- runs to span the φ dependence. 48 new runs.

Recoil Velocity Formula

(Campanelli+07a,b; Van Meter+10, Lousto+12)

$$\vec{V}_{\rm recoil}(q,\vec{\alpha}) = v_m\,\hat{e}_1 + v_\perp(\cos\xi\,\hat{e}_1 + \sin\xi\,\hat{e}_2) + v_\|\,\hat{n}_\|,$$

$$v_m = A_m \frac{\eta^2(1-q)}{(1+q)}\left[1 + B_m\,\eta\right],$$

Unequal mass non-spinning binaries produces in-plane kicks < 175 km/s

$$v_\perp = H \frac{\eta^2}{(1+q)}\left[\left(\alpha_2^\| - q\alpha_1^\|\right)\right],$$

The perp. term to L produce in-plane kicks < 500 km/s

$$\begin{aligned} v_\| \;=\; & 16\frac{\eta^2}{(1+q)}\left|\alpha_2^\perp - q\alpha_1^\perp\right| \times \\ & \left[V_{1,1} + A\left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2}\right) + B\left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2}\right)^2 \right. \\ & \left. + C\left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2}\right)^3\right]\cos(\phi_\Delta - \phi_1), \qquad (4) \end{aligned}$$

The parallel term to L, that is responsible for the superkicks (linear spins) and hang-up kick (quadratic spins)

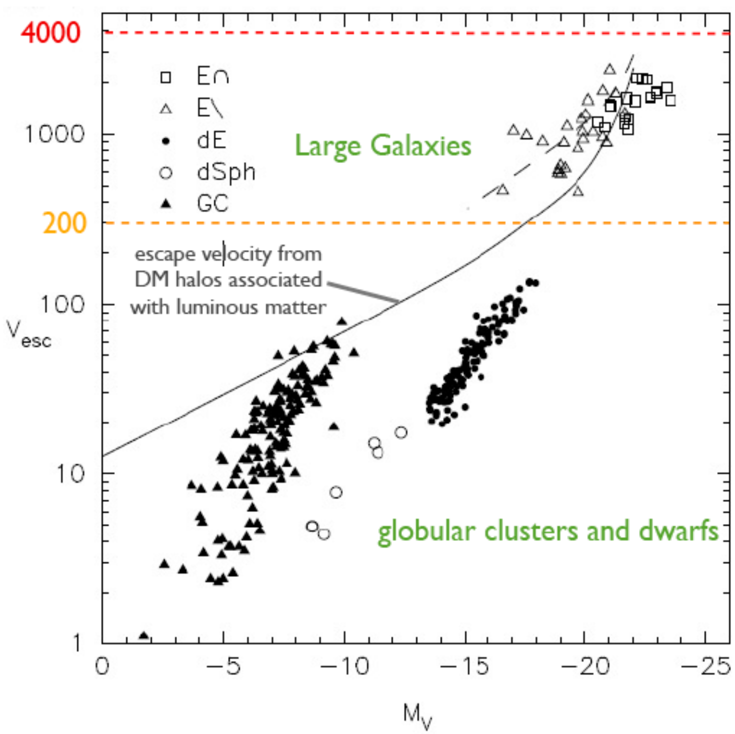
where η = q/(q+1) and q =m1/m2 < 1. A,B,H,K, ξ, and Φi are constants.

- Superkick maximum ~ 4000 km/s occurs when the spins are exactly anti-aligned and q=1.
- Hang-up kick maximum ~ 5000 km/s occurs for nearly aligned and q=1.

Probabilities to Observe Large Recoils

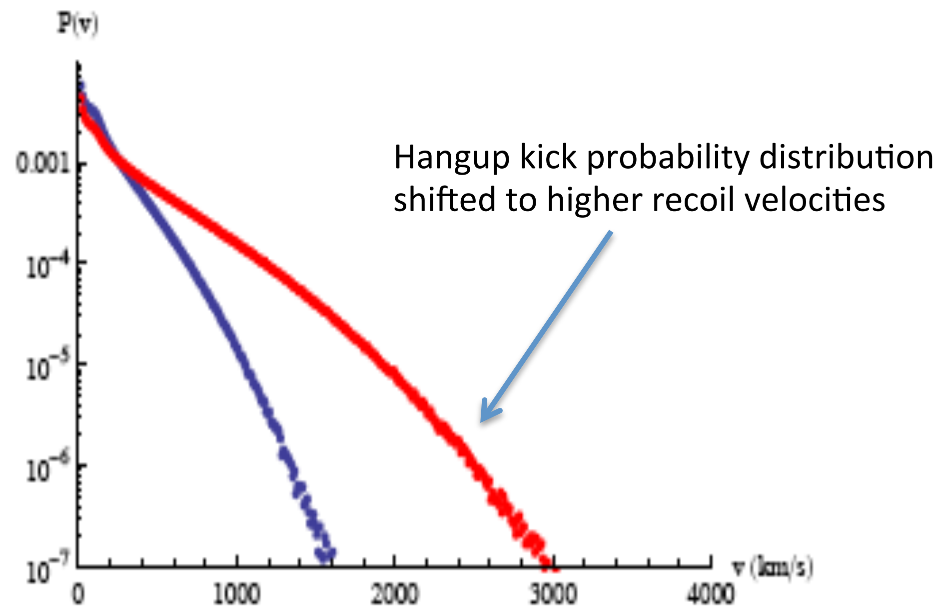
Alignment of the spins by gas accretion inhibit large recoils [Bogdanovic+07, Dotti +10)]

Kicks can have significant consequences for growth of SMBHs in galaxies and IMBH formation in globular clusters

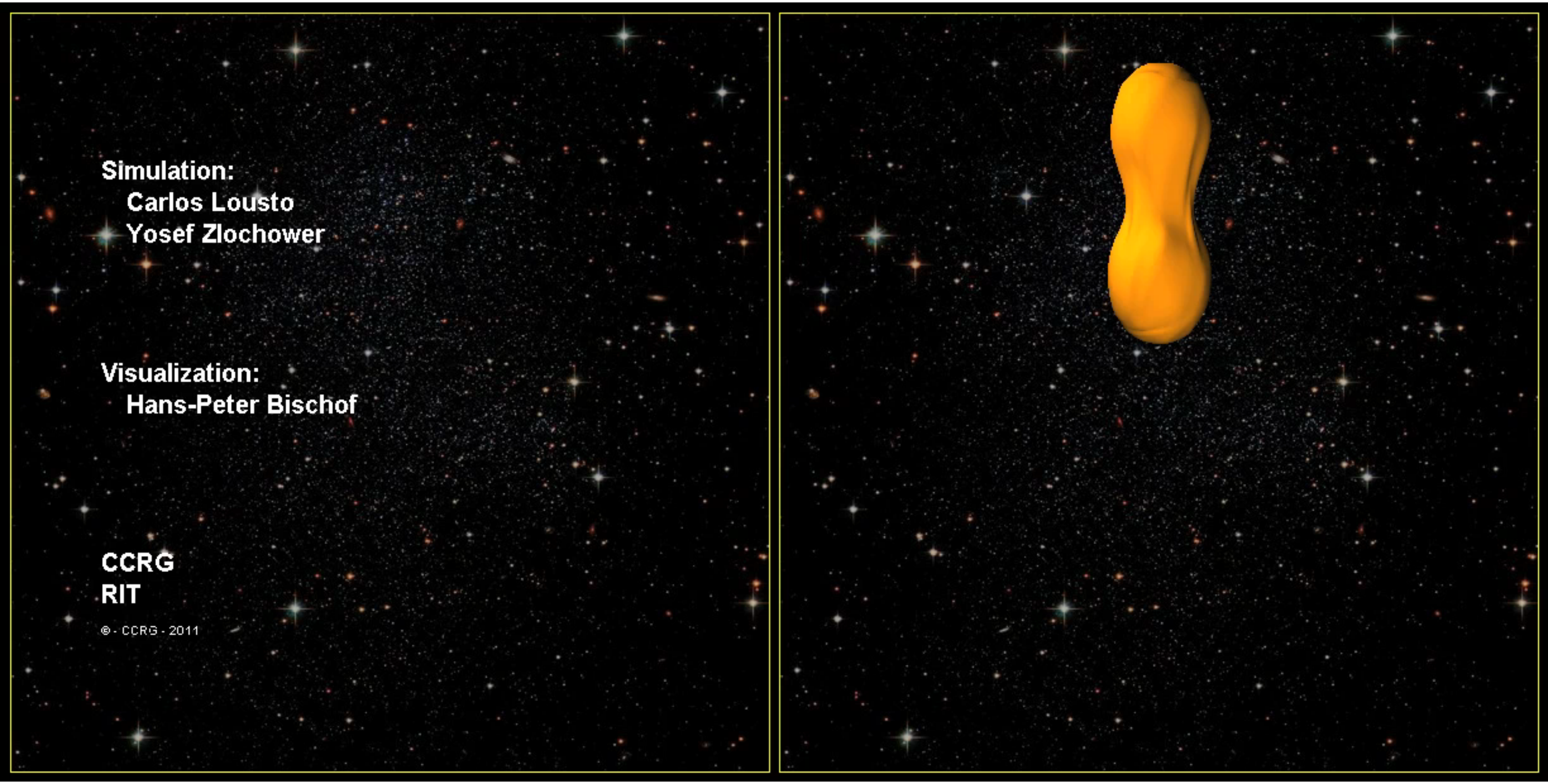


But with the hang-up kicks, probabilities that remnant BH recoils in any direction from host structure (spins from SPH simulations of hot and cold accretion models) are not small [Lousto+12]:

- 0.02% for galaxies with v_{esc} ~ 2500 km/s
- 5% for galaxies with v_{esc} ~1000 km/s
- 20% for galaxies with v_{esc} ~500 km/s



Hangup Kicks: The Movie

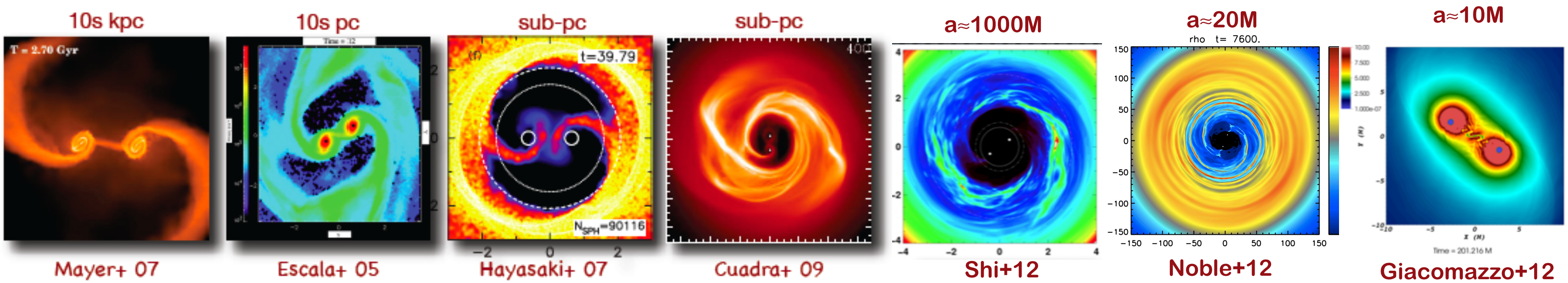


Hangup Kick (Left) and Radiated Power (Right)
[Lousto & Zlochower 11, visualization by H.P. Bischof]

Light Signatures from BBH Mergers

This require some significant amount of gas in the near vicinity of the merging BHs.

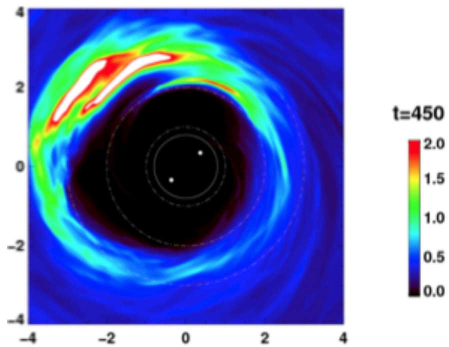
To answer the question of whether or not there is any gas present, and if so, what are its properties, one must solve a grand challenge problem because the scales ranges from 10^5 pc to 10^{-5} pc ➔ do systematic studies of each stage of the coalescence, bridging the gaps among the stages



- Realistic accretion disk physics for each stage:
  - Ideal Magnetohydrodynamics (MHD)
  - Radiative Transfer/Ray-Tracing
  - Multi-species thermodynamics
- Gravity model for BBH: Newtonian, Post-Newtonian, General Relativity

Light from the last stages of BBH Mergers

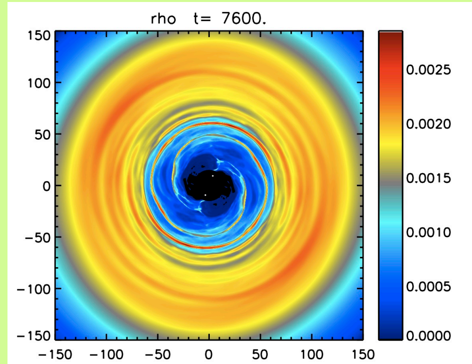
Newtonian Gravity + MHD:



t_{merger} >> t_{inflow} (a ≈ 1000s M)

- Gap formation near r ≈2a (due to binary torque) [MacFadyen & Milosavljevic 8, Cuadra+09]
- Build-up of late-time surface density maximum near the gap'edge with faster accretion (due to MHD stresses) [Shi +11].

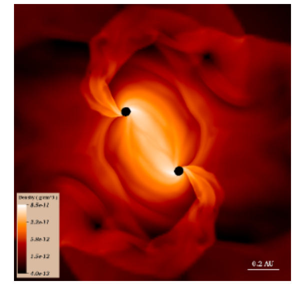
PN Gravity + MHD:

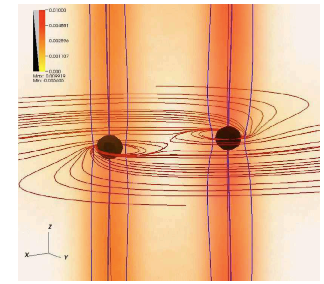


t_{inflow}≈ t_{merger} (a ≈ 10-100M)

- Evolve 3.5 PN BBH for hundreds of orbits in a radiatively efficient, circumbinary (geometrically thin) accretion disk [Noble+12]
- BBH not on the grid …
- The amount of gas available to be heated at merger depends from the balance of BBH torques and MHD stresses!

GR-MHD:





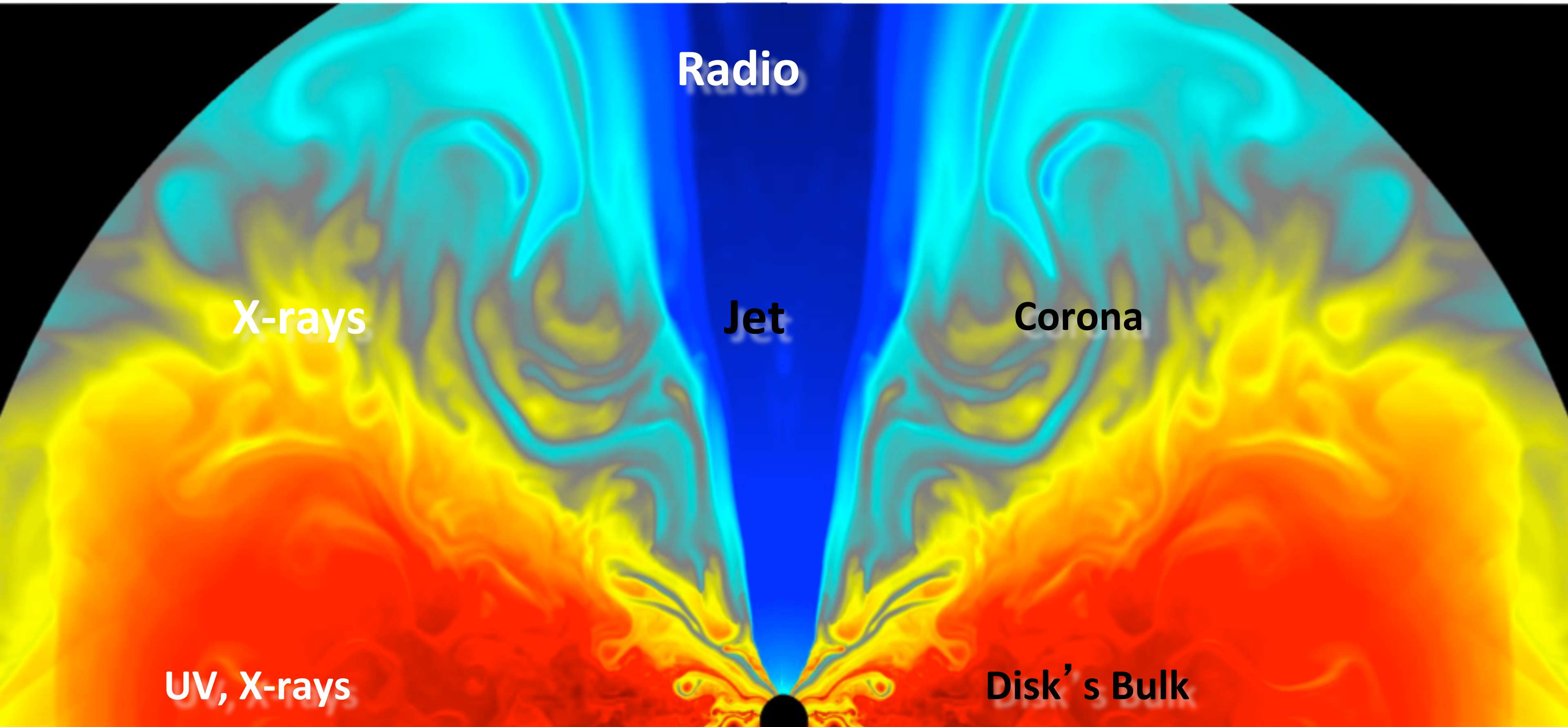
t_{merger} << t_{inflow} (a < 10M)

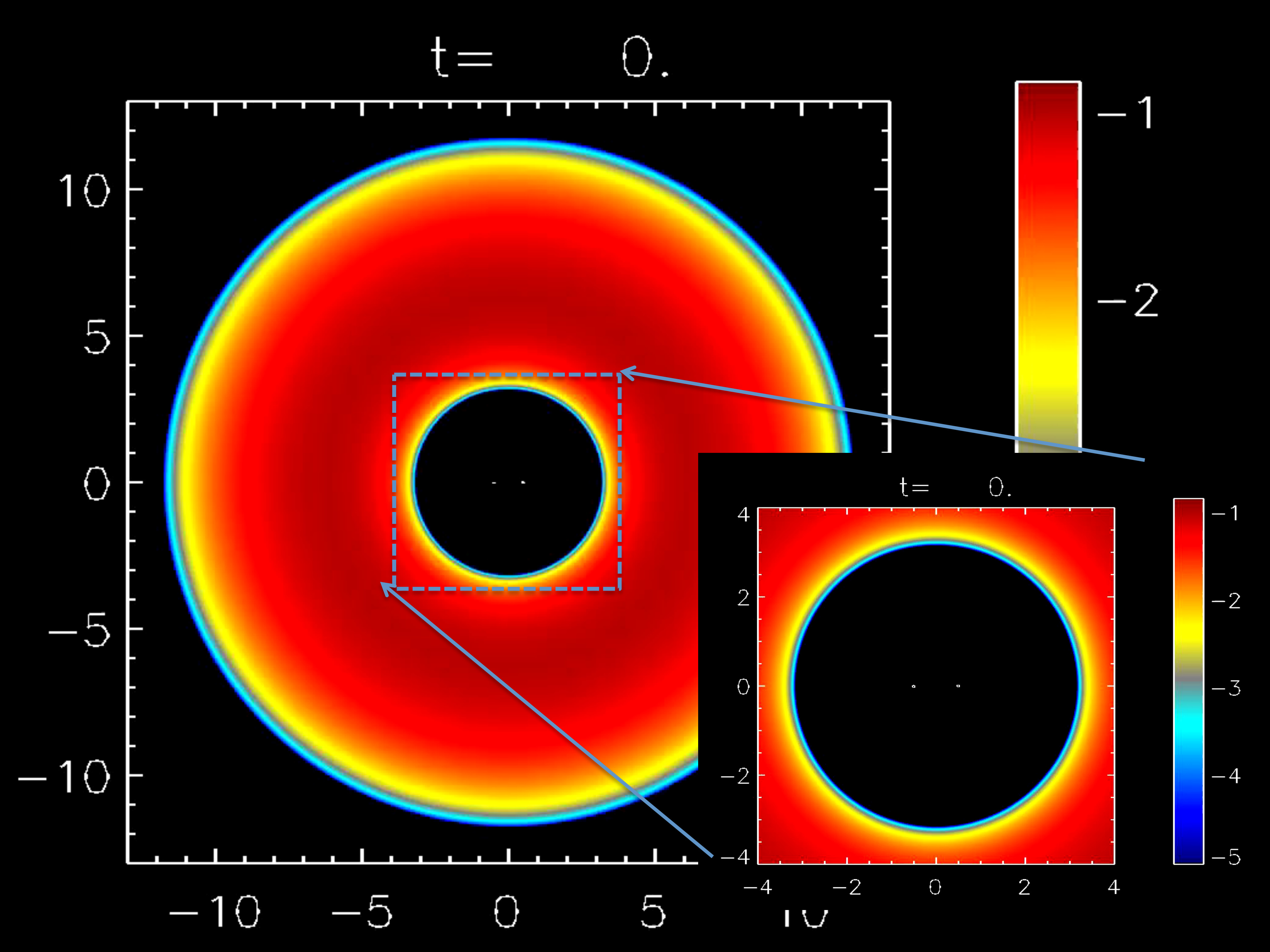
- Interesting dynamics, enhanced by BH spins[Van Meter+09]
- Double Jets [Palenzuela+10; Palenzuela+11]
- Enhanced Accreting Streams near BHs and correlation EM/ GW signals [Bode+10; Farris +10, Farris+11, Giacomazzo +12]

Black Hole Accretion Disk Anatomy

Radiatively Efficient Geometrically Thin Accretion Disk [Noble++,2009]

- Cool to constant entropy
- Thin Disk, $H/r \cong 0.1$
- Poloidal Magnetic Field following density contours
- GR-MHD grid code, based on spherical coordinates, HARM3D [Noble++,2009]

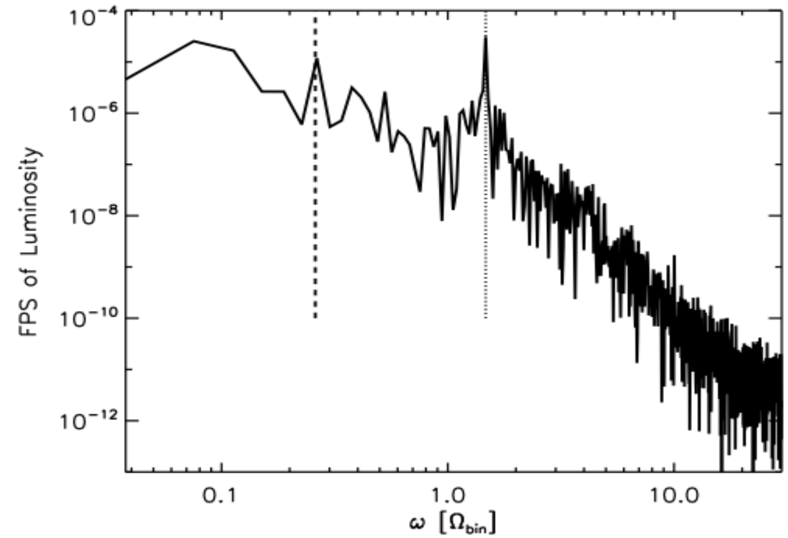




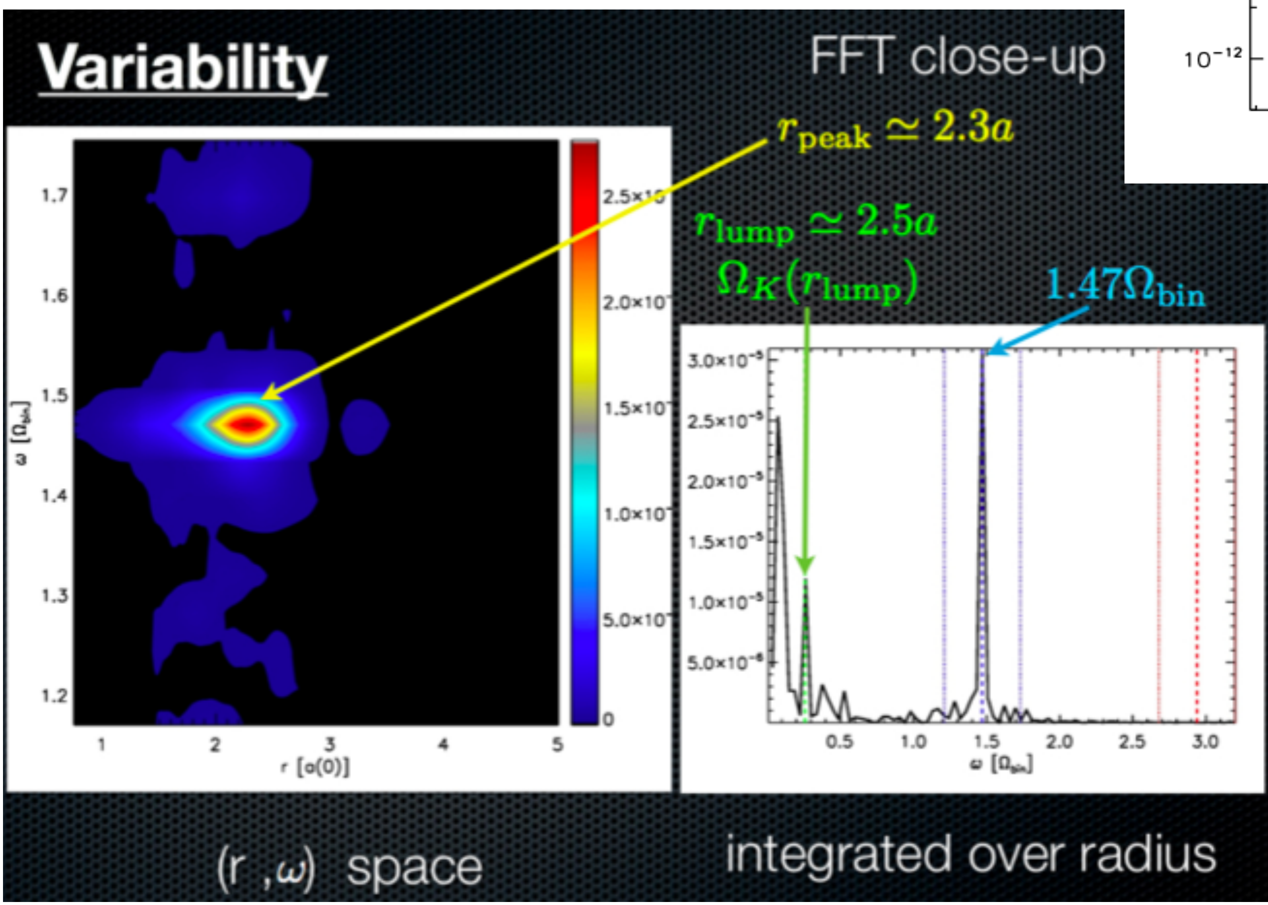
t= 0.
10
5
0
−5
−10
−10
−5
0
5
10
−1
−2
t= 0.
4
2
0
−2
−4
−4
−2
0
2
4
−1
−2
−3
−4
−5

The Lump and its Variability

- Luminosity characteristic of AGN (near Eddington, UV)
- Modulation at a frequency (ω_{peak}) that is a beat between the orbital frequency of the disk's surface density maximum (the lump) and the binary orbital frequency



Lump-BH streams Modulation



$$\omega_{peak} = 2\,(\Omega_{bin} - \Omega_{lump})$$

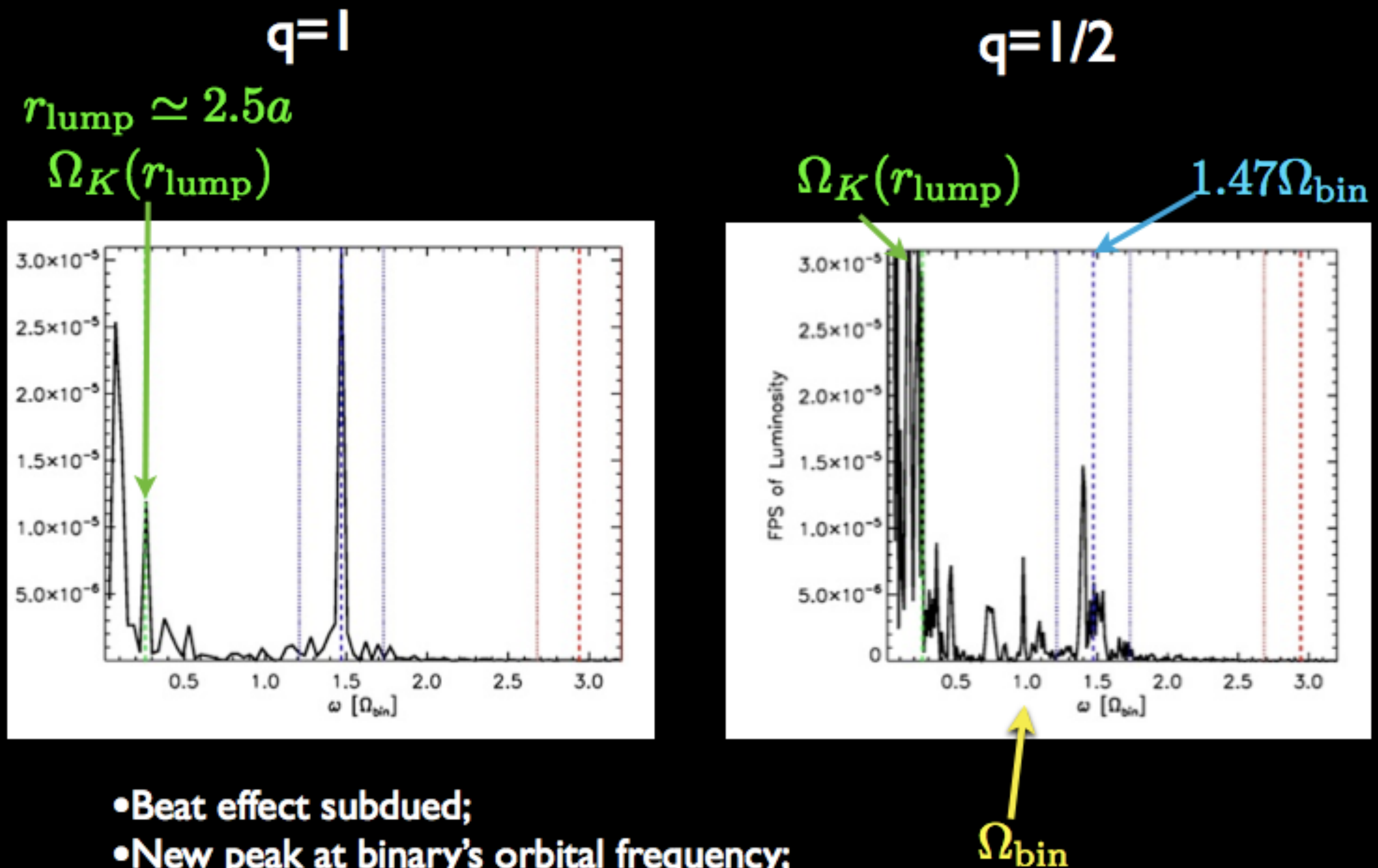
??

$$1 < \frac{\omega_{peak}}{(\Omega_{bin} - \Omega_{lump})} < 2$$

$$0 < \frac{M_2}{M_1} < 1$$

(Noble + 12, arXiv:1204.1073v1)

Variability with Mass Ratio

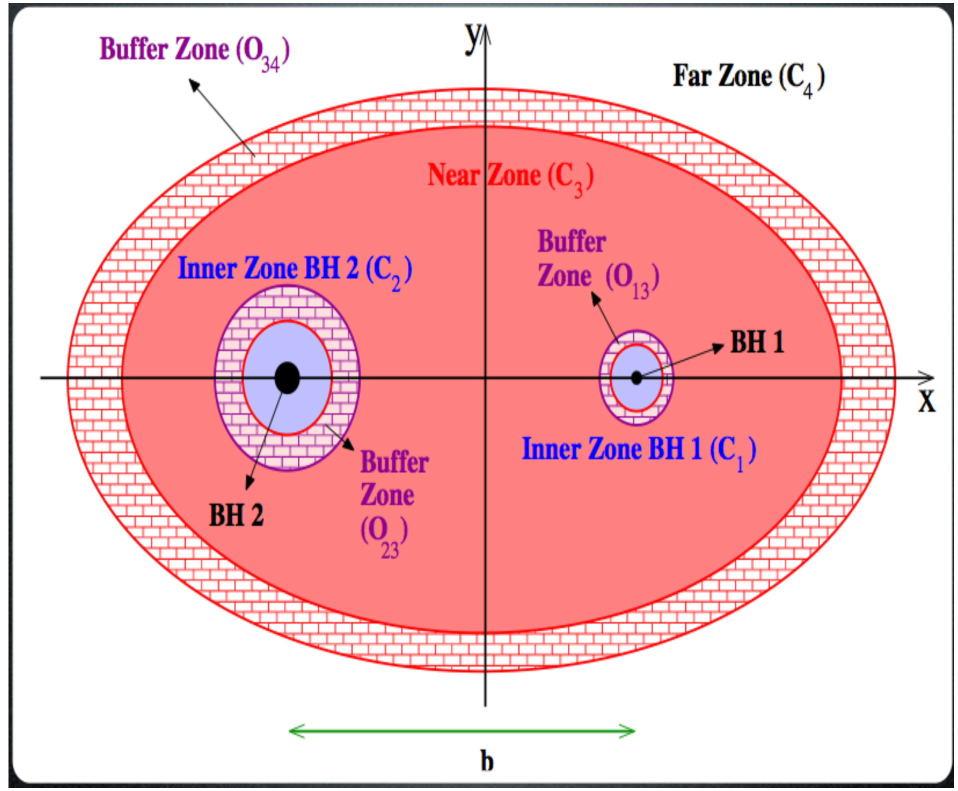


- Beat effect subdued;
- New peak at binary's orbital frequency;
- More variability on lump's orbital timescale;

A Global Approximate Two Black Hole Spacetime

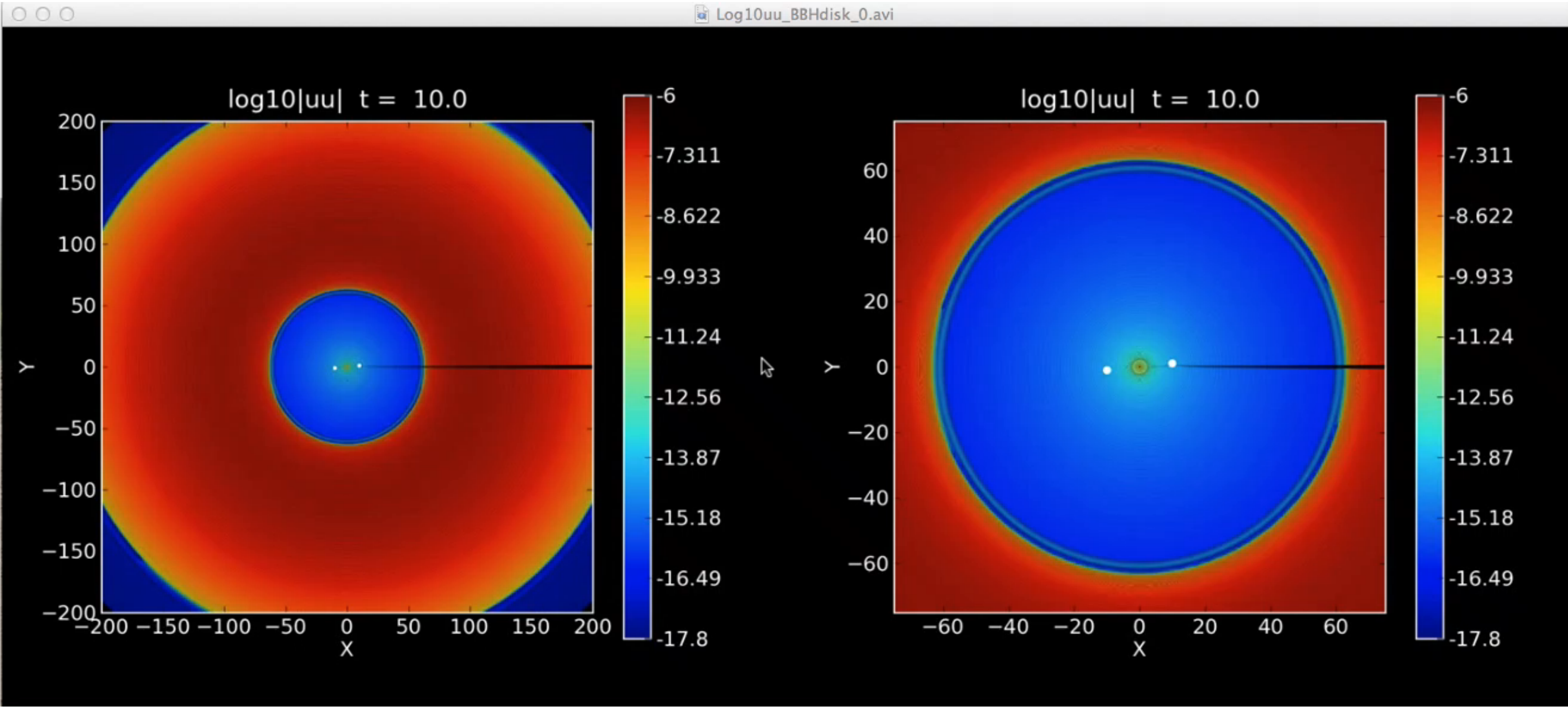
Mundim, Nakano, MC, Yunes, Noble & Zlochower, 2013

- Global, close-form, spacetime required for long-term MHD dynamical evolutions of circumbinary disks around BBH inspirals, to study the behavior of highly relativistic matter near each BH.
- Solve Einstein's Eqs approximately, perturbatively, in three different regions of the spacetime:
  - Inner-Zone (Kerr perturbations)
  - Near-Zone (Post-Newtonian)
  - Far-Zone (Post-Minkowskian)
- Joined via Asymptotic matching using of suitable Buffer-Zones regions
- Physically valid up until the last few orbits prior to merger (separation ~ 10M).



- Evolve BBH with 3.5 PN equations of motions

Simulations with Two Black Hole Spacetime on the Grid



We are now ready to start answering the question of whether or not there is any gas present, and if so, what are its properties

Summary and Conclusions

- BBH mergers are excellent laboratories for testing strong-field GR and are ideal sources for any GW detector.

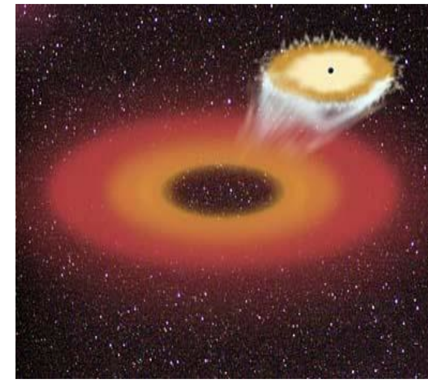
- NR calculations have already made some amazing predictions:

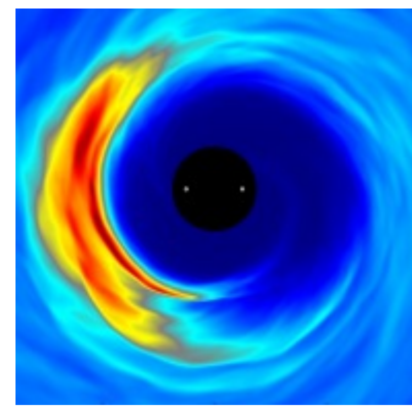
  - BBH mergers radiate up to 10% of total mass (depending spin). Many efforts to calculate waveforms from generic BBH binaries underway, including extreme BBH cases.

  - BBH merger remnants can recoil at up to 5 000 km/s ➔ astronomical recoils candidates

  - There could be enhanced, distinguishable, light signatures due to MHD accretion in strong dynamical GR (characteristic variability, jet production, etc).

  - Multimessenger astronomy is at the door!



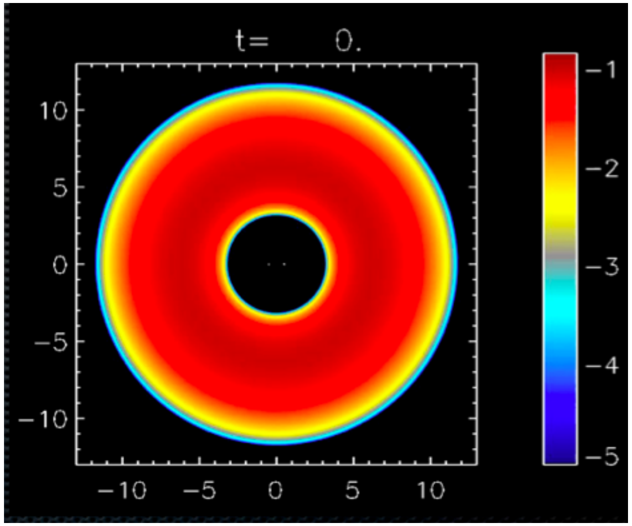


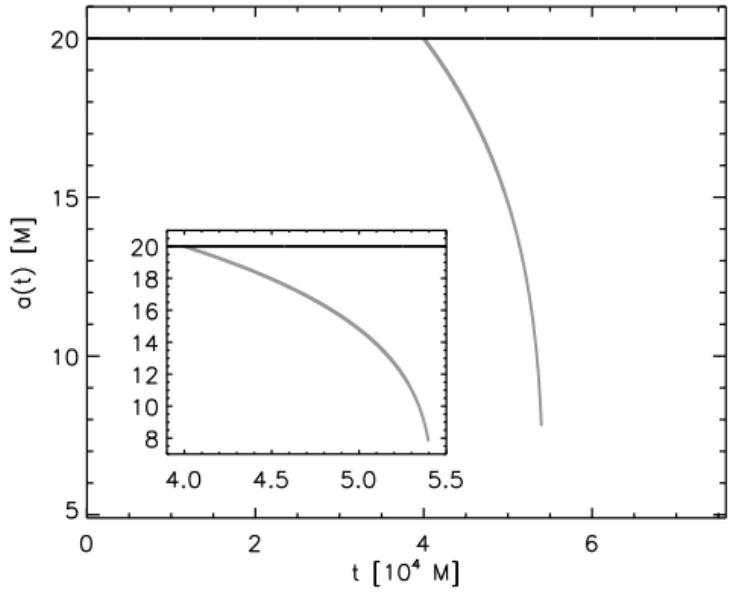
Acknowledgements: NSF (PHY-0722315, PHY-0653303, PHY-0714388, PHY-0722703, OCI-0832606, PHY-0903782, PHY-0969855, AST-1028087), NASA ATPF (07-ATFP07-0158, 08-ATFP08-0093)

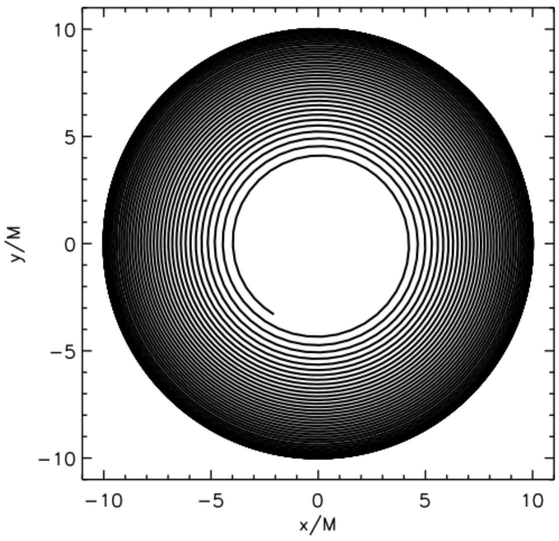
Circumbinary MHD Accretion into Inspiraling BBHs

Noble, Mundim, Nakano, Krolik, MC, Zlochower, Yunes, arXiv:1204.1073v1

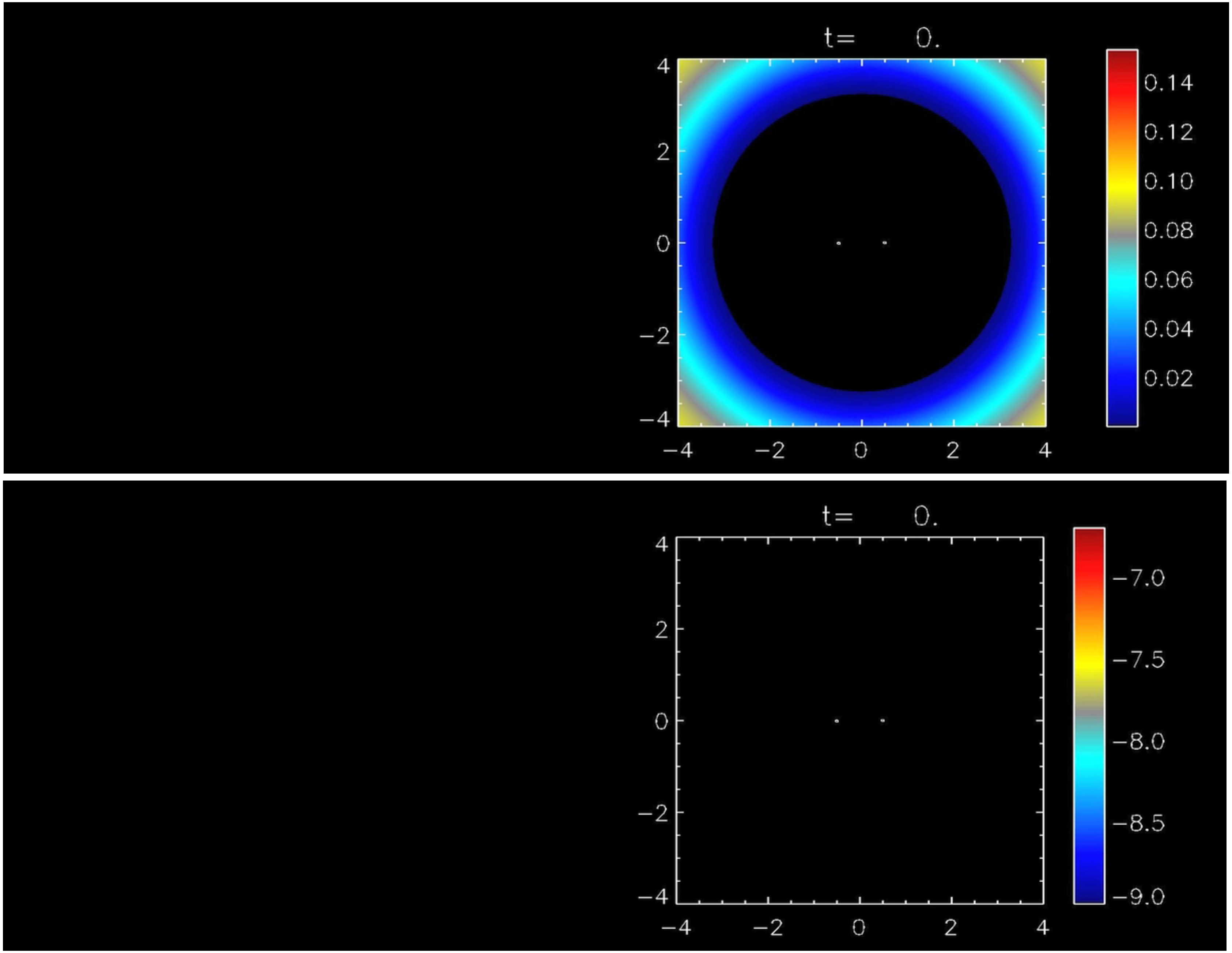
- Radiatively Efficient Geometrically Thin Accretion Disk
  - Cool to constant entropy, H/r=0.1, r=[3,10]a_0
  - Poloidal Magnetic Field following density contours
  - GRMHD code: Harm3D [Noble++,2009]
- Evolve 3.5 PN equation of motion evolution for 127 orbits
  - Initial Study M1=M2, BHs not in the grid
  - RunIN: keep binary at fixed separation (a_0 = 20M) until t = 40,000M, and then inspiral down to 8M.
  - RunSS: keep binary at fixed separation (a_0 = 20M) until t = 75,000M







Inspiral (RunIN) | Quasi Steady-State (RunSS)

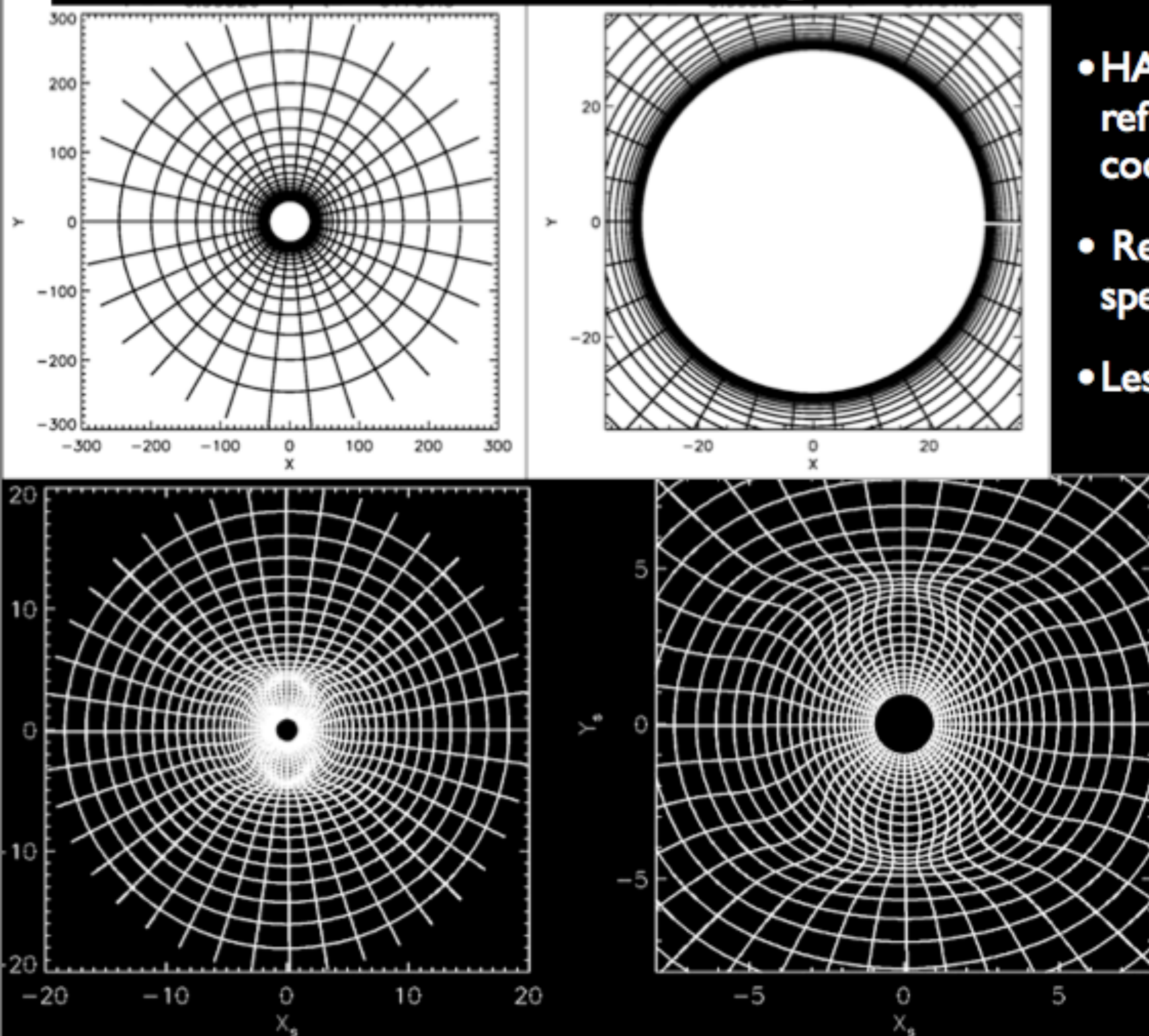


Surface Density (Linear)

Surface Brightness (Log_{10})

Movies by Scott Noble (adpated by HP. Bischof): http://ccrg.rit.edu/~scn/cmhdaiibh/

Dynamic Coordinates to Resolve Binary Black Holes on Shrinking Orbits



- HARM3D is a fixed mesh refinement GRMHD code;
- Refinement through special gridding;
- Less overhead than AMR;

The Moving Punctures Approach

Modified BSSN system (vaccum):

$$\partial_0 = \partial_t - \mathcal{L}_\beta,$$
$$\tilde{\Gamma}^i = -\partial_j \tilde{\gamma}^{ij}.$$

$$\begin{aligned}
\partial_0 \tilde{\gamma}_{ij} &= -2\alpha \tilde{A}_{ij}, \\
\partial_t \chi &= \frac{2}{3}\chi\left(\alpha K - \partial_a \beta^a\right) + \beta^i \partial_i \chi, \\
\partial_0 \tilde{A}_{ij} &= \chi\left(-D_i D_j \alpha + \alpha R_{ij}\right)^{TF} + \\
&\quad \alpha\left(K \tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j\right), \\
\partial_0 K &= -D^i D_i \alpha + \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right), \\
\partial_t \tilde{\Gamma}^i &= \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \\
&\quad \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - 2\tilde{A}^{ij}\partial_j\alpha + \\
&\quad 2\alpha\left(\tilde{\Gamma}^i{}_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\phi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K\right),
\end{aligned}$$

Dynamical Gauge:

Replace ϕ $(O(\log r))$ with $\chi = e^{-4\phi}$ $(O(r^4))$

$\partial_0 \alpha = -2\alpha K$

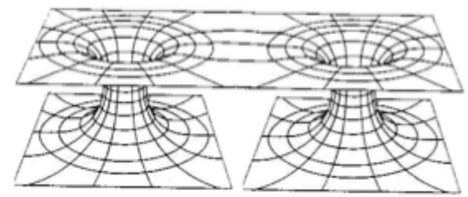
$\partial_t \beta^a = B^a$, $\partial_t B^a = 3/4\partial_t \tilde{\Gamma}^a - \eta B^a$

$\alpha(t=0) = \psi_{BL}^{-2} \qquad \beta^i = B^i = 0.$

Punctures

- Key idea of the puncture approach (Brandt & Bruegmann, 1997):
  - Use Brill-Lindquist two-sheeted topology to represent BHs (at t=0)
  - Factor out the singular part of ψ via following ansatz for N BHs

$$\psi = \psi_{BL} + u, \quad \psi_{BL} = 1 + \sum_{i=1}^{N} \frac{m_i}{2|\vec{r} - \vec{r_i}|}$$



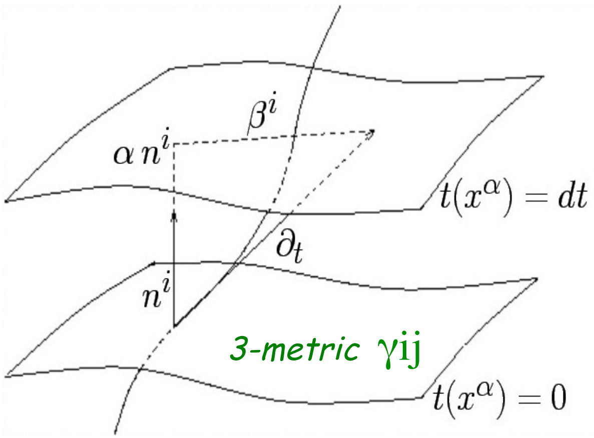
m_i = *BH* bare masses and r_i = *BH* locations

  - Solve numerically the HC for *u* everywhere on R^3 with N "punctures" removed (no inner boundaries at r=r), where $\tilde{A}_{ij}$ is the BY extrinsic curvature

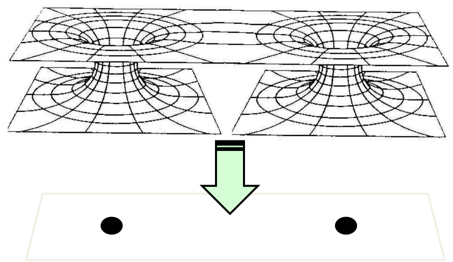
$$D^2_{flat} u + \eta \left(1 + \frac{u}{\psi_{BL}}\right)^{-7} = 0, \quad \eta = \frac{1}{8\psi^7_{BL}} \tilde{A}_{ij}\tilde{A}^{ij}$$

Some details about the moving puncture simulations …

- Use the 3+1 formulation of GR:
  – spacetime sliced in 3-D (*t* = constant) slices
  – gauge: lapse *α*, shift vector β^i
- Einstein's eqs split into 2 sets:
  – Constraint equations (only spatial derivatives)
  – Evolution equations (time derivatives)



- Set (constrained) initial data at t = 0
  – choose free data (masses, spins, orbital parameters)
  – represent BHs with "punctures" …
  – Solve constraints



- Evolve forward in time, from one slice to the next:
  – solve nonlinear, coupled PDEs for 17 BSSN vars: *g*ij, *A*ij ~ ∂t *g*ij ,, Φ, K, Γ^i
  – with devised gauge conditions to move the punctures across the grid …

- Extract the physics from the data:
  – BHs horizons, masses, linear momenta and spins, …
  – radiation waveform, energy, angular and linear momenta

Waveform:

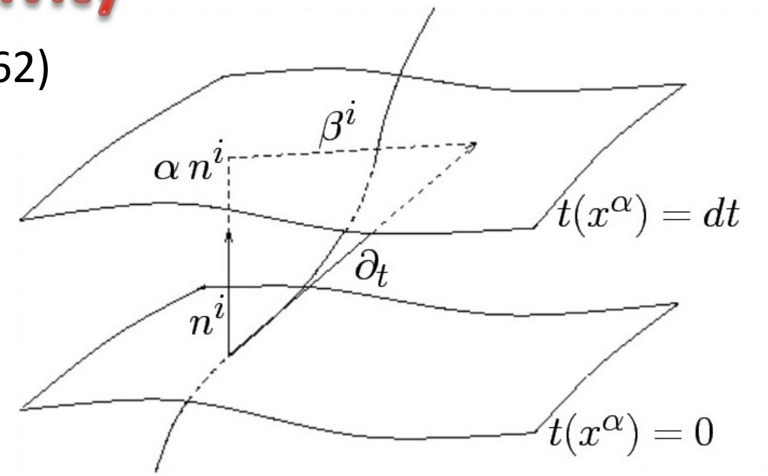
$$\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$$

$$h(t) = \sum_{lm} A_{lm}^{-2} Y_{lm}(\theta,\phi) e^{im\omega t}$$

3+1 Numerical Relativity

(Arnowitt, Deser, Misner, 1962)

• Slice the spacetime $g_{\mu\nu}(t, x_i)$ *metric* into 3-D spacelike (t = const) hypersurfaces (slices)



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$K_{ij} = -\frac{1}{2\alpha}(\partial_t \gamma_{ij} - D_i \beta_j - D_j \beta_i)$$

Einstein's equation split in 2 sets:

$$^{(3)}R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$D_j(K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

} Constraints → Initial data

Gauge:
relate coords on neighboring slices (4 degree of freedom),

α → lapse function
β^i → shift vector

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\begin{aligned} \partial_t K_{ij} &= \beta^k \partial_k K_{ij} + K_{ki}\partial_j \beta^k + K_{kj}\partial_i \beta^k \\ &- D_i D_j \alpha + \alpha[^{(3)}R_{ij} + K K_{ij} - 2K_{ik}K^k_j] \\ &+ 4\pi\alpha[\gamma_{ij}(S-\rho) - 2S_{ij}] \end{aligned}$$

} Evolution equations

$\{\gamma_{ij}, K_{ij}\}$: 12 independent vars = 4 constraints + 8 free quantities (4 dynamical + 4 gauge)

Conformal Transverse-Traceless decomposition

- Key question: The determination of initial data is highly non-trivial due to the constraints, particularly to set "astrophysically realistic" conditions encoded in the choice of the "free data". Which of the 12 $\{\gamma_{ij}, K_{ij}\}$ do we specify freely at the initial time, and which do we determine from the constraints?

- York-Lichnerowicz: $$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$ $$K_{ij} = \psi^{-10} \tilde{A}_{ij} + \frac{1}{3}\gamma_{ij} K \qquad \tilde{A}_{ij} = (\tilde{L}\tilde{V})^{ij} + \tilde{M}^{ij}$$

- Hamiltonian and Momentum Constraints: 4 quasi-linear, coupled elliptic PDEs for the 4 gravitational potentials $\{\psi, \tilde{V}^i\}$ with free data $\{\tilde{\gamma_{ij}}, \tilde{M}_{ij}\}$

HC: $$8\tilde{D}^2\psi - \tilde{R}\psi + \psi^7 \tilde{A}_{ij}\tilde{A}^{ij} - \frac{2}{3}\psi^5 K^2 + 16\pi\psi^5\rho = 0$$

MC: $$\tilde{\Delta}_{\tilde{L}}\tilde{V}^i + \tilde{D}_j\tilde{M}^{ij} - \frac{2}{3}\psi^6\tilde{D}^i K - 8\pi\psi^{10} j^i = 0 \qquad \tilde{\Delta}_{\tilde{L}}\tilde{V}^i = \tilde{D}^2\tilde{V}^2 + \frac{1}{3}\tilde{D}^i\tilde{D}_j\tilde{V}^j + \tilde{R}^i_j\tilde{V}^j$$

For vacuum, conformal flat metric and time symmetric data ($K_{ij}=0$ *at* $t=0$) the MC is trivially satisfied and HC is a Laplace eq.:

$$\tilde{D}^2_{flat}\psi = 0 \quad \longrightarrow \quad \psi = 1 + \frac{const}{r}$$

Cook, 2004: http://relativity.livingreviews.org/Articles/lrr-2004-5/

Bowen-York Initial Data

- Vacuum: $\rho = j^i = 0$
- Conformal flat metric: $\tilde{\gamma}_{ij} = \delta_{ij}$
- Maximal slicing: $K = 0$
- "Minimal radiation": $\tilde{M}_{ij} = 0$

HC: $\tilde{D}^2\psi + \frac{1}{8}(\tilde{L}\tilde{V})_{ij}(\tilde{L}\tilde{V})^{ij}\psi^7 = 0$

MC: $\tilde{\Delta}_{\tilde{L}}\tilde{V}^i = 0$ (linear)

- The MC can be solved analytically (Bowen & York 1980) to produce BH data with given P^i = linear mom. and S^j = ang. mom. (with clear physical interpretation at ∞)

$$\tilde{V}^i = \frac{1}{4r}[7P^i + n^i n_j P^j] + \frac{1}{r^2}\epsilon^{ijk} n_j S_k$$

→ BY extrinsic curvature K_{ij}

$$P^i_{ADM} = \frac{1}{8\pi}\lim_{r\to\infty}\oint_\sigma (K^i_j - \delta^i_j K)d\sigma^j$$

$$J^i_{ADM} = \frac{1}{8\pi}\lim_{r\to\infty}\oint_\sigma \epsilon^{ijk} x_j K_{kl} d\sigma^l$$

$$d\sigma^i = n^i dA$$

- These solutions can then be superimposed to generate solutions of MC representing multiple holes

- The HC must then be solved numerically, and one must deal with singular behavior of ψ as $r \to 0$

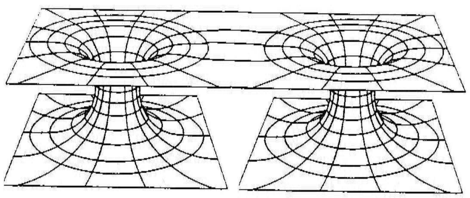
Cook, 2004: http://relativity.livingreviews.org/Articles/lrr-2004-5/

Puncture Initial Data

- Traditional BY approach (Cook, 1994) introduced inner boundaries at r_i around each hole and inversion symmetry which require black-hole excision, but in context of finite difference methods, this is a complication …

- Key idea of the puncture approach (Brandt & Bruegmann, 1997):
  - Use Brill-Lindquist two-sheeted topology to represent BHs (at t=0)
  - Factor out the singular part of ψ via following ansatz for N BHs

$$\psi = \psi_{BL} + u, \quad \psi_{BL} = 1 + \sum_{i=1}^{N} \frac{m_i}{2|\vec{r} - \vec{r_i}|}$$



m_i = *BH* bare masses and r_i = *BH* locations

  - Solve numerically the HC for u everywhere on R^3 with N "punctures" removed (no inner boundaries at r=r), where $\tilde{A}_{ij}$ is the BY extrinsic curvature

$$D^2_{flat} u + \eta \left(1 + \frac{u}{\psi_{BL}}\right)^{-7} = 0, \quad \eta = \frac{1}{8\psi^7_{BL}} \tilde{A}_{ij} \tilde{A}^{ij}$$

- Technique has become very popular, primarily due to its ease of implementation in 3D Cartesian coordinates codes