FROM PERTURBATION TO OBSERVATION: MEASURING THE RESPONSE OF NEUTRON STARS

JOCELYN READ

Need GR to calculate structure of neutron stars since they're so dense.

Equation of state: described by fluid equation. Density goes up, but we don't know which is correct. On outside, neutrons, protons, electrons, but inside, where it's more dense, it could be quarks. But for this talk, it's just an unknown function we want.

We get a family of stars, the radius and masses, for each model equation of state. Can go back and forth between the two.

We do some perturbations for GR, can only been done when they're far away from each other. λ is a constant, somewhat like the love number.

We can then relate tidal deformability with equations of state.

More tidal effects mean they fall faster, so bigger (i.e. higher radius) (since more tides) means inspiral quicker.

Post-Newtonian approximation doesn't work very well for late times of inspiral, so this only works for a while. So, then we have to do full inspiral numerically.

Less compact versions of equations of state have this hyper-massive object that sits there for a while, while more dense ones immediately collapse to BHs.

Frequency of peak amplitude has tight dependence on perturbative constant. That's odd.

Consider space of signals, parameterized by something like a line (actually 7 parameters). We have a true signal, and a measured signal (thanks to noise). Define a measured loudness. How well can we distinguish a signal from nothing? Define an inner product. Use that to define a norm. We say things are distinguishable from noise if the norm is bigger than 1. In reality want more.

You fit the observed to the closest parameters, but there're errors, of course, because of noise. May not be linear scaling of error with linear amount of noise. There's also the fact that our model waveform family is different from the true waveform family.

The spikes in sensitivity are resonant modes of the equipment.

From perturbation to observation: measuring the response of neutron stars

Jocelyn Read
CSUF

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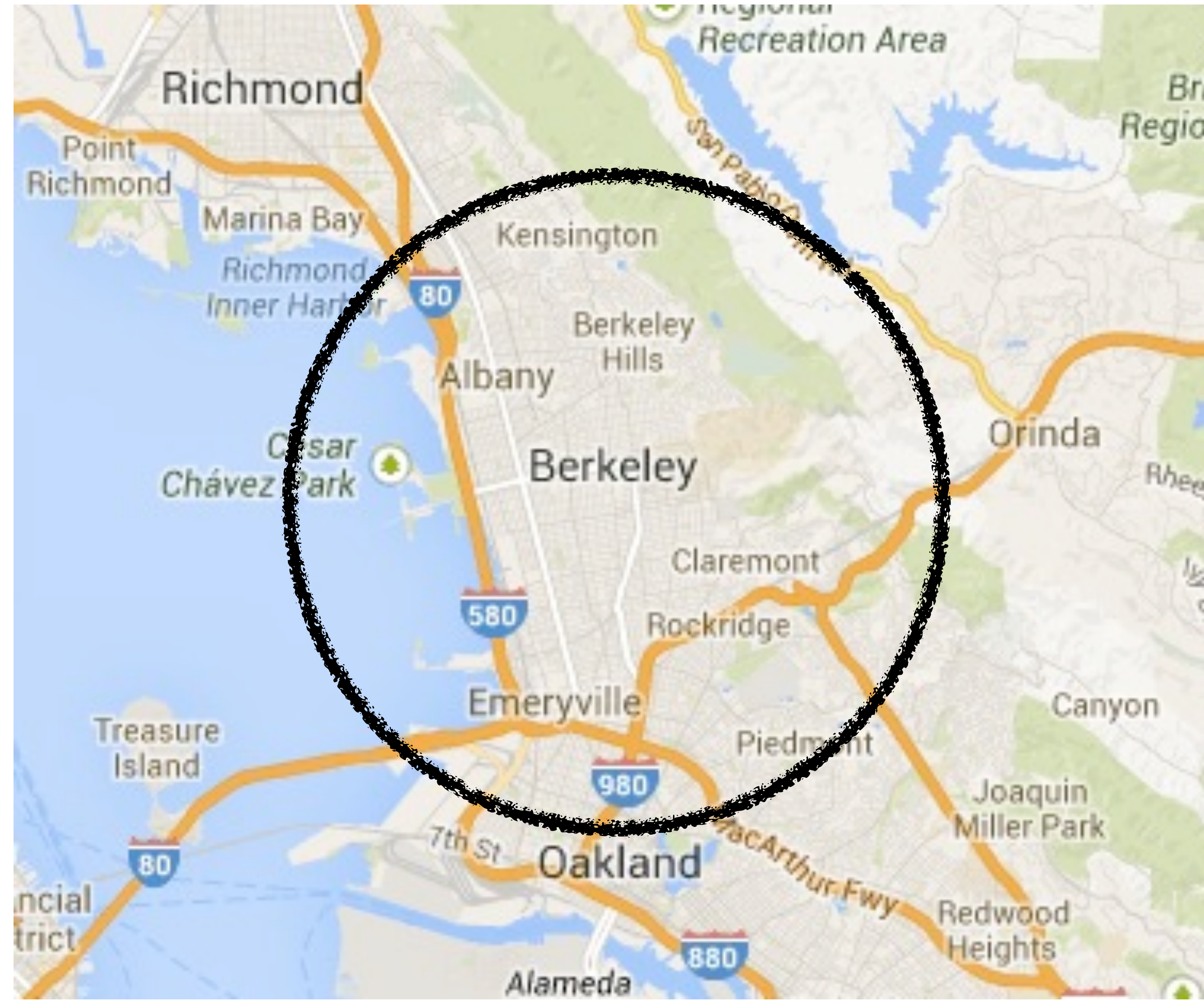
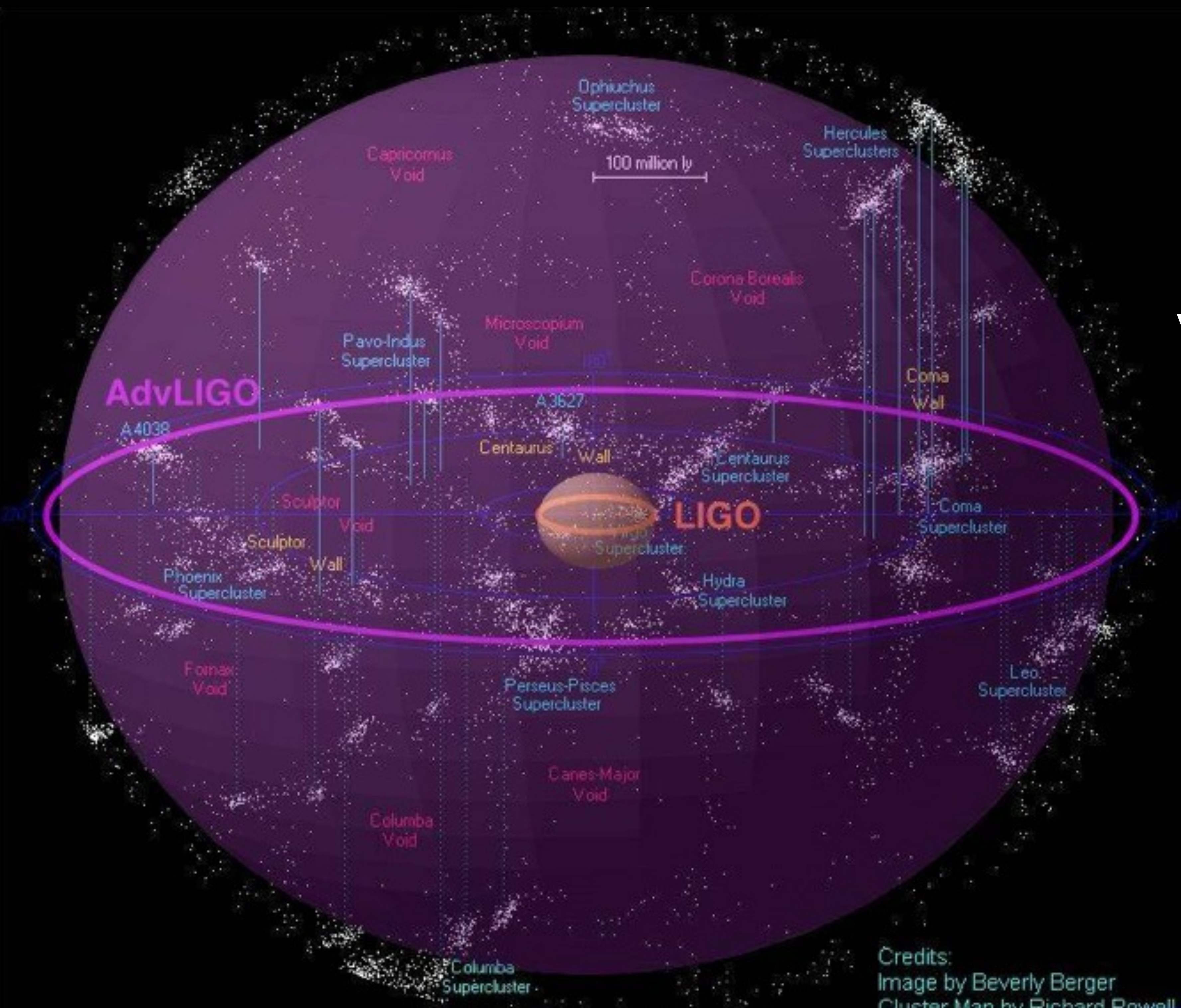
Outline

- Neutron stars
- Inspiral effects and characteristic parameters
- Numerical simulation of merger
- Gravitational wave detection and parameter estimation
- Measurements in Advanced LIGO and impact of current model uncertainties

Neutron stars

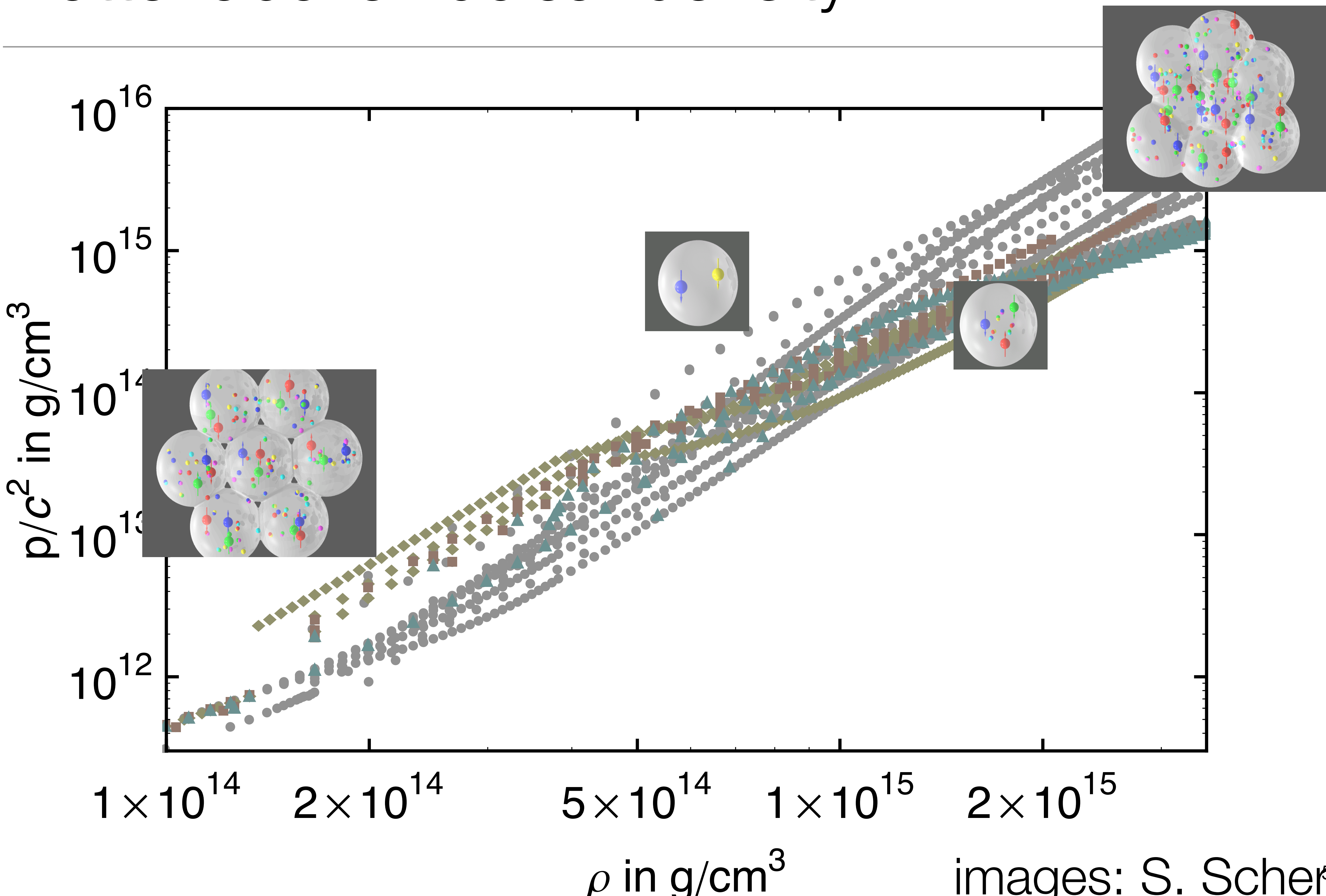
- 1-2 solar masses
- ~10-15 km radius
- average density of matter higher than nuclear density
- Observed: radio pulsars, isolated blackbody emission, high-energy flares
  - Challenging to determine exact size





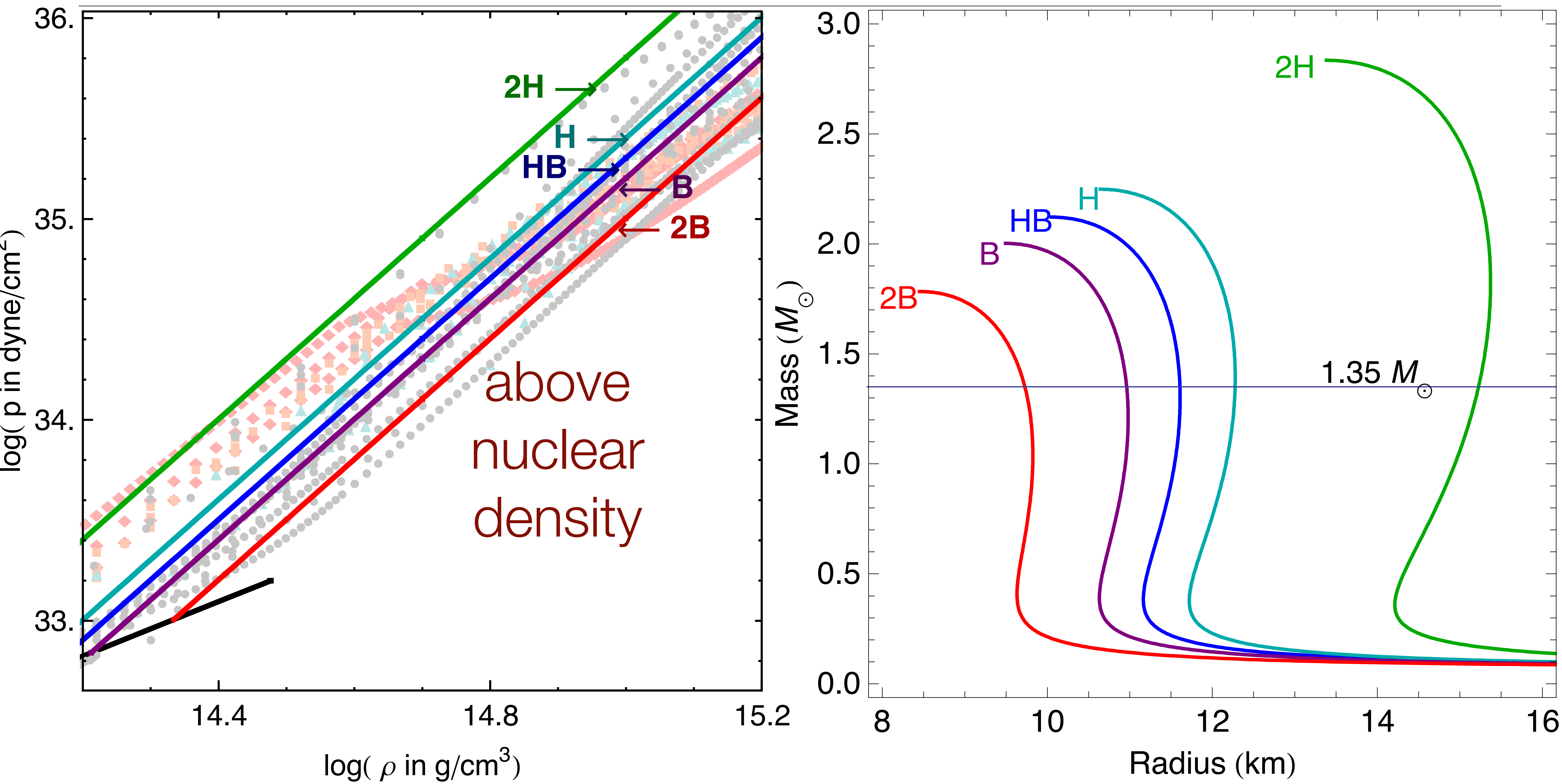
Advanced LIGO/ VIRGO expects to see ~40 NS-NS coalescences each year (0.4 to 400) http://arxiv.org/abs/1003.2480v2

Matter above nuclear density?



images: S. Scherer

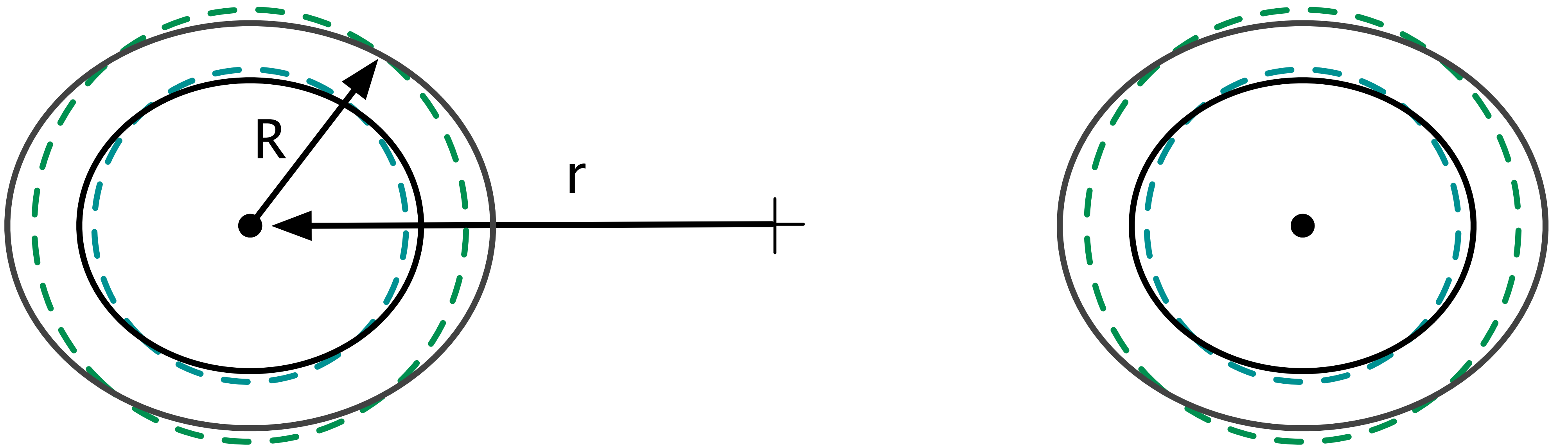
Equation of state ↔ radius(mass)



Astrophysical question: what is NS EOS?

EOS in inspiral: the contribution of tides

Consider two extended bodies in orbit or free-fall:



Residual gravitational effect is tidal deformation.

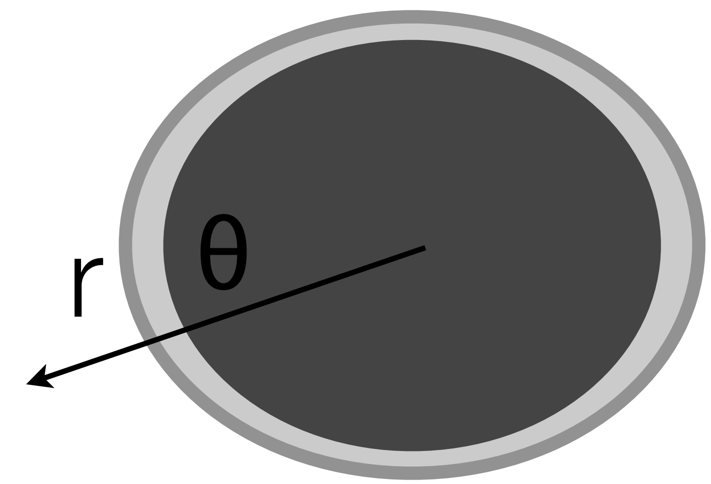
Amount of deformation depends on size and matter properties.

Deformations induce changes in the gravitational potential.

Newtonian Love number k

Love number k determines quadrupole moment Q of deformed body

$$Q = km\frac{R^5}{a^3}$$



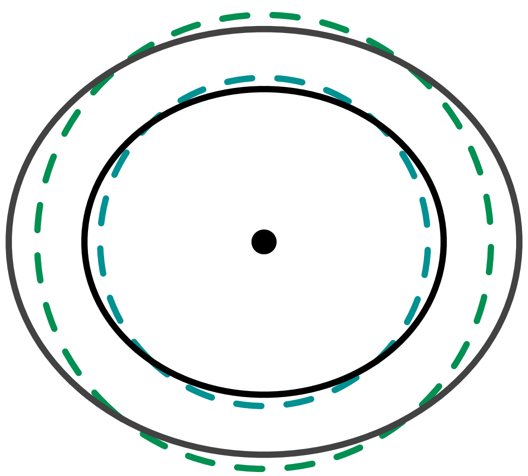
Q determines the gravitational potential around the deformed body

$$U = -\frac{M}{r} - \frac{3}{2}\frac{Q(\cos^2\theta - 1)}{r^3}$$

e.g. Moon deformed
M mass of moon
m mass of Earth
R radius of moon
a distance to Earth

This tells us about things like satellite movement around the body and tidal locking

Calculate in GR:



Perturb a spherically symmetric neutron star
impose Y_{20} angular dependence

(0711.2420, 0906.1366, 0906.0096)

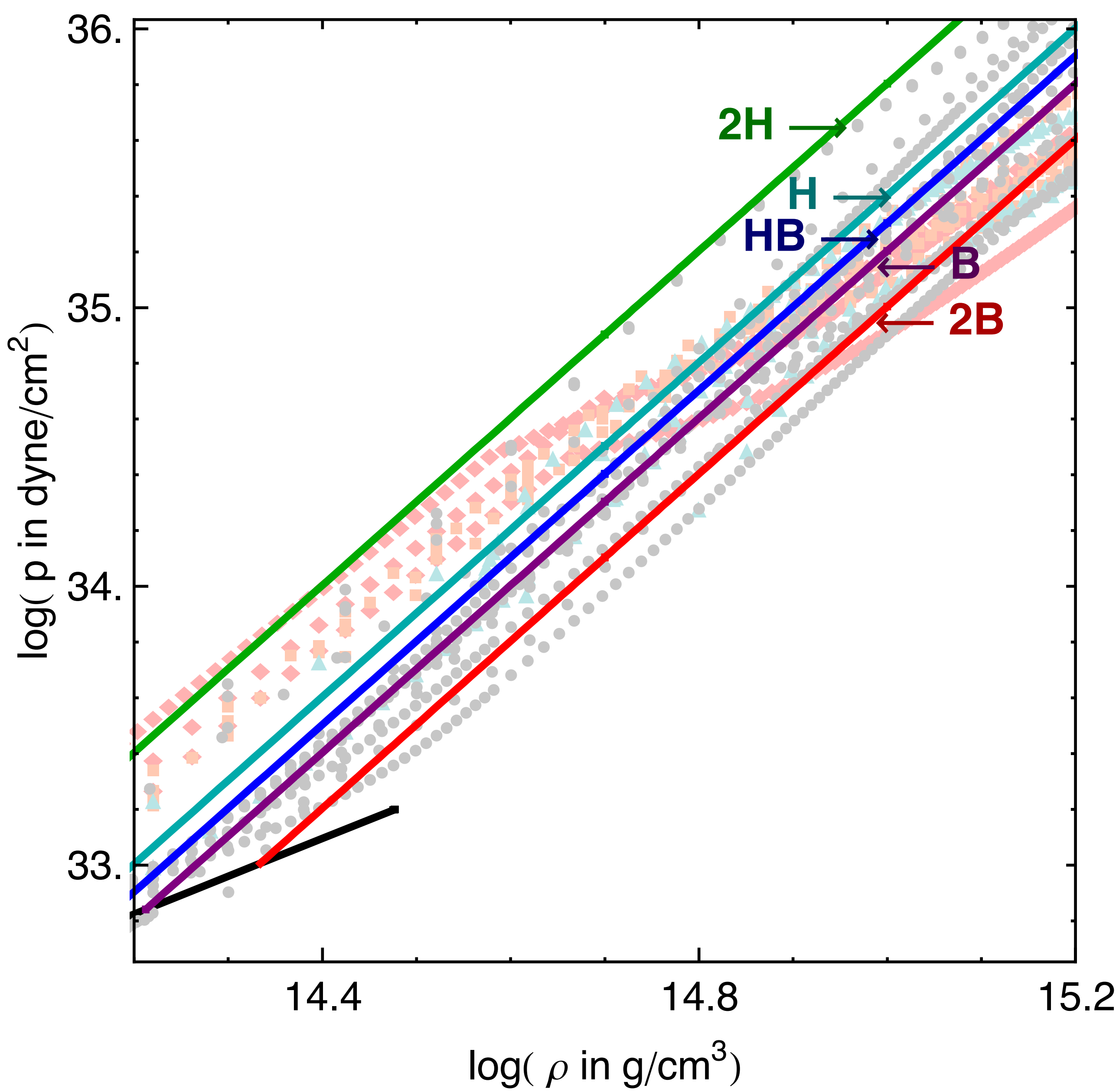
$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}} \quad \begin{matrix} (\sim r^{-3}) \\ (\sim r^{2}) \end{matrix}$$

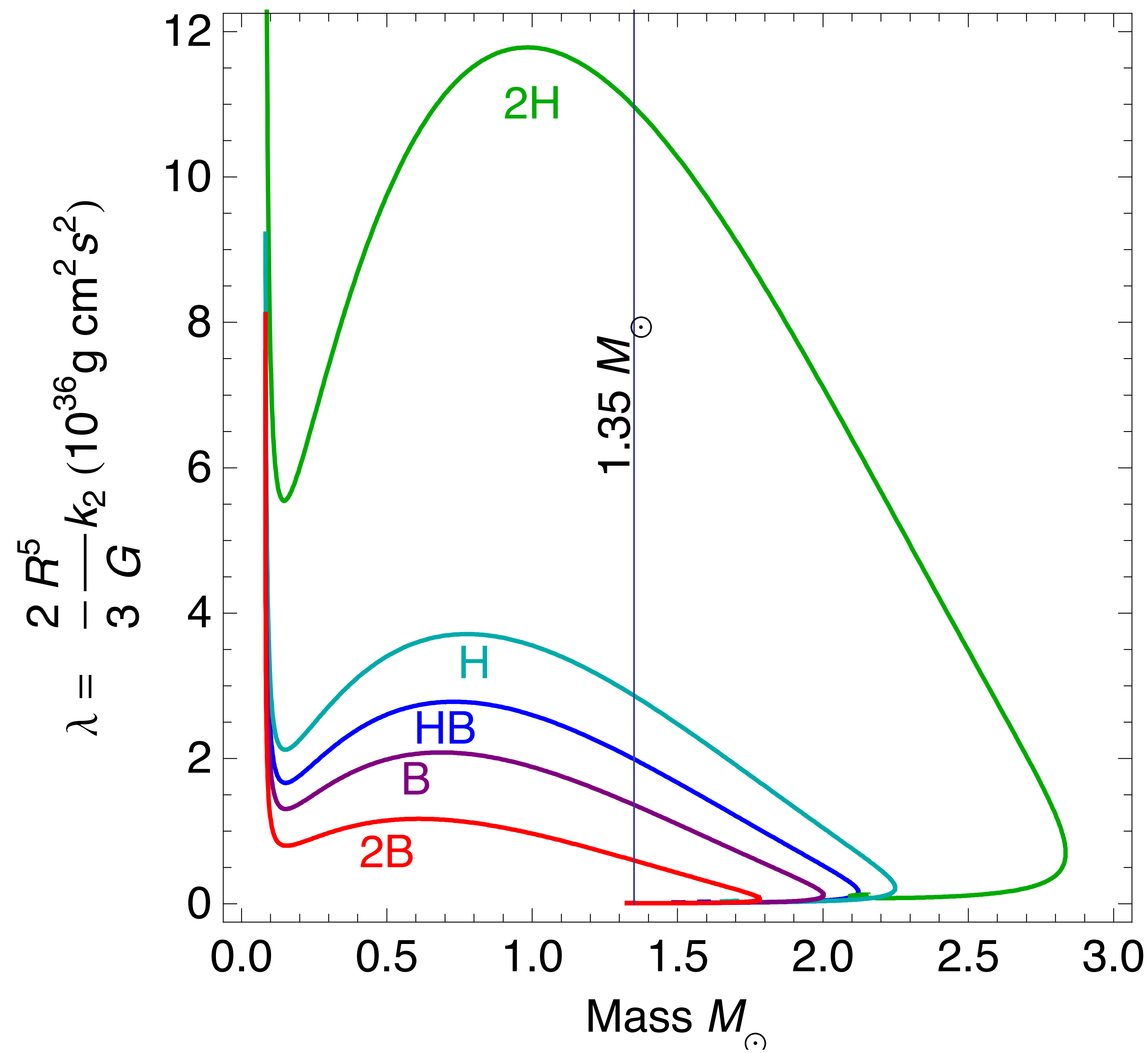
Love number k_2

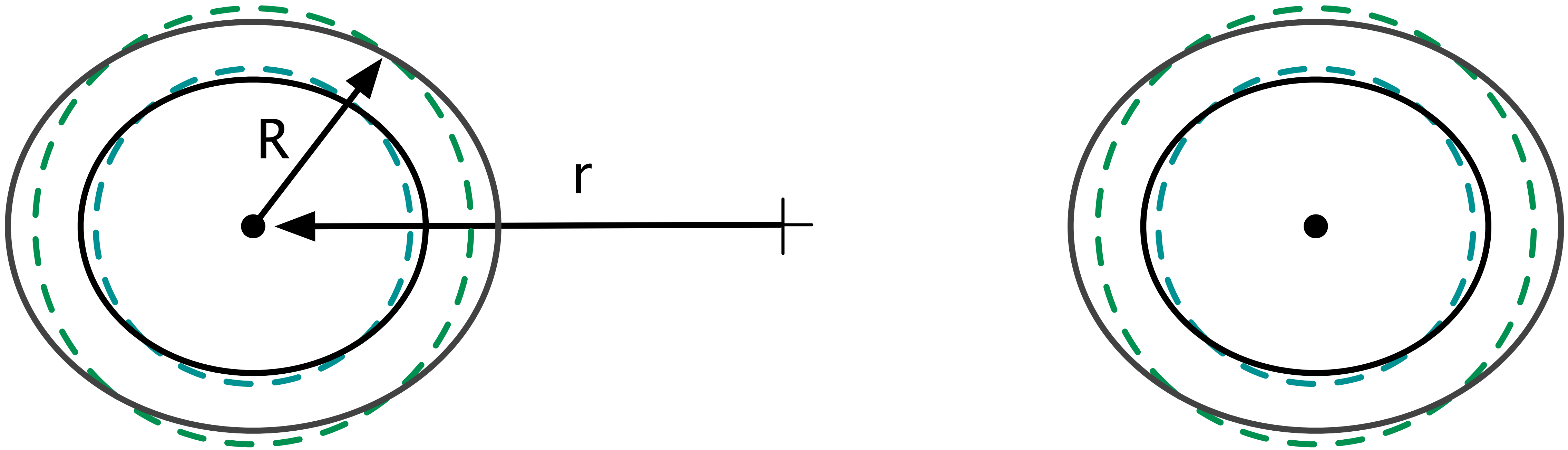
Radius R

$$\lambda = \frac{2}{3} k_2 R^5 \quad (G = c = 1)$$

Equation of state determines λ(mass)







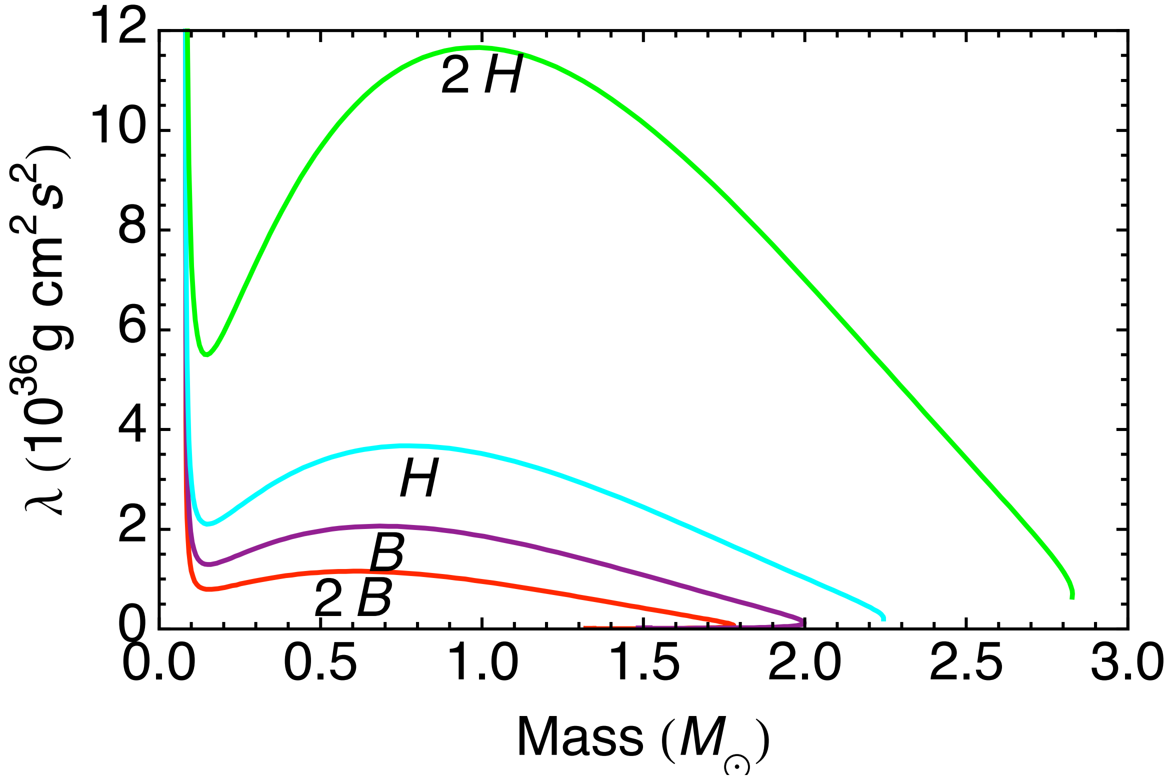
Early tidal interactions:

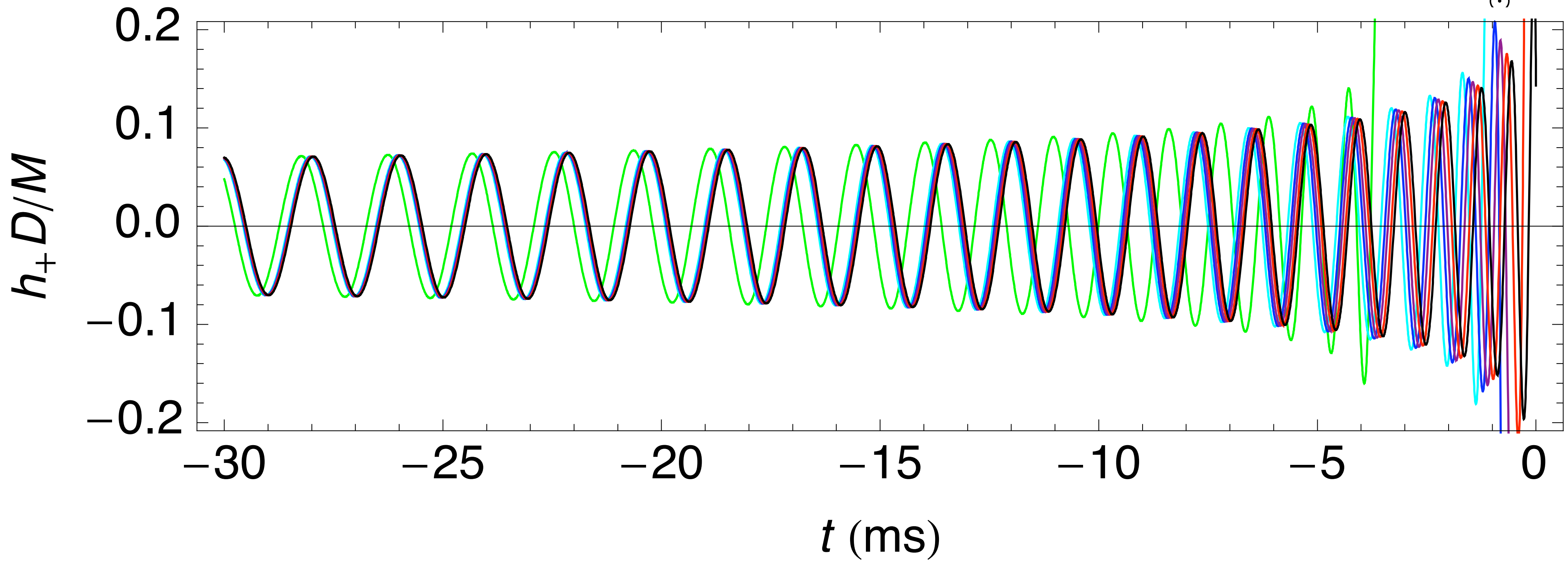
Some energy goes into deforming the stars

Moving tidal bulges add a bit to the gravitational radiation

Effect on waveforms in PN approximation

Newtonian and 1PN tides contribute to waveform at 5 and 6 PN
Vines et al (arXiv:1101.1673)



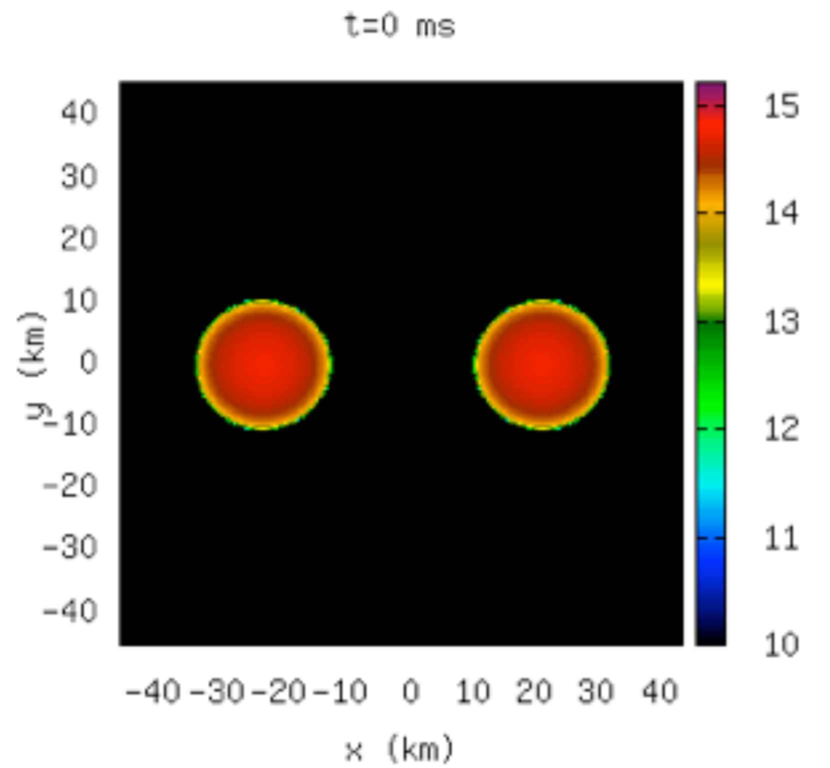


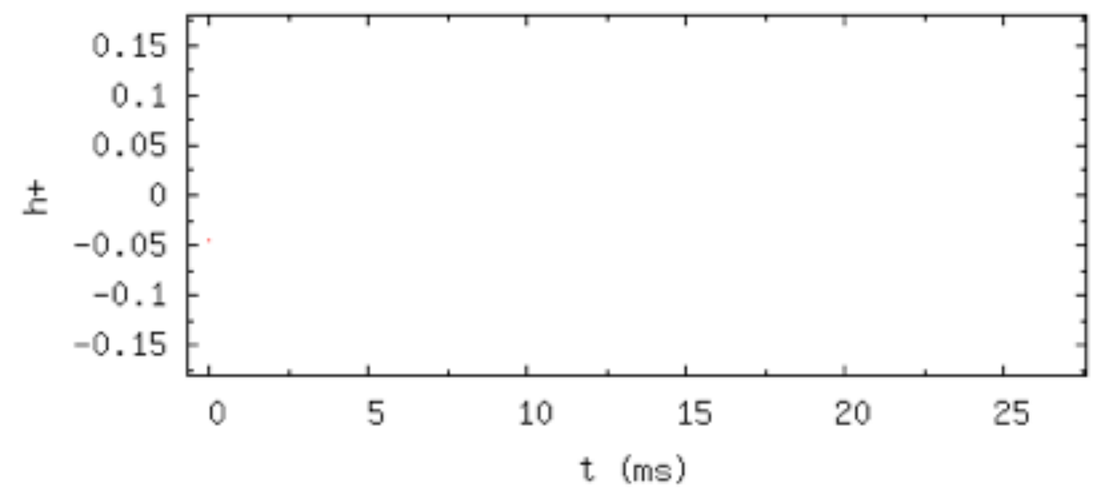
Impact on waveforms from numerical simulations

Similar simulation/movies by K Hotokezaka,

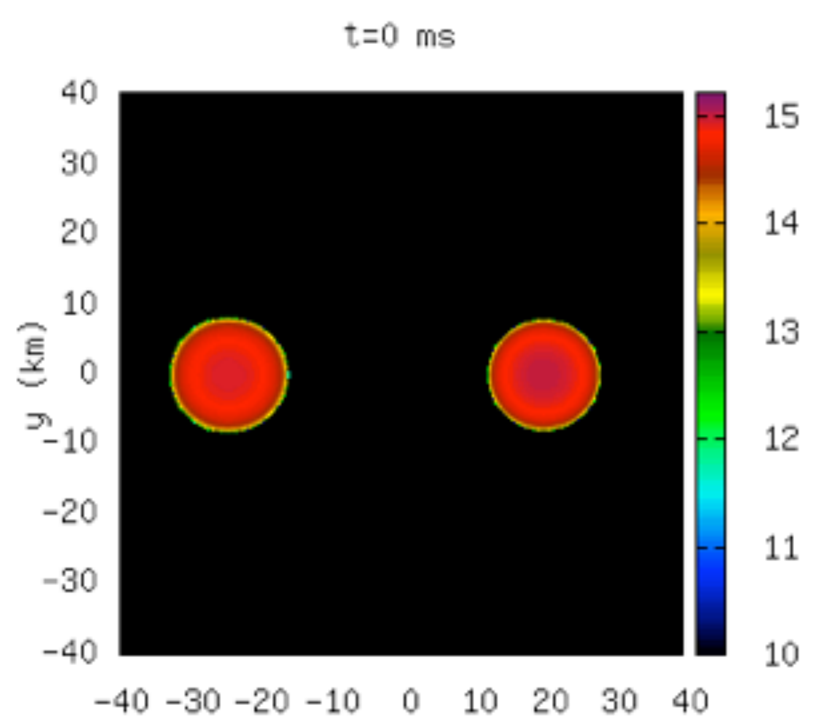
http://www2-tap.scphys.kyoto-u.ac.jp/~hotoke/research.html

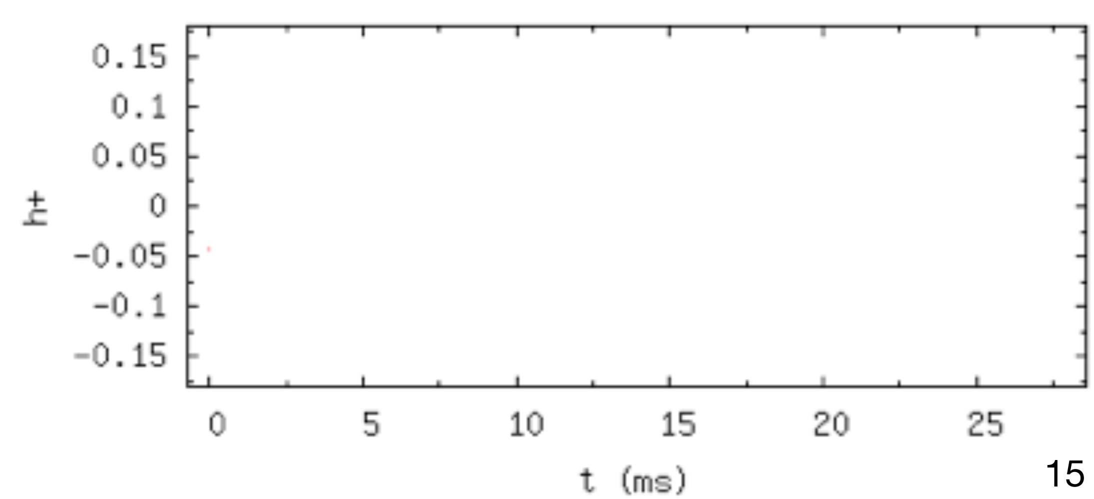
Large stars





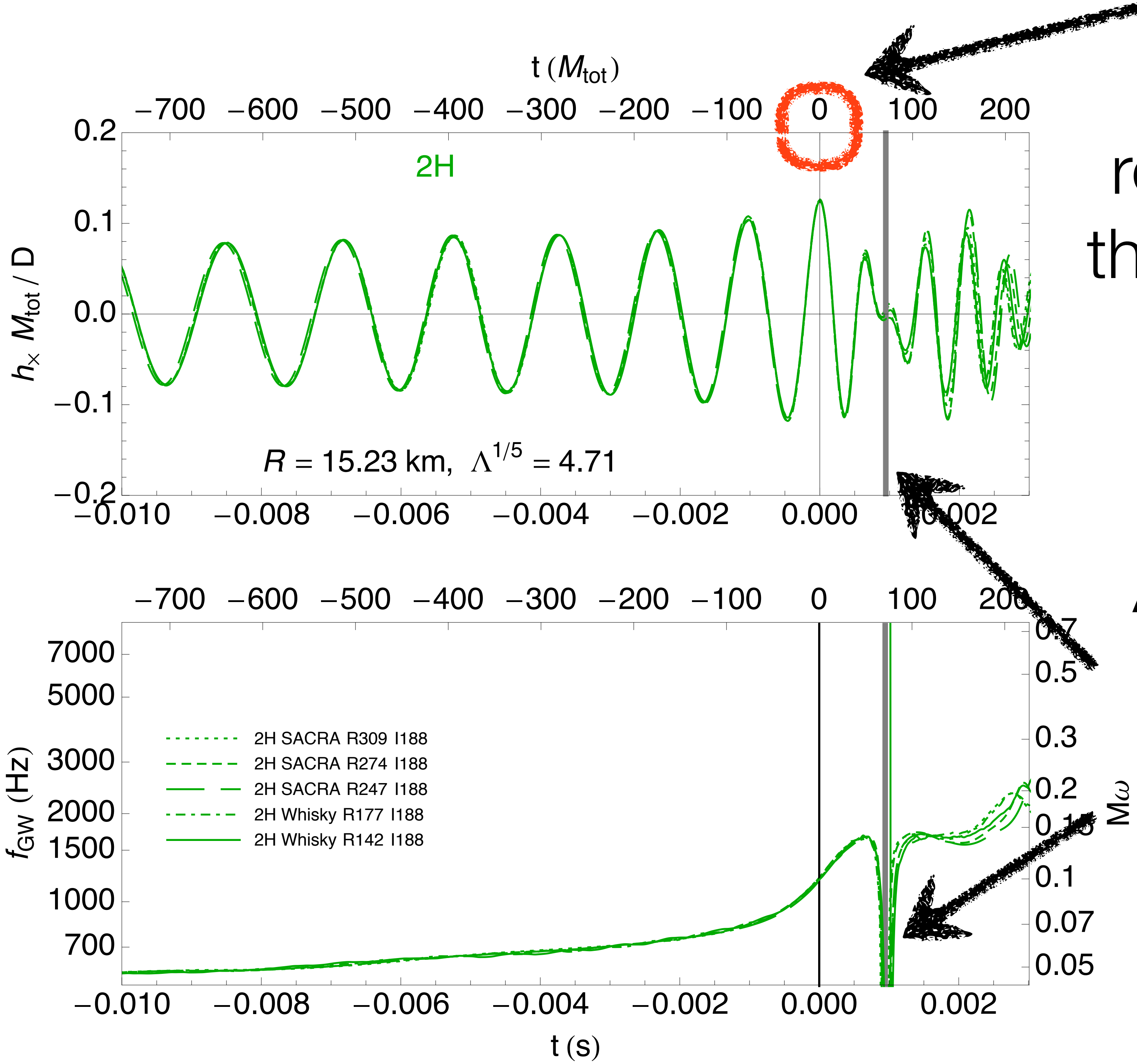
Compact stars





EOS Effect on Merger in SACRA/Whisky codes

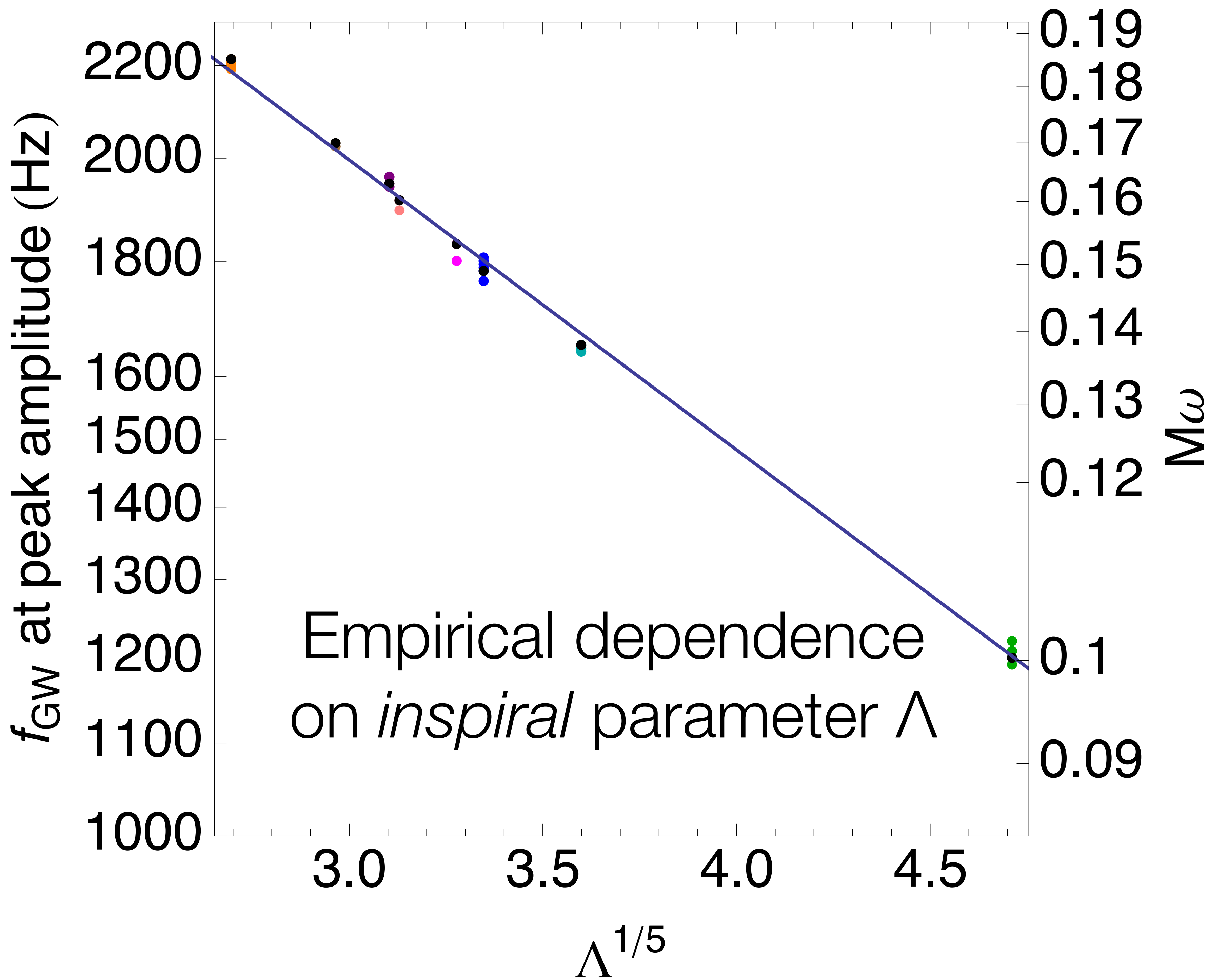
J Read et al arxiv.org/abs/1306.4065



Maximum amplitude: roughly where the stars collide

Amplitude later goes through minimum; frequency undefined

Frequency of peak amplitude: characteristic of *merger*



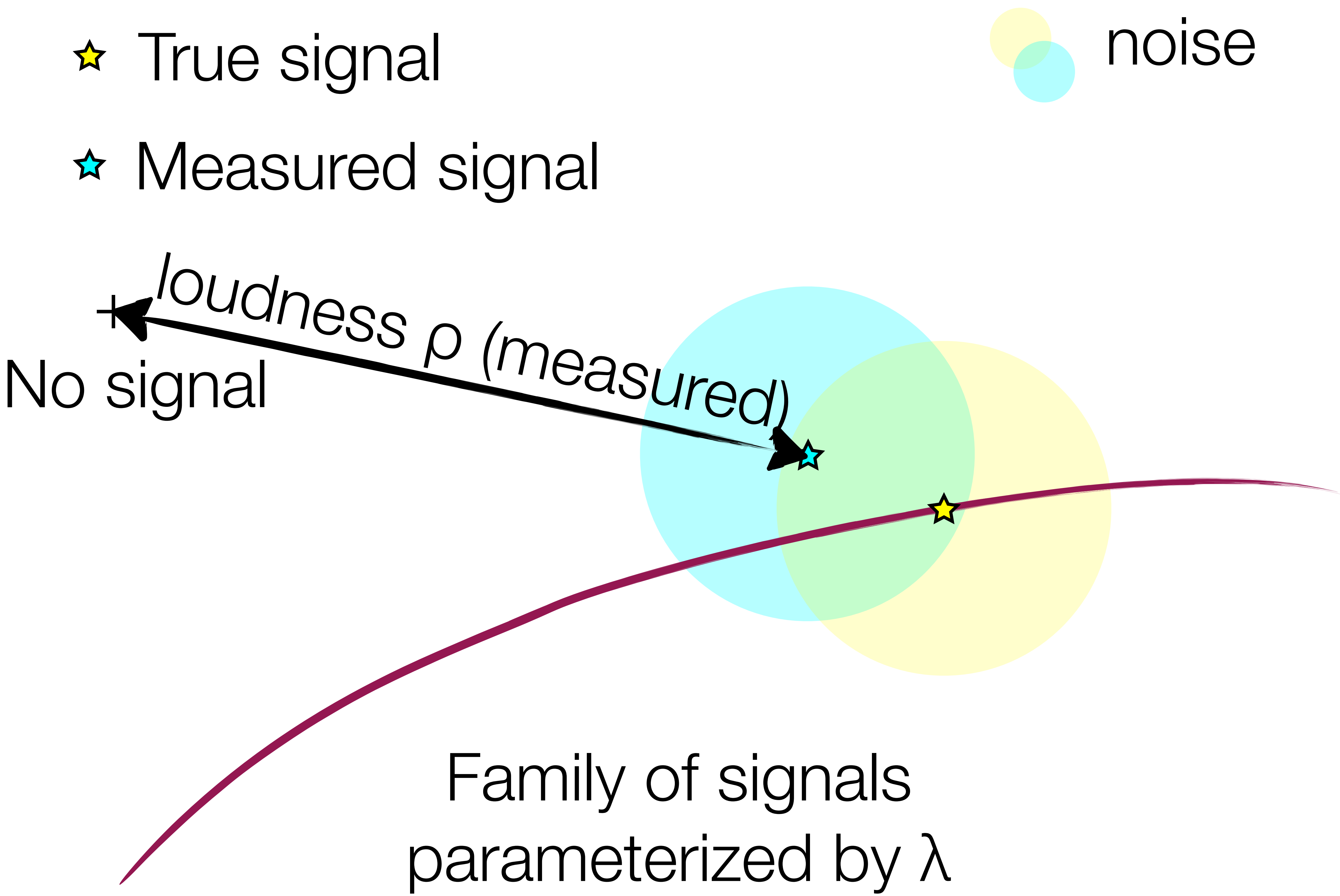
Why?

We often see such relations between NS quantities

(fast rotation, I-Love-Q 1302.4499, 1304.2052)

Detection and parameter estimation

Signal in gravitational-wave detectors



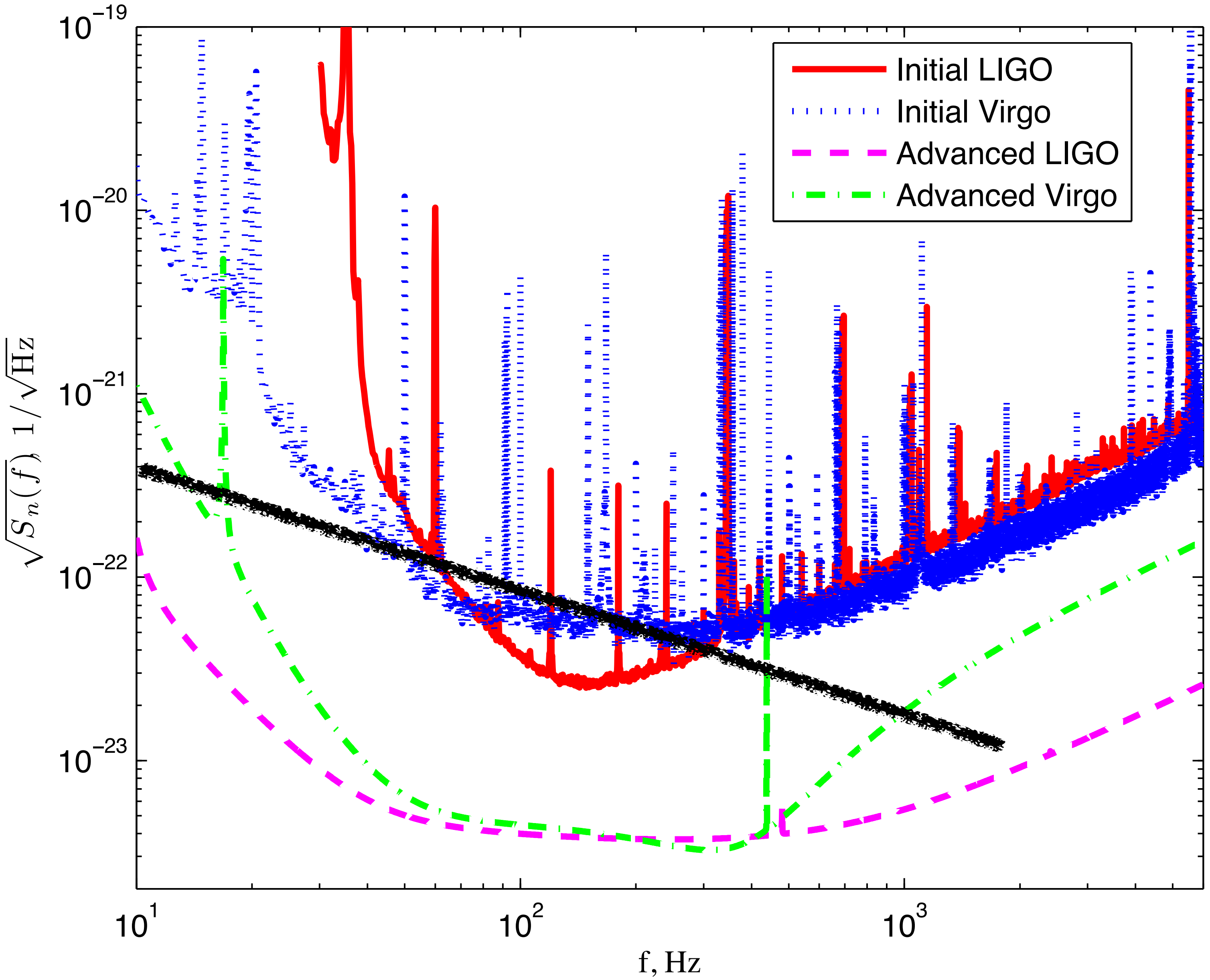
Inner product on the space of signals

- Define product $\langle h_1 | h_2 \rangle$ such that that noise has unit amplitude $\langle n | n \rangle = 1$

$$\langle h|g\rangle = 4\mathrm{Re}\int_0^\infty \frac{\tilde{h}(f)\tilde{g}^*(f)}{S_h(f)}df$$

- Expected loudness of signal h
  - SNR = $\langle \rho \rangle = \| h \| = \langle h | h \rangle^{1/2}$
- Distance *between two signals* $\| h_1 - h_2 \| = \langle h_1 - h_2 | h_1 - h_2 \rangle^{1/2}$
  - Consider two signals "distinguishable" if $\| h_1 - h_2 \| > 1$
  - marginally distinguishable if $\| h_1 - h_2 \| = 1$
  - (see e.g. Lindblom et al arXiv:0809.3844)

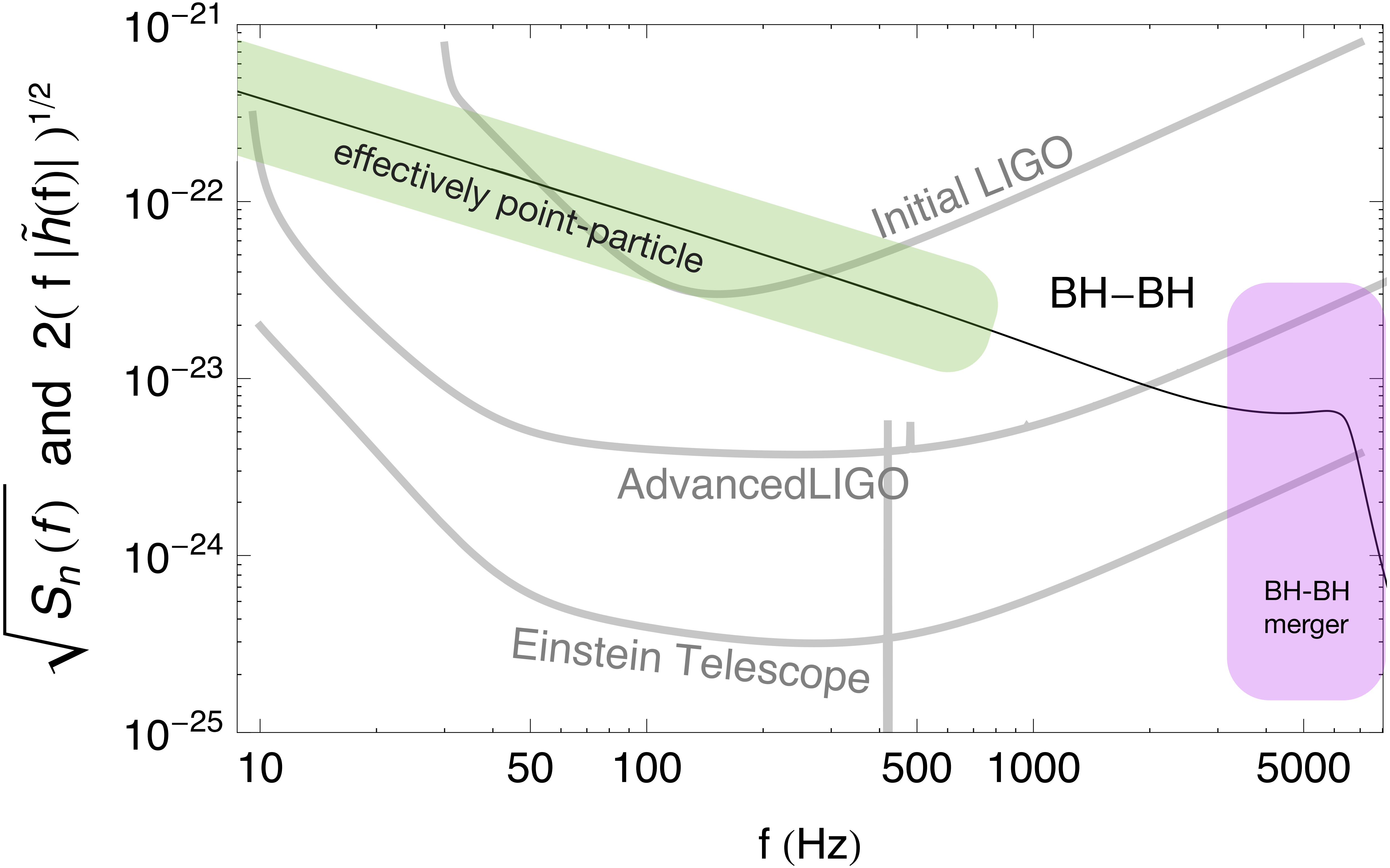
Noise in ground-based detectors



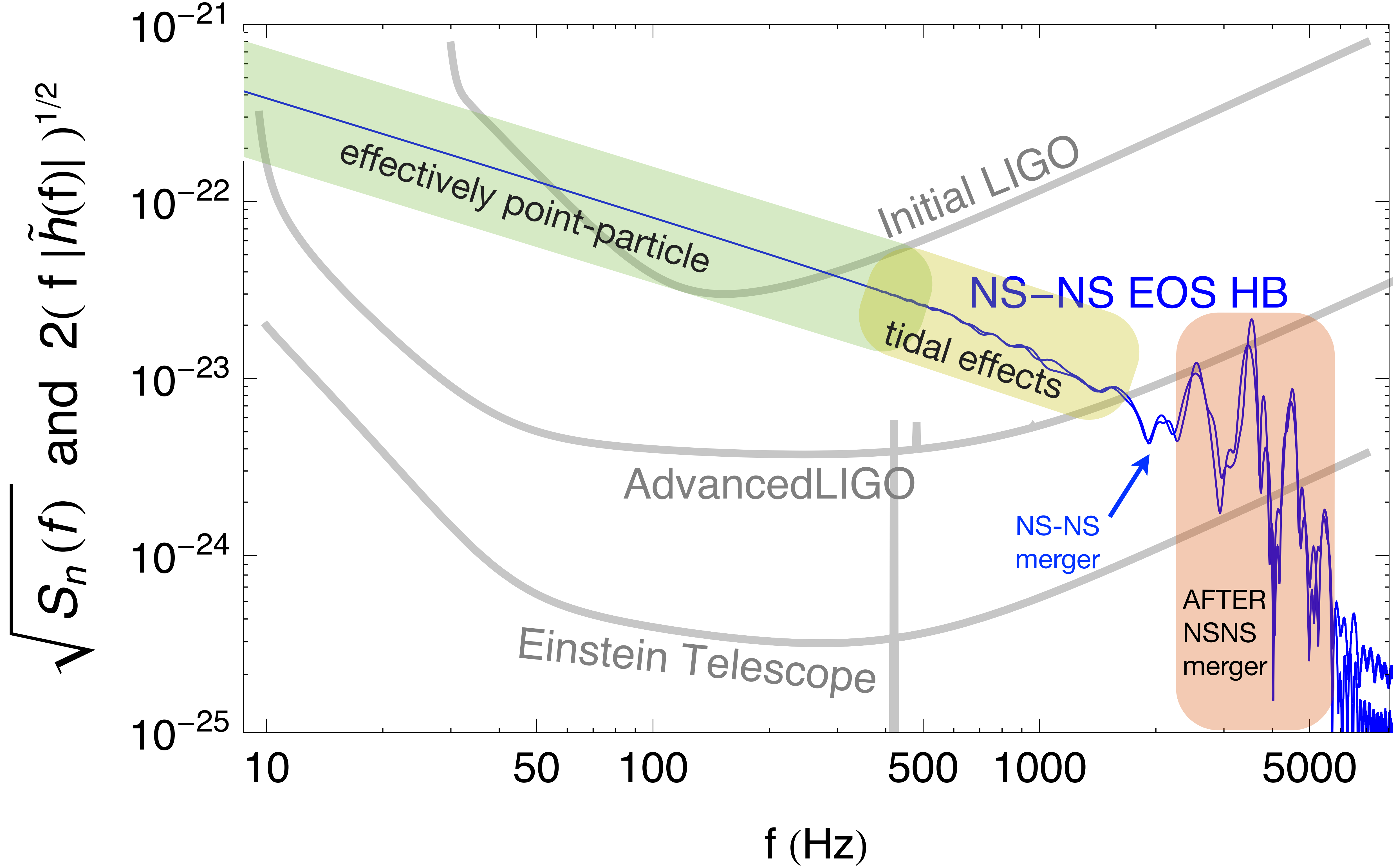
Amplitude of NSNS binary at 100 Mpc

SNR~3 in Initial LIGO
SNR~30 in Advanced LIGO

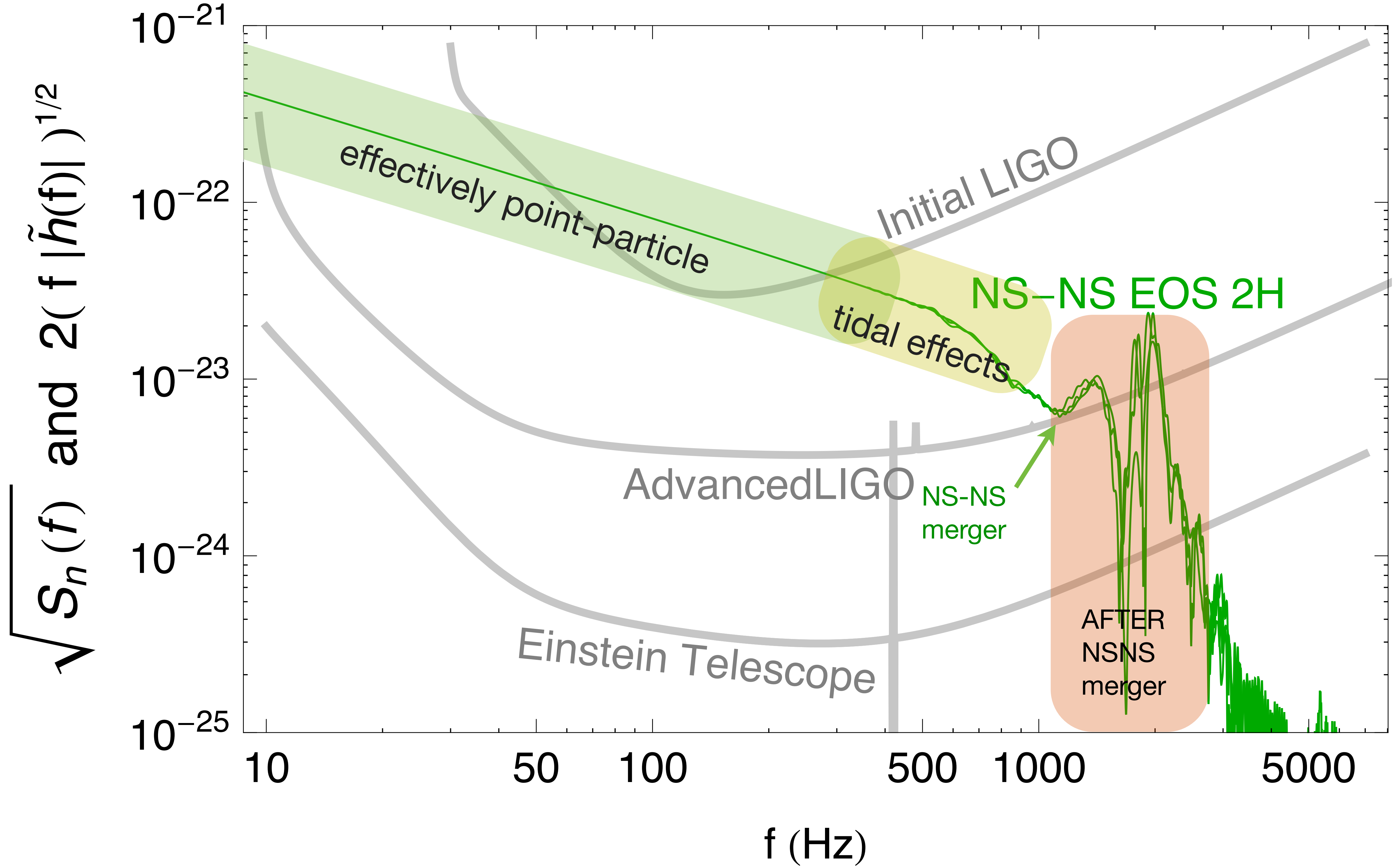
Advanced LIGO rates: http://arxiv.org/abs/1003.2480v2
Advanced LIGO/Virgo possible observing timeline: http://arxiv.org/abs/1304.0670



10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
$\sqrt{S_n(f)}$ and $2(f\,|\tilde{h}(f)|)^{1/2}$
effectively point-particle
Initial LIGO
BH−BH
AdvancedLIGO
Einstein Telescope
BH-BH merger
10
50
100
500
1000
5000
f (Hz)

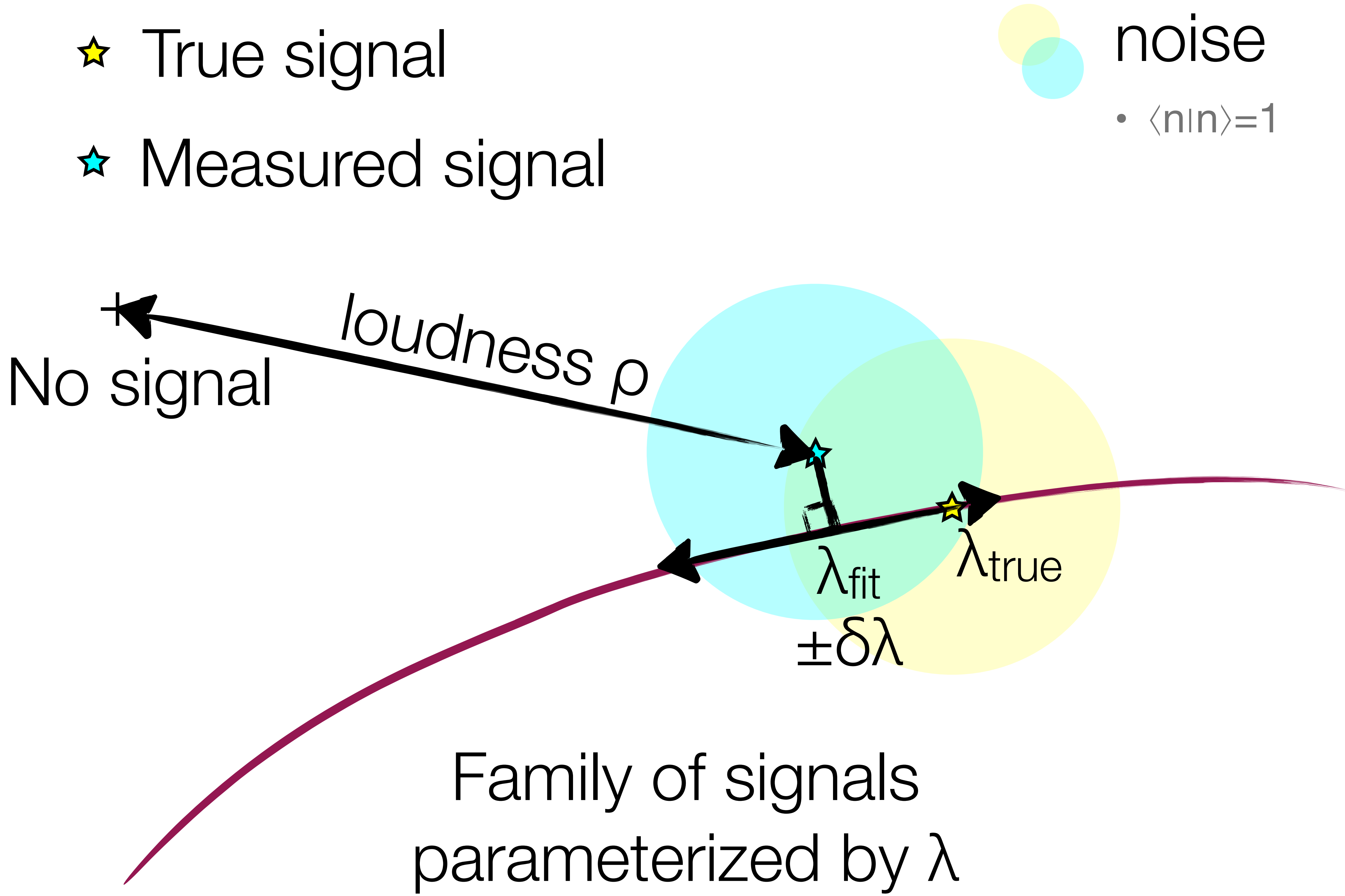


$\sqrt{S_n(f)}$ and $2(f\,|\tilde{h}(f)|)^{1/2}$
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10
50
100
500
1000
5000
f (Hz)
effectively point-particle
Initial LIGO
NS−NS EOS HB
tidal effects
AdvancedLIGO
NS-NS merger
AFTER NSNS merger
Einstein Telescope



$\sqrt{S_n(f)}$ and $2(f\,|\tilde{h}(f)|)^{1/2}$
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10
50
100
500
1000
5000
f (Hz)
effectively point-particle
Initial LIGO
tidal effects
NS−NS EOS 2H
AdvancedLIGO
NS-NS
merger
Einstein Telescope
AFTER
NSNS
merger

Parameter estimation

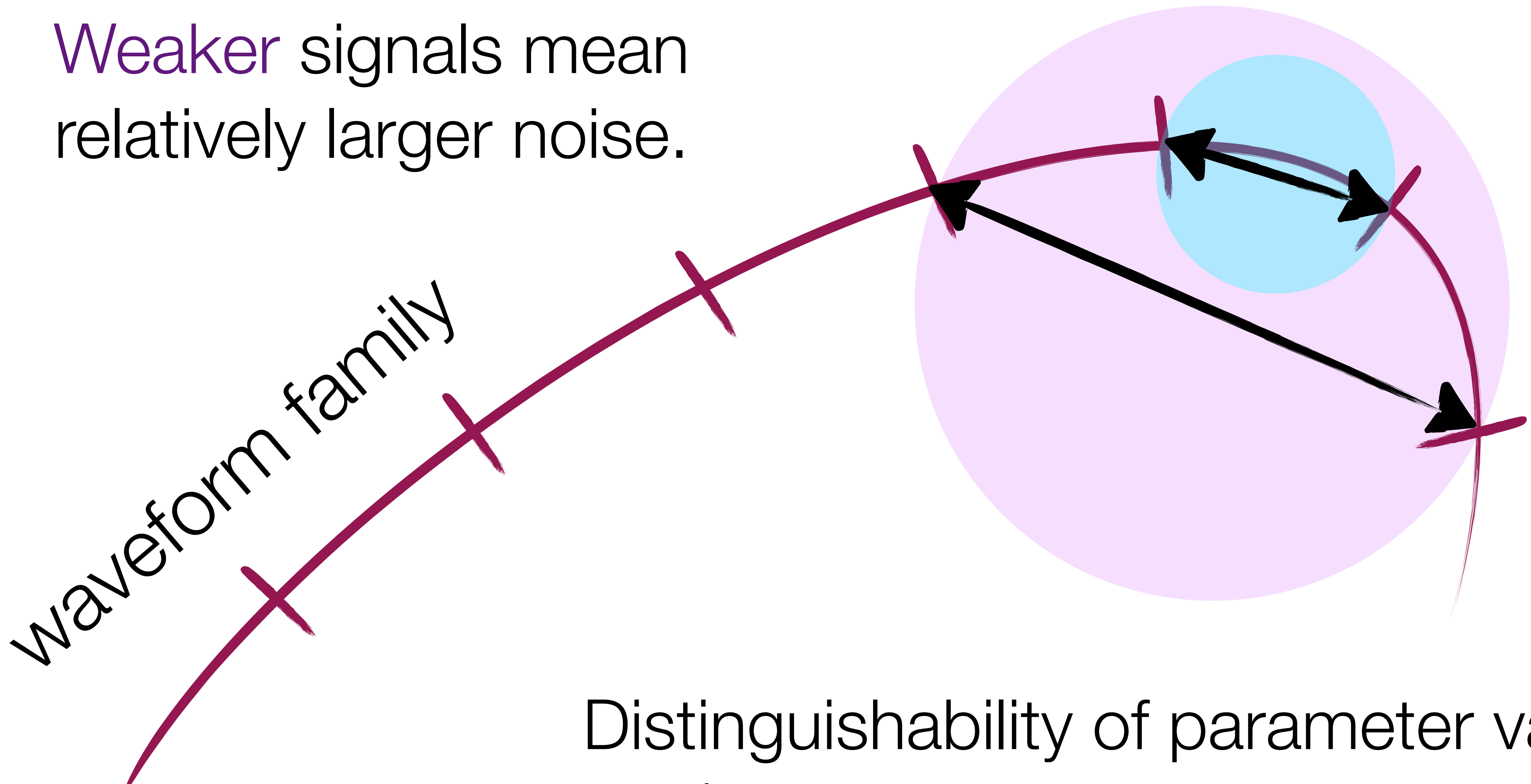


Parameter estimation

- Consider waveform as a function of parameter λ:
  - h_1 is $h(\lambda_1)$ and h_2 is $h(\lambda_2)$
- $\| h_1 - h_2 \| = 1$ for *marginally distinguishable* parameter values
  - Expected measurement uncertainty is then $\langle \delta\lambda \rangle = | \lambda_2 - \lambda_1 |$
- Can linearize around a particular parameter value for small differences
  - $h_2 = h_1 + (\lambda_2 - \lambda_1)\, \partial h/\partial\lambda$
  - the $\delta\lambda$ above then equals the Fisher matrix estimate
- Generalizable to multiple-parameter waveform family; can account for correlations between different parameters

Beyond linear approximation of error
(Cho et al arXiv:1209.4494)

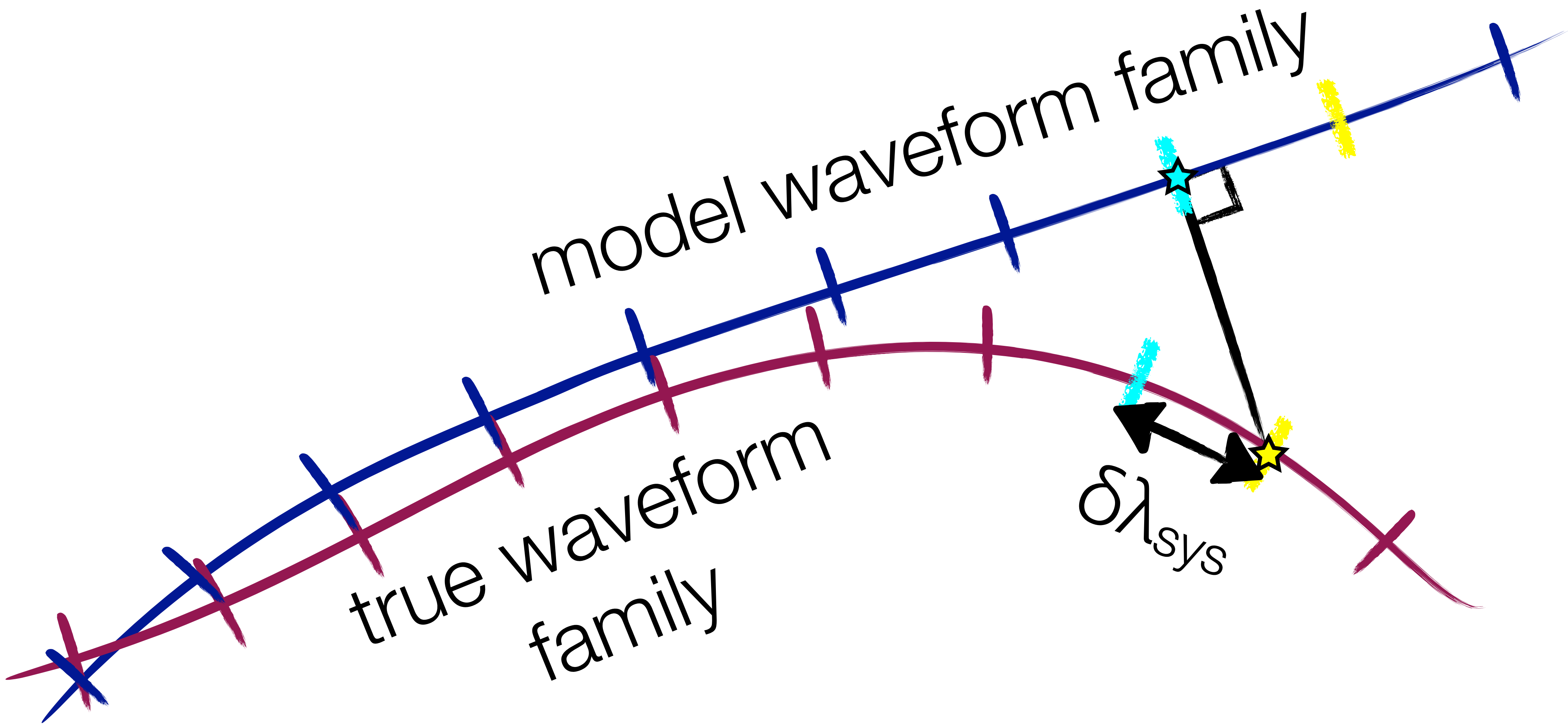
Weaker signals mean relatively larger noise.



Distinguishability of parameter values
(and thus measurement error)
may scale nonlinearly
depends on relative scale of noise

Systematic error

- ☆ True waveform
- ☆ Best-fit model
- True parameter value
- Best-fit parameter value



Estimate systematic error

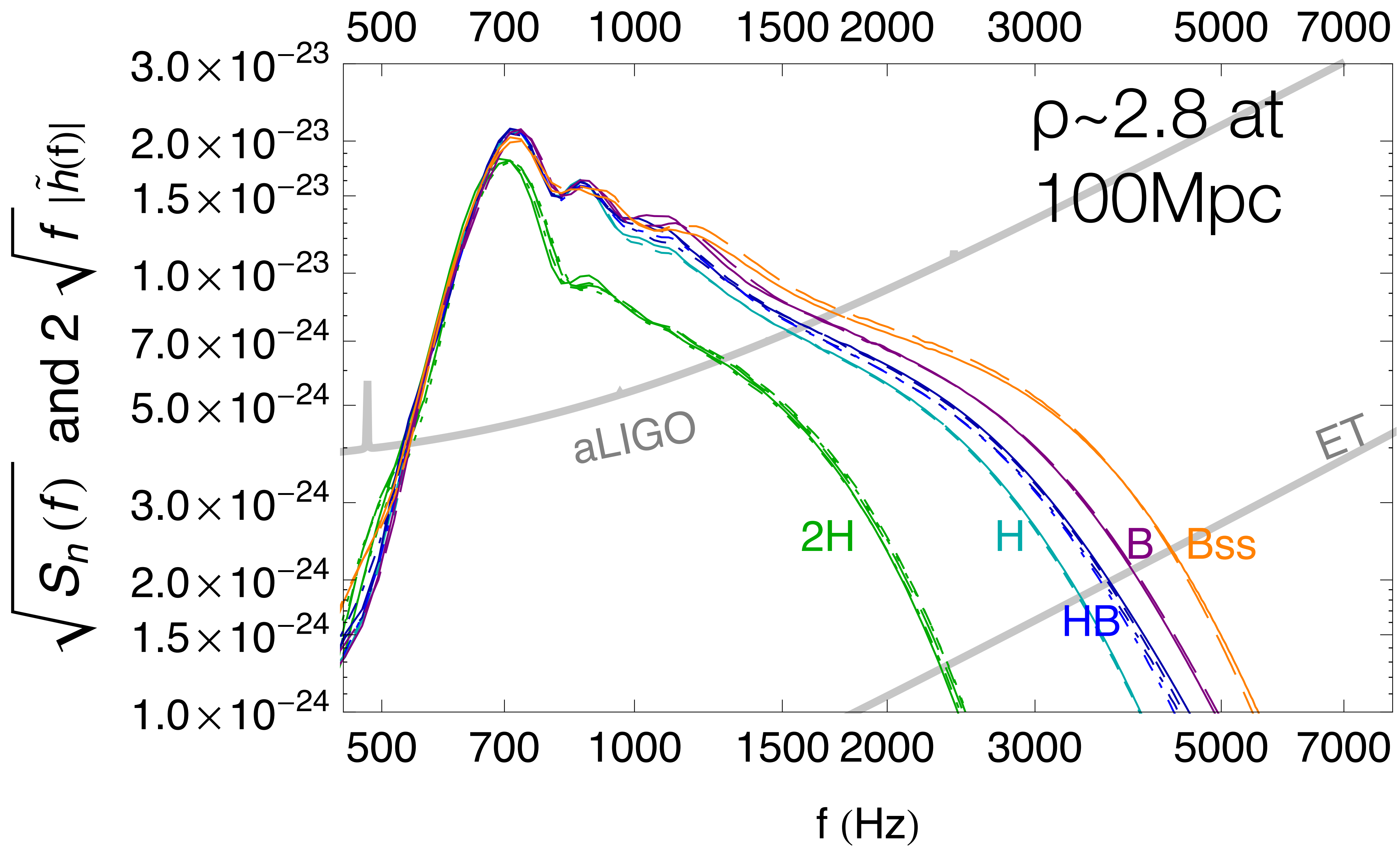
- Example: changing a numerical simulation for given parameter from h_1 to g_1
- Systematic error estimate
    - $\langle \delta\lambda \rangle_{sys} = (\lambda_2 - \lambda_1) \langle h_1 - g_1 \mid h_1 - h_2 \rangle / \| h_1 - h_2 \|$
    - $\langle \delta\lambda \rangle_{sys} / \langle \delta\lambda \rangle_{rand} \leqq \| h_1 - g_1 \| / \| h_1 - h_2 \|$
- Compare

$\| h_1 - g_1 \|$ for variant waveform of same parameter value

to

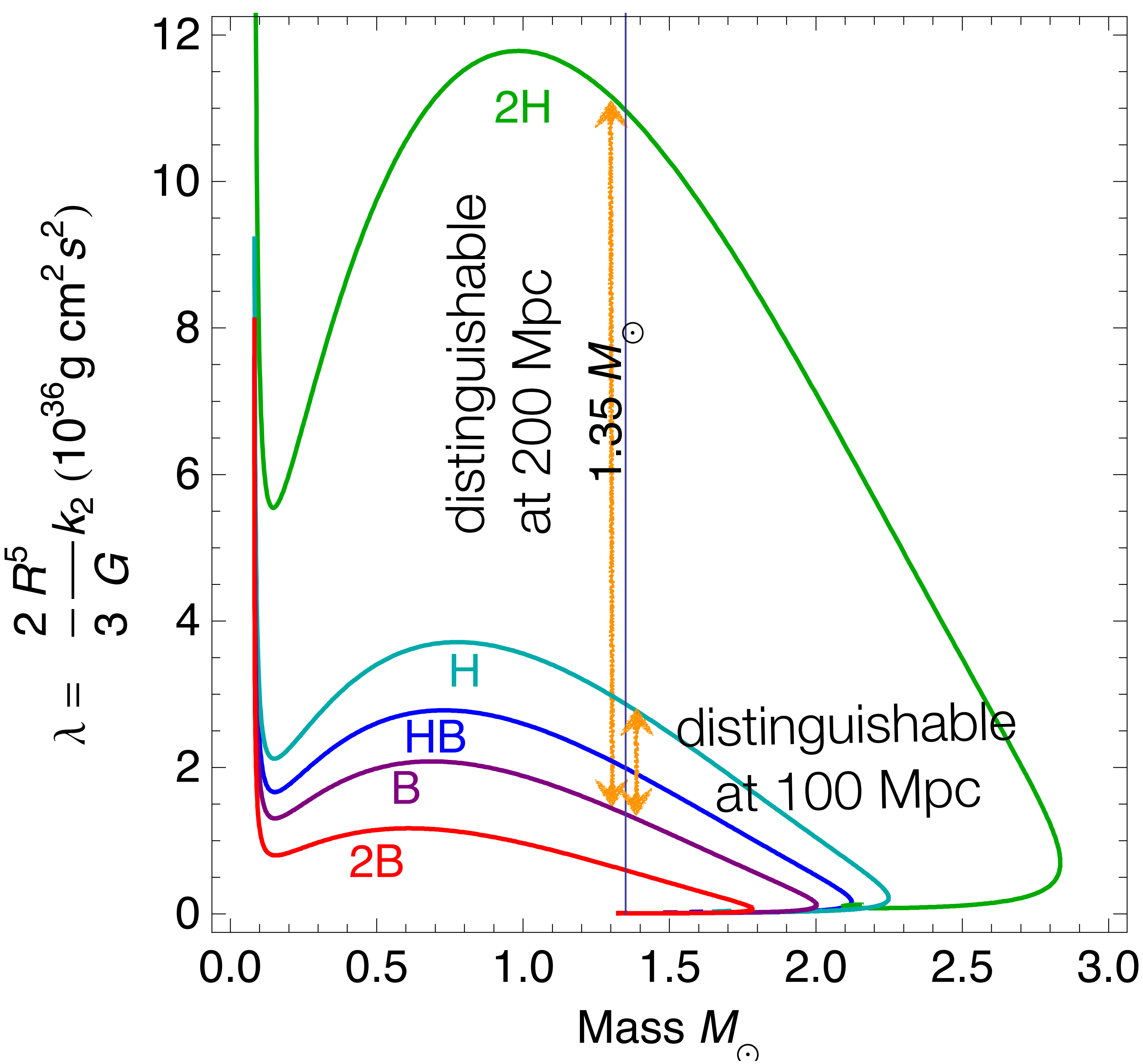
$\| h_1 - h_2 \|$ for waveforms of differing parameter value

Numerical inspiral waveform templates: 600Hz to merger



Can we tell these apart?

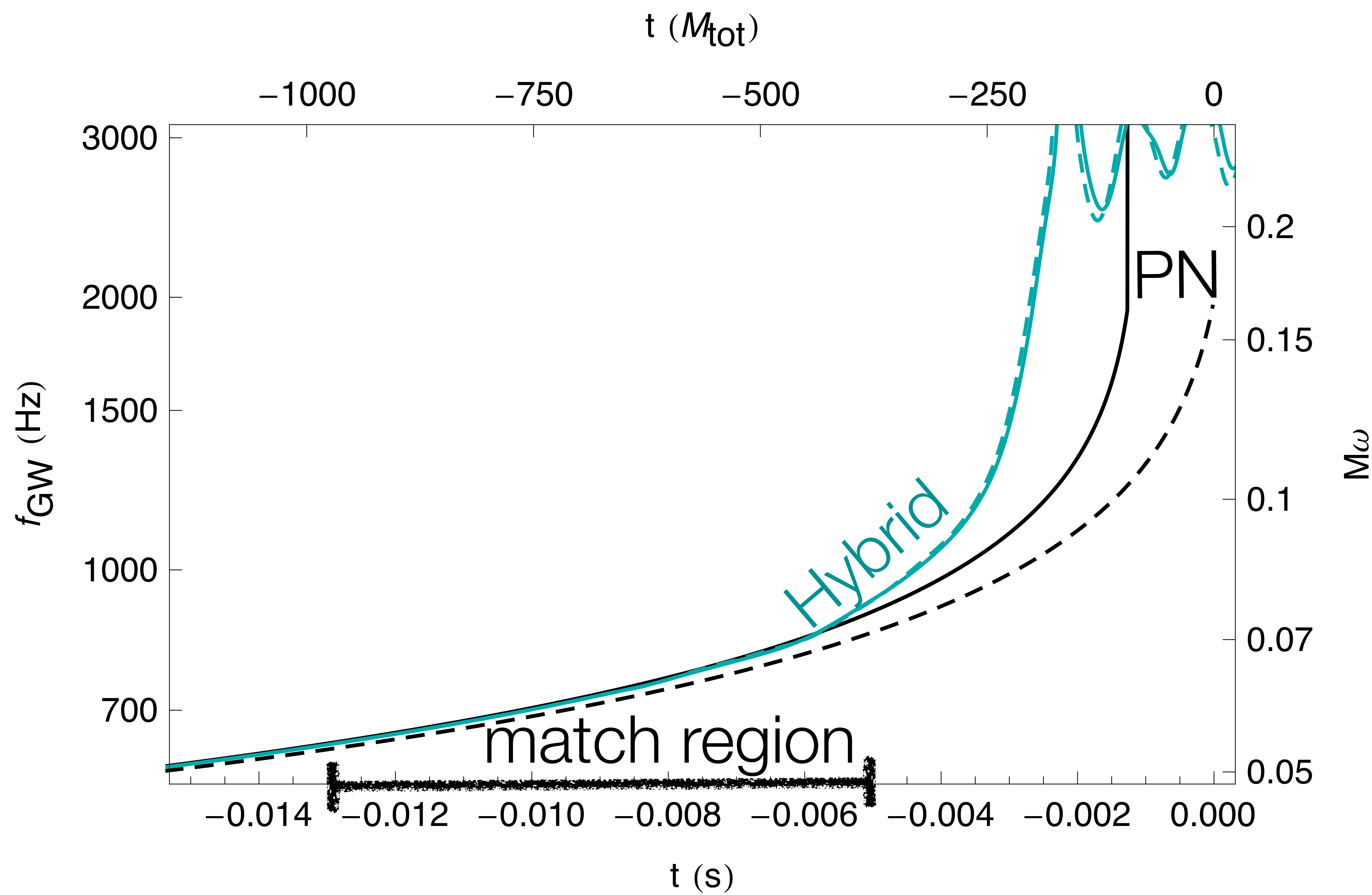
Distinguishable when || h_1 - h_2 ||>1



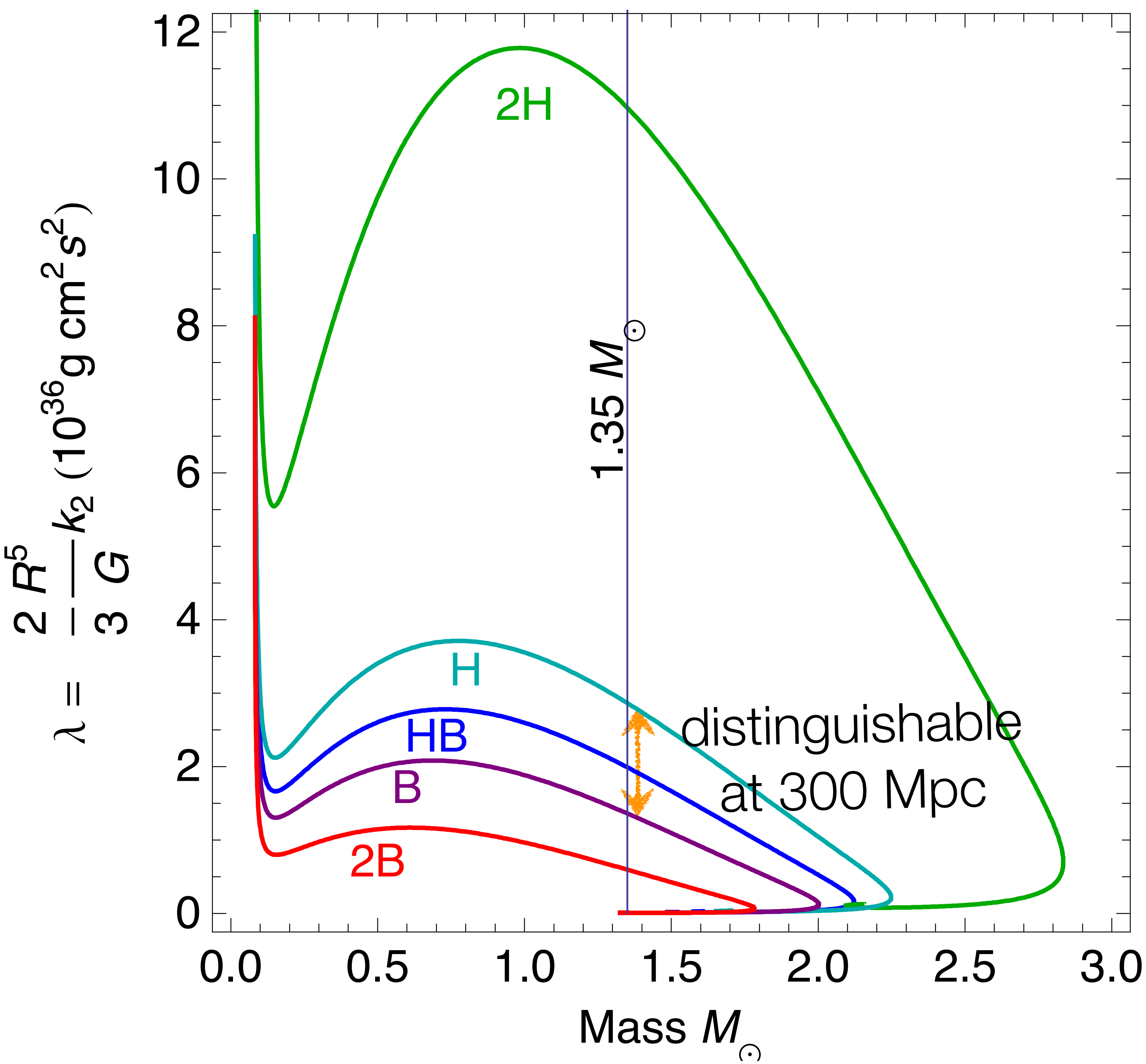
systematic error
20% @ 200MPc

systematic error
40% @ 100MPc

Hybrid waveforms



Measurement using hybrid waveforms

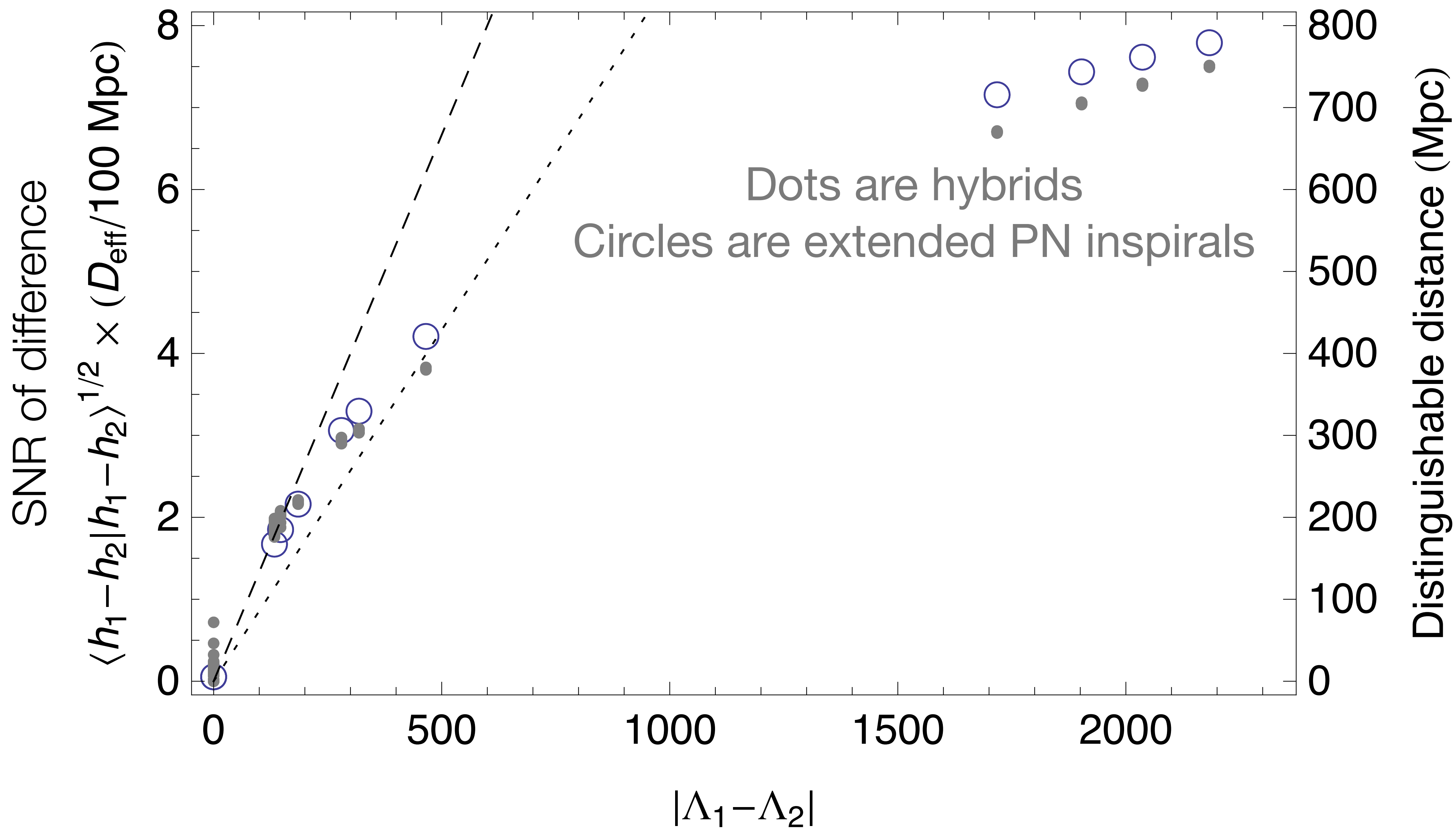


systematic error 60% with these simulations

current work:

- long and accurate simulations
- analytic waveforms which extend into merger

Usefulness of inspiral parameter



Analytic IMR waveform

- http://arxiv.org/abs/1303.6298 Lackey et al: BHNS waveform is approximately a 1-parameter deviation from a BBH waveform
  - includes varying mass ratio, spin as in BBH model