Dafermos- Cosmic Censorship.

Fig 1

$\times_{r^2} S^2$

Fig 2

Fig 3

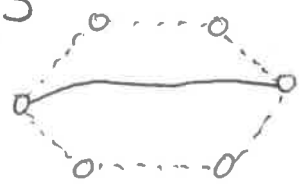

Fig 4

given data

Fig 5

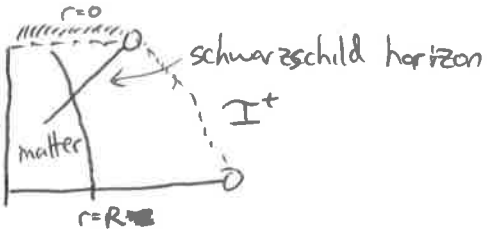

Fig 6

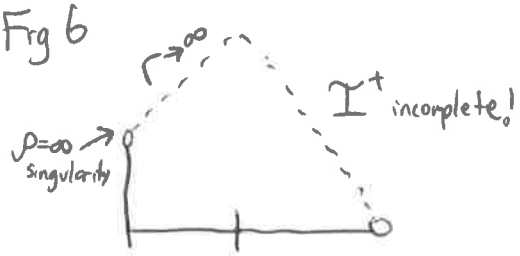

Fig 7

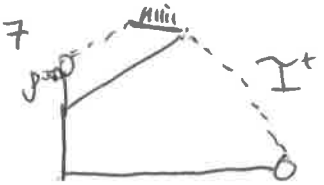

Fig 8

but also

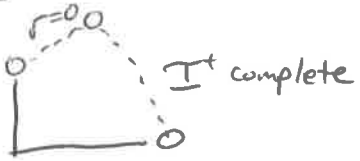

Fig 9

disperses

Fig 10

Fig 11

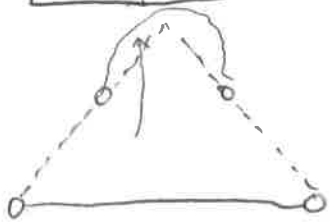

Fig 12

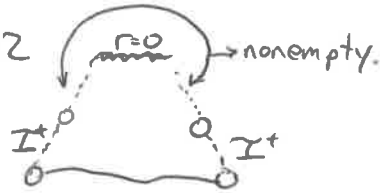

Fig 13

THE COSMIC CENSORSHIP CONJECTURE

MIHALIS DAFERMOS

This will be a kind of overview of what we currently know. Honestly, it'll be more like a picture book rather than an encyclopedia.

Weak cosmic censorship (WCC): For generic asymptotically flat (AF) vacuum data, future null infinity, $\mathcal{I}^+$ is complete.

We can define this without actually defining $\mathcal{I}^+$, and so this has a precise formulation.

Strong cosmic censorship (SCC): For generic AF vacuum data, the maximal Cauchy development is inextendible as a suitably regular Lorentz manifold.

There's a lot of physics and math that goes into what "should be" the choice for "suitably regular". There will be more on that later.

Most of our intuition comes from reduced versions of these conjectures under symmetry. For example: for both version, we can assume spherical symmetry. There aren't that many spherically symmetric spacetimes... They're true for this case since only the Schwarzschild family is spherically symmetric [Birkhoff's Theorem].

Ok, so let's get rid of the vacuum assumption too, so that we have something left to do. We'll instead say we want "reasonable matter." This means we have equations for matter coupled now.

Why is one version weak and one strong? One might get the impression that one implies the other. Actually, they're logically independent. Christodoulu says we should call WCC the global existence, SCC the global uniqueness theorems.

We will be using Penrose diagrams, and so we'll describe them a bit. A spherically symmetric spacetime is one where you can write the metric as $-\Omega^2(u,v)dudv+r^2(u,v)d\sigma$, where $d\sigma$ is standard on unit sphere. We can think of this defined as a warped product between a domain in the u,v plane and S^2, as in figure 1. It is easy to show that if the initial data is spherically symmetric that the maximal development is also. It can have 1 or 2 ends, modeled by Minkowski (see figure 2) and Schwarzschild (see figure 3, [Kruskal extension]) respectively. The figures show the part of the u,v plane that they use in the warped product. [The S^2 factor is suppressed.] They also show the Cauchy surface as a horizontal line.

We say M is the overall manifold, and we have to be careful about which boundary is in the manifold and that which isn't (the left side is for Minkowski, but the rest aren't). You might think the rest of the boundary has no natural meaning, but it does, in a coordinate independent sense.

We can consider, in general, 1 or 2 ended data (see fig 4). A priori, the upper boundary is unknown. However, for spherically symmetric spacetimes, we can a priori define null infinity, $\mathcal{I}^+$, and so we can identify part of this boundary as null infinity. It is simply defined as all points of the boundary that are limit points of null curves in spacetime (as sketched), such that our function $r \to \infty$ along them. Note that r is not a coordinate function. $\mathcal{I}^+$ may or may not include it's future endpoint, but it will be connected. It could also be empty, but if the energy conditions are reasonable, and the asymptotic convergence is fast enough, then it will be non-empty.

If we define the asymptotics a bit better, we can get an affine parametrization of $\mathcal{I}^+$, and say if it has infinite affine length. More pedestrianly, we can redo the coordinates, using U, such that $\partial r/\partial U = -1$. We can then ask if the coupled U coordinate ranges from $-\infty$ to ∞, which is equivalent to asking if $\mathcal{I}^+$ is complete. An interpretation is that far away observers will observe for all time.

The story begins with Oppenheimer-Snyder spacetimes in 1939. They considered the Einstein dust model, which is an Einstein-Euler system, but the equation of state of the fluid is very easy; it is jus pressure $p = 0$. Also, they considered data with a ball of dust near center of radius $r = R$. (see fig 5). The ball is of constant density, and is surrounded by vacuum. The inside of the "star" is homogeneous even! We can think of the matter as a collapsing star. The radius goes to 0 as we evolve. Since the outside is exactly Schwarzschild, we have that $\mathcal{I}^+$ is complete. This was the first data that evolved into a black hole.

The top boundary of this spacetime is singular! We have observers getting obliterated at the upper boundary. There are curvature blow ups of all types at that boundary, and Jacobi fields get all twisted up as they approach it. This is related to the folk theorem that it can't be extended even as just a continuous spacetime, though it can't be extended for sure in C^2.

Chirstodoulou in '84 kept the model, but didn't just use homogenous matter, though he kept spherical symmetry. See fig 6. If using increasing density ρ, it was known that you get shell crossing singularities. These are known to be inessential singularities, but they are ugly to look at, so he ignored this case. He showed that there is an open set of initial data such that the diagram looks like drawn in fig 6 AND another open set with diagram like fig 7, with closure of the second family including the Oppenheimer family. For the first family, $\mathcal{I}^+$ is incomplete, *and* you can smoothly extend the solution! Thus both censorships fail... for this model of matter.

For the second family $\mathcal{I}^+$ is complete, but we can extend the metric over the dotted line in the upper left. So SCC fails for this one. Thus SCC fails even just around Oppenheimer-Snyder. It's prediction of SCC is not stable, even though we were trying to use it to form intuition about the full case.

For both, we have that there is a $\rho = \infty$ singularity at the upper left dot.

So shouldn't we just kill the conjectures? Well, it may be that the model of no pressure is just a bad model, since when we evolve, we get infinite density and so clearly there shouldn't just be no pressure.

Einstein-Scalar field model: Einstein coupled with wave equations for matter. Motivations: it is the simplest type of matter, it satisfies a linear equation, and it models the degrees of freedom of the vacuum in some sense. There's even an equation of state for fluid that gives an equivalent formulation. It was studied by Christodoulou and others 1970-2001.

We might hope that for this model, that figure 6 could never happen. But... there are solutions such that fig 8 holds. The second one has naked singularity. There are more examples like the other bad ones. For the first one, if WCC is going to be true, we need generic for this statement, since we have an example. Because of this, and because the scalar field is expected to act kind of like vacuum, we expect bad solutions like this to exist for vacuum data.

But he also showed that for generic initial data, the diagram looks like one of Fig 9, where for the 2nd one the BH is the right kind singularity, and all observers going to the BH die in the BH. Thus WCC and SCC are true for this model (still assuming spherically symmetric). The set of solutions that don't look like this is codimension 1 in the set of all solutions.

What is generic? Well... Let's give cartoon of proof, because the definition of generic really depends on proof. The hope before was to find some sort of global estimate, but that's doomed to fail because of the bad solutions.

We start with a solution that is bad, as in figure 10, and show that it is in a 1 dimensional family in the moduli space that is nice. How? If want to perturb that upper left singularity, we might think to change the part of the initial data that collapses to there, but that turns out not to work. We instead need to change the outer part. These waves want to collect near this. We show that the family has trapped surfaces in solutions near it, and we can use them to show regularity.

For the full CC, Kerr is a weird problem solution. It is much worse than that first one of Fig 8, since the upper left dotted line is niceish. But for Kerr we have Fig 11. There's no first singular point, unlike the other! Any observer could get into the weird extended region.

Well, back to spherically symmetric case, we can construct similar solutions for more complicated matter models, for instance in Einsstein-Maxwell we have the Reissner-Nordstrom solutions, which still look like fig 11. It is the unique spherically symmetric which is AF with EM. Thus the SCC is false for this "matter" model.

We could also couple in *also* scalar field by just adding the stress energy tensors together. We could do worse by coupling them to each other by taking a charged scalar field. In this model we need 2 ends for completeness, so we also have to think about that. It's much like the bottom of fig 12. Kommeni considered strongly "tame models," and showed that $\mathcal{I}^+$ is complete. Also, for tame models,

the boundary of spacetime will have at most the pieces in Fig 12. It is inextendible through the top piece, so if the two side pieces are empty, we also have SCC.

We hope that they are both empty. Consider figure 13. We have an observer on outside, A, that sends info to the guy on the inside B. The frequency of the info sent is infinitely blue shifted as we approach the Cauchy horizon, and so something should blow up at Cauchy horizon, or perhaps before because of the non-linearities of the Einstein equations. This is why we conjectured SCC in the first place.

However, the null components in Fig 12 are proved to be non empty. Also, we can extend the metric beyond them. So SCC is false. But it's singular in the C^2 sense modulo some extra fact, so is *true* if say C^2 metric.

Is the top boundary there also? If the data is near Reissner-Nordstrom, then there is no top part. The conjecture is that for small neighborhoods of Kerr, for vacuum, you would get the same thing. So there may be no spacelike singularities (generically).

In some sense, a C^0 continuation is too weak, since we might expect we want weak solutions of Einstein with L^2 Christoffel symbols in some sense.

The same holds near the Reissner-Nordstrom solution in asymptotically de Sitter space as well.