Perseverance and Other Challenges in Problem Solving

MELANIE SMITH

EYAL WALLENBERG

MATH FOR AMERICA

THE URBAN ASSEMBLY SCHOOL FOR LAW AND JUSTICE

Investigate . . .

If all the blood from everyone in the world was poured over Central Park, what depth would the blood reach?

- adapted from *Innumeracy* by John Allen Paulos

Turn and talk . . .

What are some pitfalls or challenges that a teacher may face when implementing complex tasks?

Challenges

- Students give up completely
- Students need constant reassurance that their answer is right or that their solution path is ok

Standards for Mathematical Practice

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.

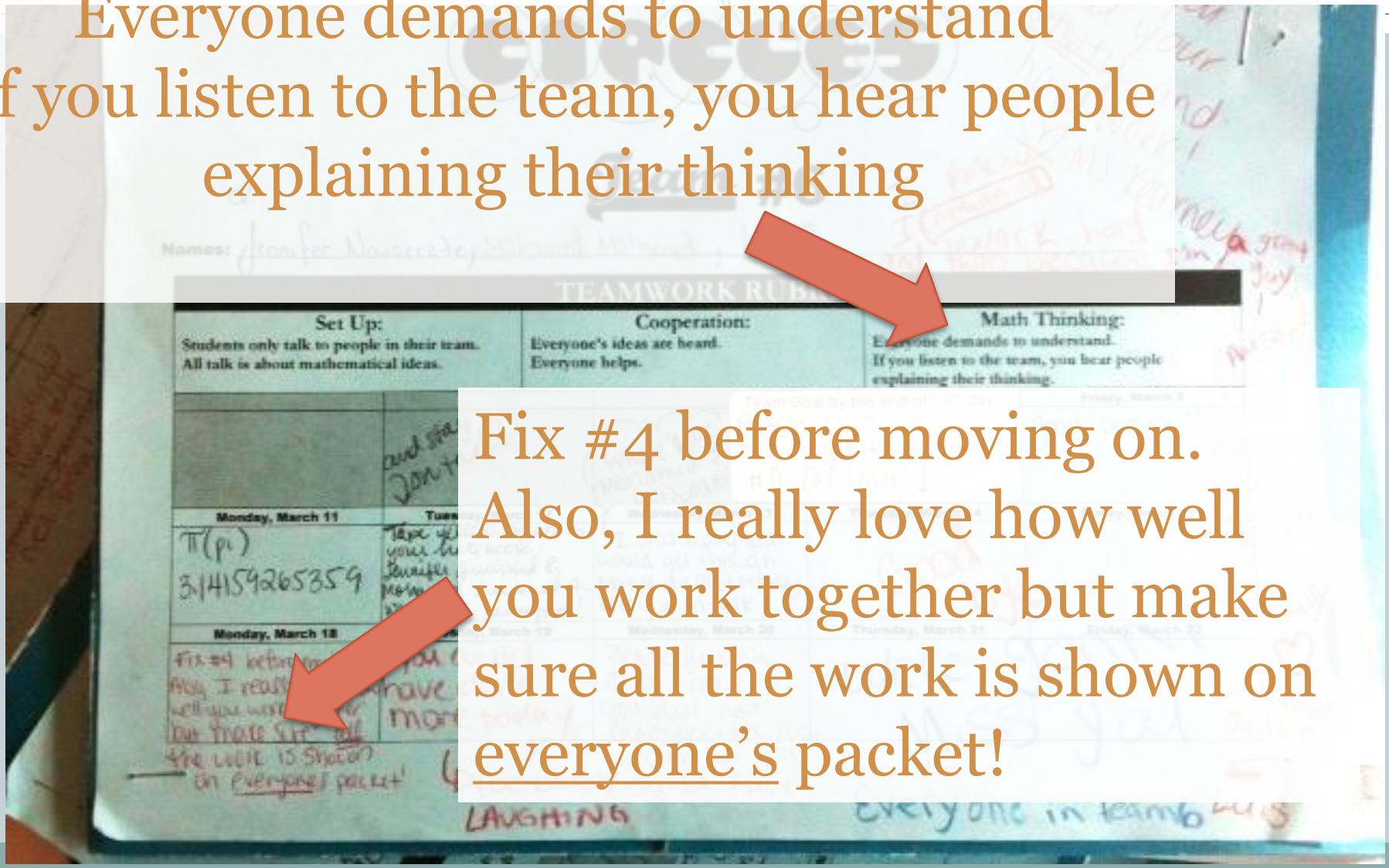
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Make sense of problems and persevere in solving them

Math Thinking

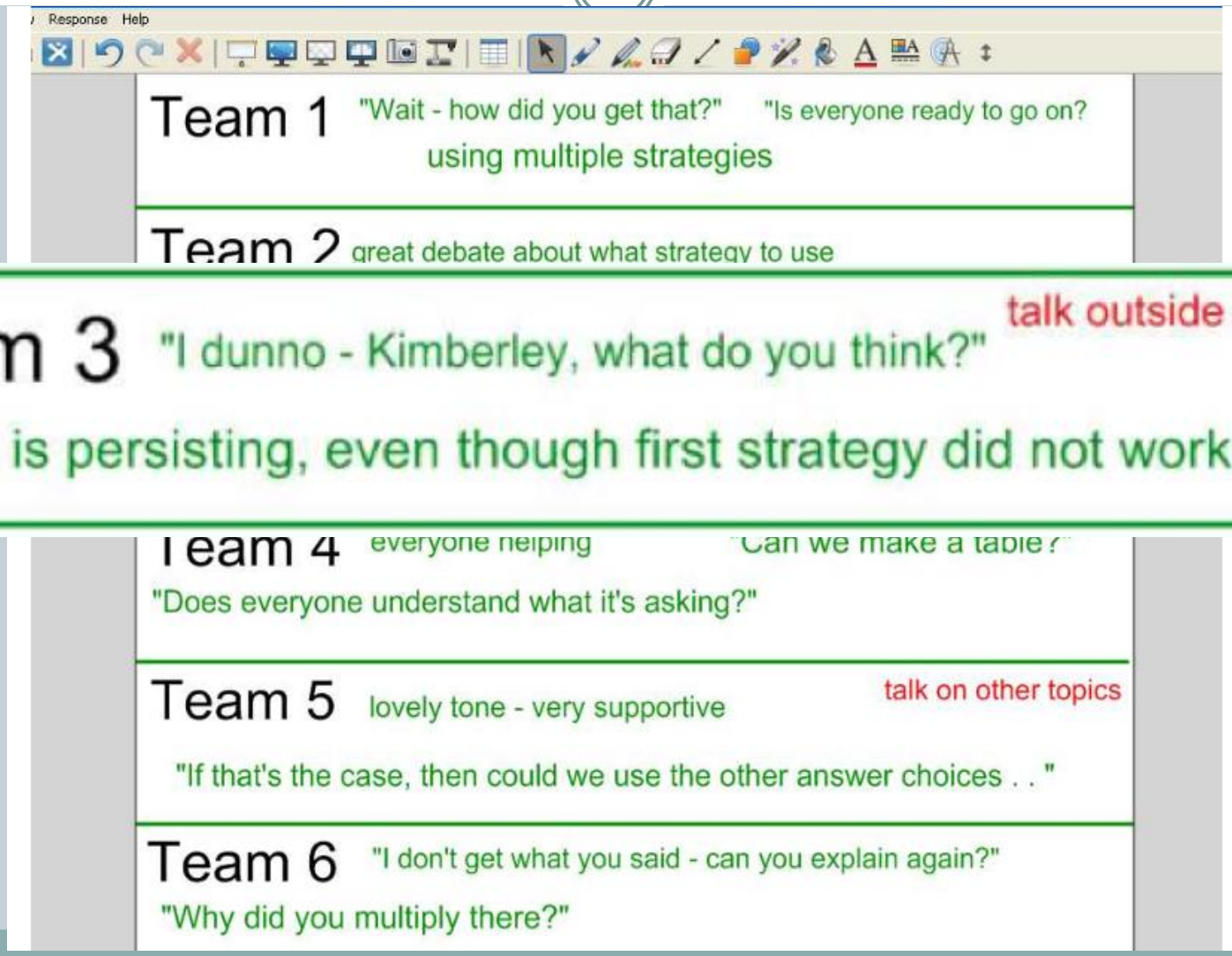
Everyone demands to understand

If you listen to the team, you hear people explaining their thinking

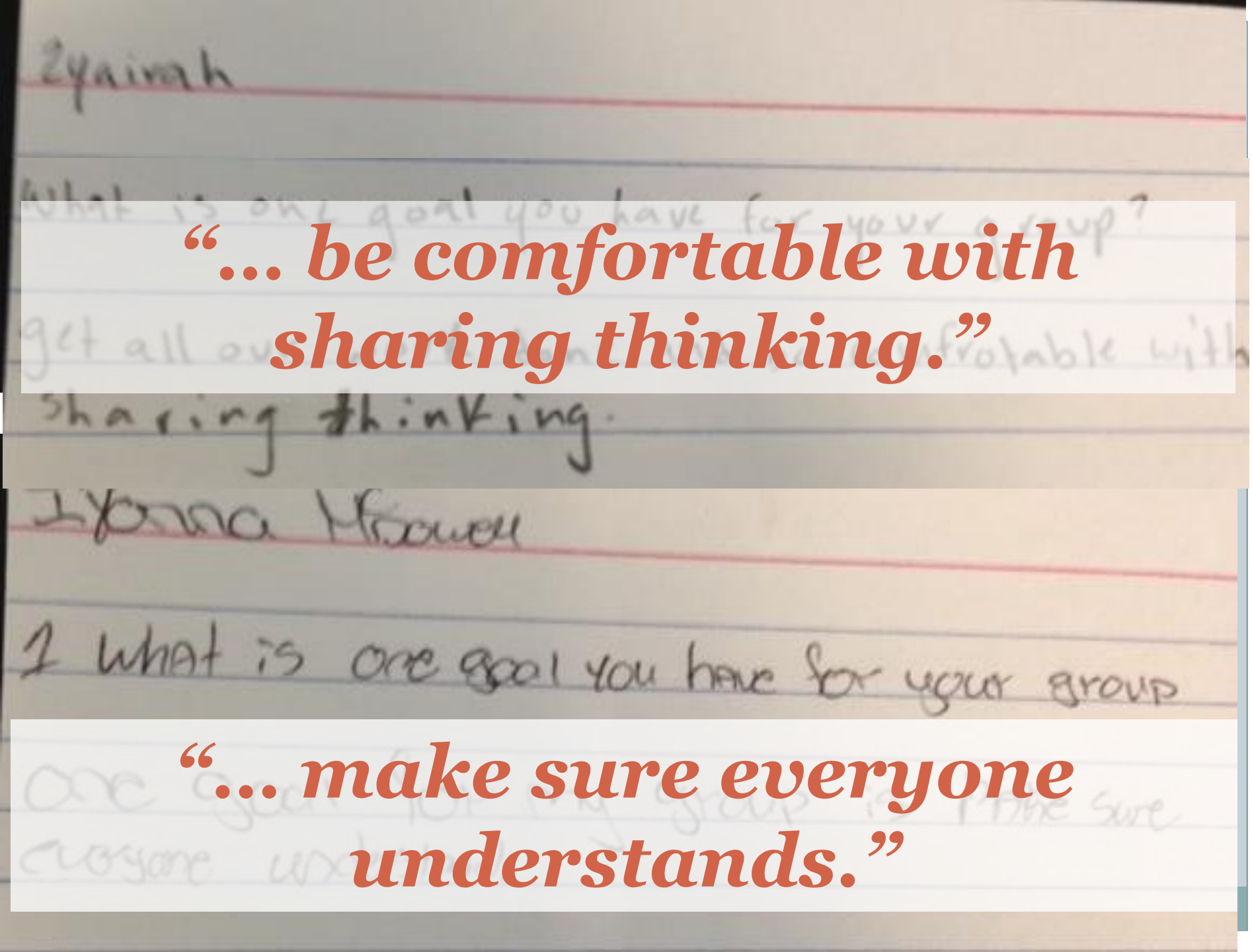


Fix #4 before moving on. Also, I really love how well you work together but make sure all the work is shown on <u>everyone's</u> packet!

Live “tweeting” of quotes and observations

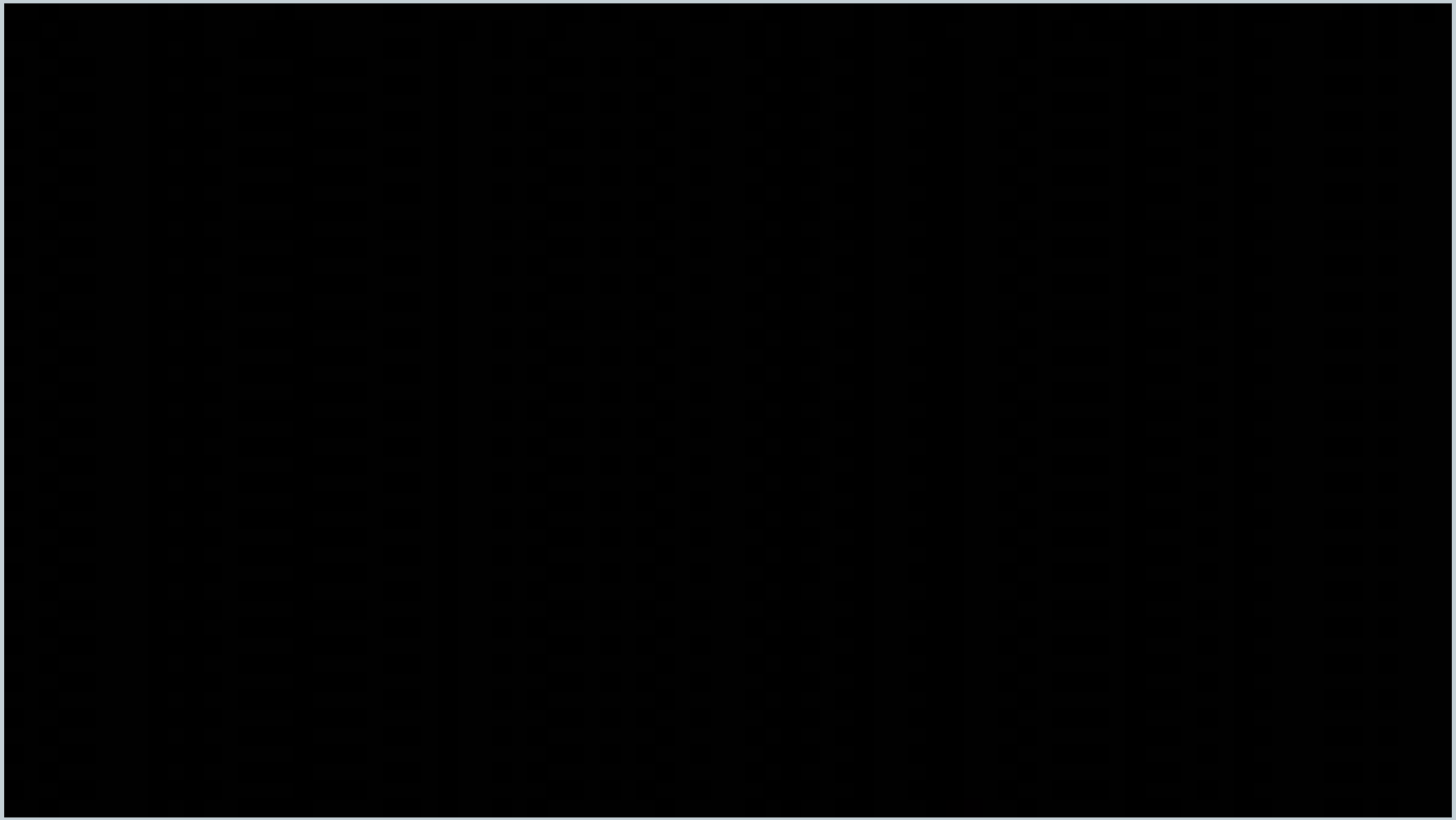


Student-to-student feedback

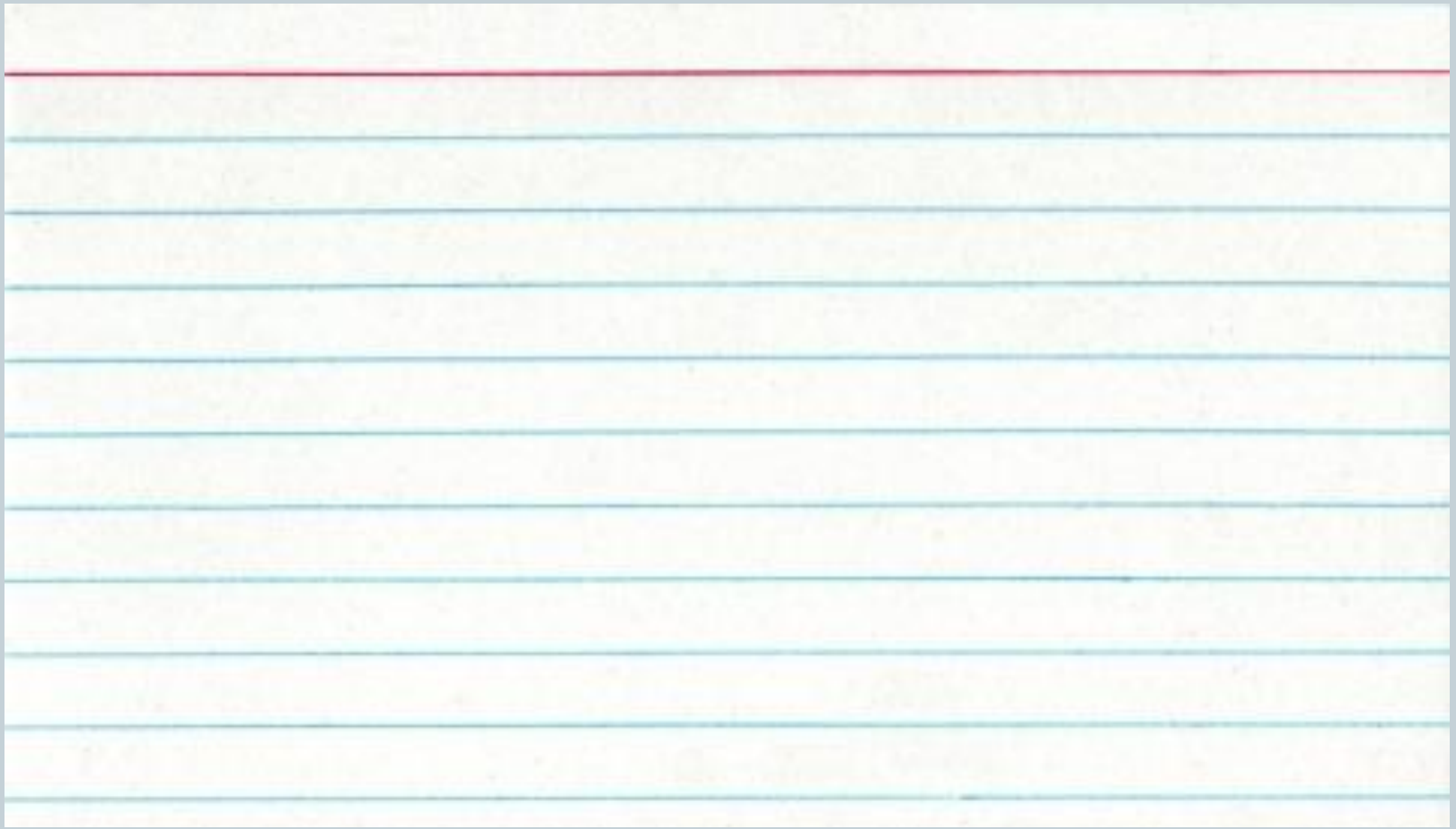


"... be comfortable with sharing thinking."

"... make sure everyone understands."



What is one goal you have for your group?



Challenges

- Students give up completely
- Students need constant reassurance that their answer is right or that their solution path is ok

Standards for Mathematical Practice

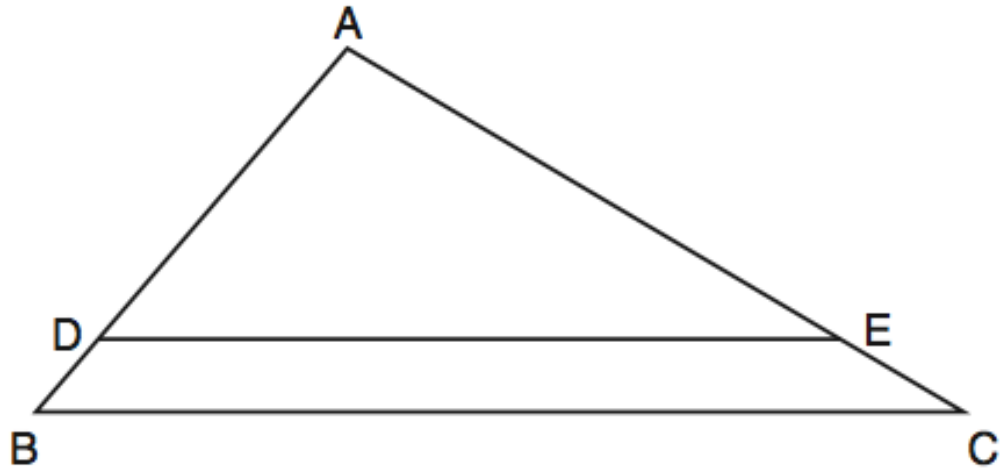
CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Construct viable arguments and critique the reasoning of others.

NYS Geometry Regents, June 2012

16 In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$.



If $AB = 10$, $AD = 8$, and $AE = 12$, what is the length of $\overline{EC}$?

(1) 6

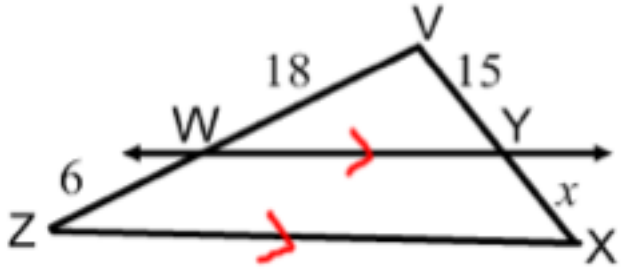
(2) 2

(3) 3

(4) 15

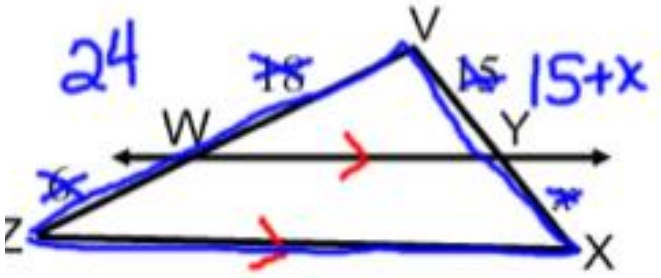
Alan's Answer:

$$\frac{18}{6} = \frac{15}{x}$$



Noah's Answer:

$$\frac{24}{18} = \frac{15 + x}{15}$$



playing the skeptic

assessing hard to assess competencies

Knots and Ropes

Imagine that you and your friend each own a dog and you like to take them for walks together. You and your friend would like to shorten your dogs' leashes so that it is easier for you to pull the dogs apart when they tangle.

The leashes are different lengths and thicknesses.

Can you and your friend tie the same number of knots in each leash and end up with leashes that are the same length?

Jabari's work

I discovered that the **best way to solve this was to find a steady drop of the average** so I found the averages of both ropes.

Drop Rate	Thick& Long	Drop Rate	Thin & Short
3.5	55.5	3.5	35.5
5	52	4	32
5	47	3	25
4	38	4	22

After finding the drop rate of each number I **took all the drop rates added them up and then divided them** . . .

Now that I found the average for both ropes **the next step was to choose a length to start from for both ropes** . . . I picked the numbers 80.1 and 74.5 as my starting points.

I steadily subtracted the average drop rate for the Thick& Long (4.3) and the average drop rate for the Thin & Short (3.5) until I got to an equal number of Knots and an equal length.

Novice-Expert Score	
	Problem Formulation
	Hypothesize
	• Hypothesis contains cause-and-effect or thesis statement • Hypothesis or thesis is sufficient to formulate a potential solution • Hypothesis or thesis makes sense and is complete
	Strategize
	• Strategy or strategies are appropriate to the subject area • Strategy or strategies will likely be effective for solving the problem
	Research
	Identify
	• Search method or methods are geared to the problem • Sources identified are related to the problem • Potential sources identified are sufficient to address the hypothesis or thesis
	Collect
	• Sources are collected systematically • Actual information collected is sufficient to address the hypothesis or thesis
	Interpretation
	Analyze
	• Analysis method or methods used are appropriate to the problem • Analysis method or methods are sufficiently systematic to reveal patterns in the data or information • Analysis generally helps support or call into question the hypothesis or thesis
	Evaluate
	• Findings selected are of value to completing the task • Findings are prioritized in a way that is useful to addressing the hypothesis or thesis • Findings are sufficient to help support or call into question the hypothesis or thesis
	Communication
	Organize
	• Final work product uses a logically consistent organizational structure • Final work product uses formats and conventions appropriate to the subject area
	Construct
	• Drafts of work product are of increasing quality and incorporate feedback • Results from Problem Formulation, Research, and Interpretation are incorporated and integrated into the final product
	Precision and Accuracy
	Monitor
	• Precision follows the subject area's rules and conventions • References are documented properly
	Confirm
	• Technical and grammatical accuracy is confirmed • Final product is consistent with the task's requirements and directions

Characteristics of Work Product					
Insight	**Efficiency**	**Idea Generation**	**Concept Formation**	**Integration**	**Solution Seeking**
Emerging Expert – 7					
Work product shows strong evidence of an intuitive sense of the use of subject-area rules to demonstrate insight	Work product treats task highly efficiently, few ways it could be done more efficiently	Work product shows strong evidence of novel or creative use of conventional ideas and/or strong evidence of unique or innovative ideas	Work product shows strong evidence of conscious design around a set of core concepts to organize and explain findings	Work product uses integration and connection among its elements in a highly effective fashion that is readily apparent	Work product shows strong evidence of a cogent, coherent solution strategy for the problem
Accomplished Strategic Thinker – 6					
Work product shows evidence of a more intuitive use of subject-area rules to gain insight beyond literal application of rules	Work product treats task efficiently, with a few minor or inconsequential inefficiencies	Work product shows strong evidence of novel or creative use of conventional ideas and/or clear evidence of original ideas	Work product is purposely and intentionally structured around a set of core concepts to organize and explain findings	Work product is integrated and connected in an effective fashion	Work product shows evidence of a cogent, coherent solution strategy for the problem
Strategic Thinker – 5					
Work product shows evidence of applying subject-area rules in an insightful fashion beyond literal application of rules	Work product is predominantly efficient in its treatment of the task, but some inefficiency may still be apparent	Work product shows strong evidence of proper use of conventional ideas and some evidence of original or novel ideas	Work product uses and incorporates a set of core concepts to organize and explain findings	Work product shows convincing evidence of integration or connection among all its elements	Work product shows evidence of a full and complete solution strategy for the problem
Emerging Strategic Thinker – 4					
Work product shows some evidence of applying subject-area rules in an insightful fashion beyond literal application of rules	Work product shows evidence of efficiencies in its treatment of the task, but has several areas where efficiency could be improved	Work product shows consistent evidence of proper use of conventional ideas and at least some evidence of original ideas or novel variations on	Work product uses and incorporates concepts to organize and explain findings but with some inconsistency	Work product shows evidence of integration or connection among all its elements with some places that are not well integrated or onnected	Work product comes very close to a complete solution strategy
		Accomplished Novice – 3			
Work product applies subject-area rules correctly and uses rules to demonstrate limited insight into subject area	Work product has areas of efficiency in its treatment of the task, but also contains significant inefficiencies	Work product shows consistent evidence of proper use of conventional ideas	Work product uses and incorporates concepts in a limited fashion to organize and explain findings	Vork product shows mited evidence of ntegration or onnection among ll its elements and ne or more places here lack of ntegration or onnection is a roblem	Work product fails short of a complete solution strategy
		Novice – 2			
Work product applies subject-area rules in a procedural (literal) fashion	Work product is inefficient in its treatment of the task	Work product shows some evidence of proper use of conventional ideas	Work product organizes and explains findings in a way that does not use concepts in any significant fashion	Vork product shows ttle evidence of ntegration or onnection among ll its elements and any places where ack of integration or onnection is a roblem	Work product fails well short of a complete solution strategy for the problem
		Emerging Novice – 1			
Work product applies wrong rules, applies rules inefficiently, or not at all	Work product is highly inefficient, redundant, or confused in its treatment of the task	Work product shows little or no evidence of proper use of conventional ideas	Work product does not use or incorporate concepts and/or does not explain findings coherently	Vork product shows ssentially no vidence of ntegration or onnection among ll its elements	Work product fails to show a solution strategy for the problem

Accomplished Novice – 3	
Work product shows consistent evidence of proper use of conventional ideas	Work product uses and incorporates concepts in a limited fashion to organize and explain findings
Novice – 2	
Work product shows some evidence of proper use of conventional ideas	Work product organizes and explains findings in a way that does not use concepts in any significant fashion
Emerging Novice – 1	
Work product shows little or no evidence of proper use of conventional ideas	Work product does not use or incorporate concepts and/or does not explain findings coherently

Modeling Standards

- **Making assumptions to simplify a situation**
- **Representing relationships between quantities with tables, graphs, & equations**
- **Generalizing**

Questions that we are left with . . .

- Training and support for teachers?
- Making expectations clear to teachers and students?
- Standardizing the standards?

EXIT SLIP

Please answer one . . .

- Share one teacher move that you use to teach students perseverance in problem solving.

- Pose a question to the group.