Critical Issues in Mathematics Education

Assessment of Mathematical Proficiencies in the Age of the Common Core

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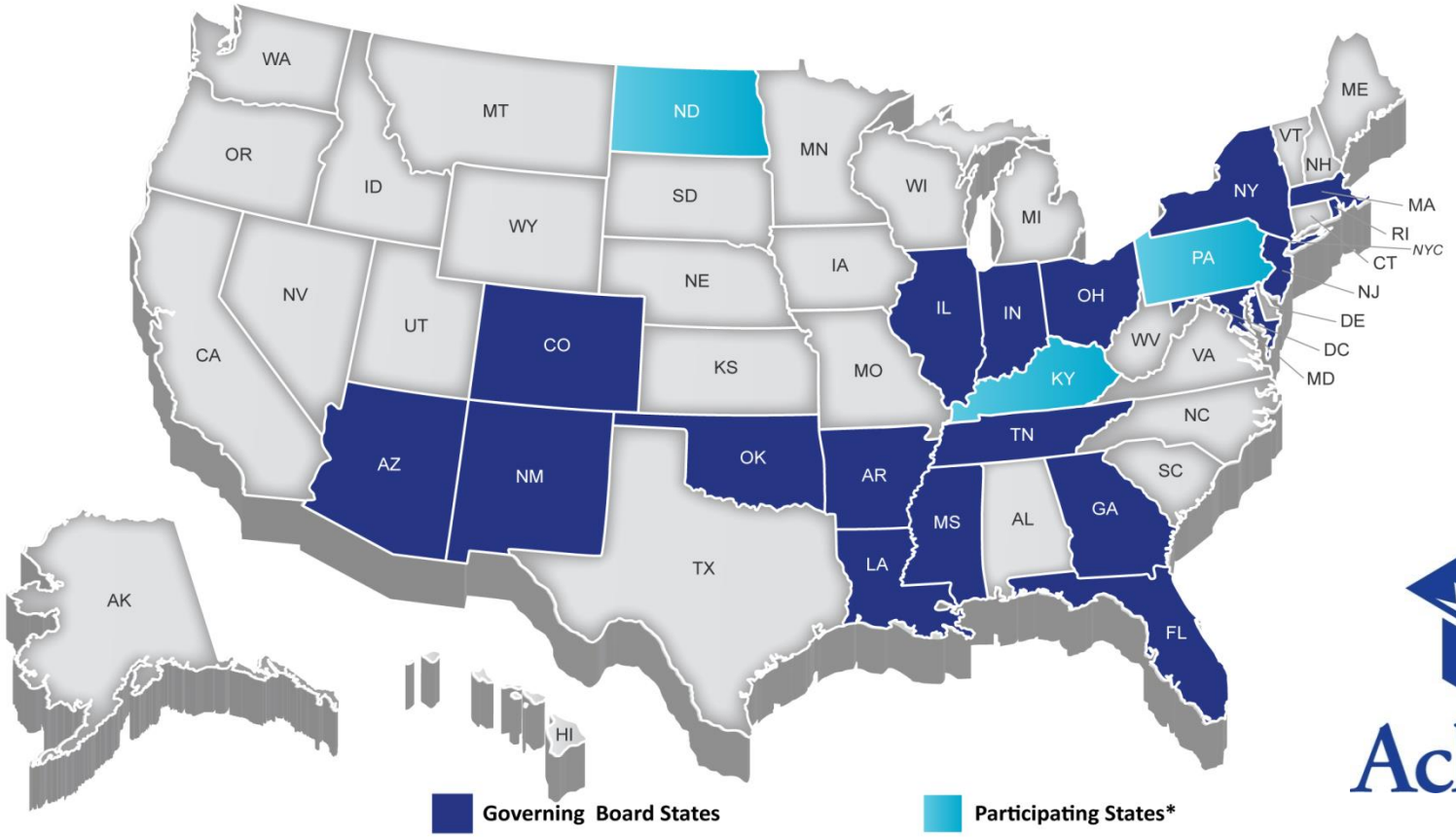
PARCC
Partnership for Assessment of Readiness for College and Careers

Overview

- The PARCC Design
- Developing PARCC: An Interconnected Approach
- Successes
- Challenges

Partnership for Assessment of Readiness for College and Careers (PARCC)



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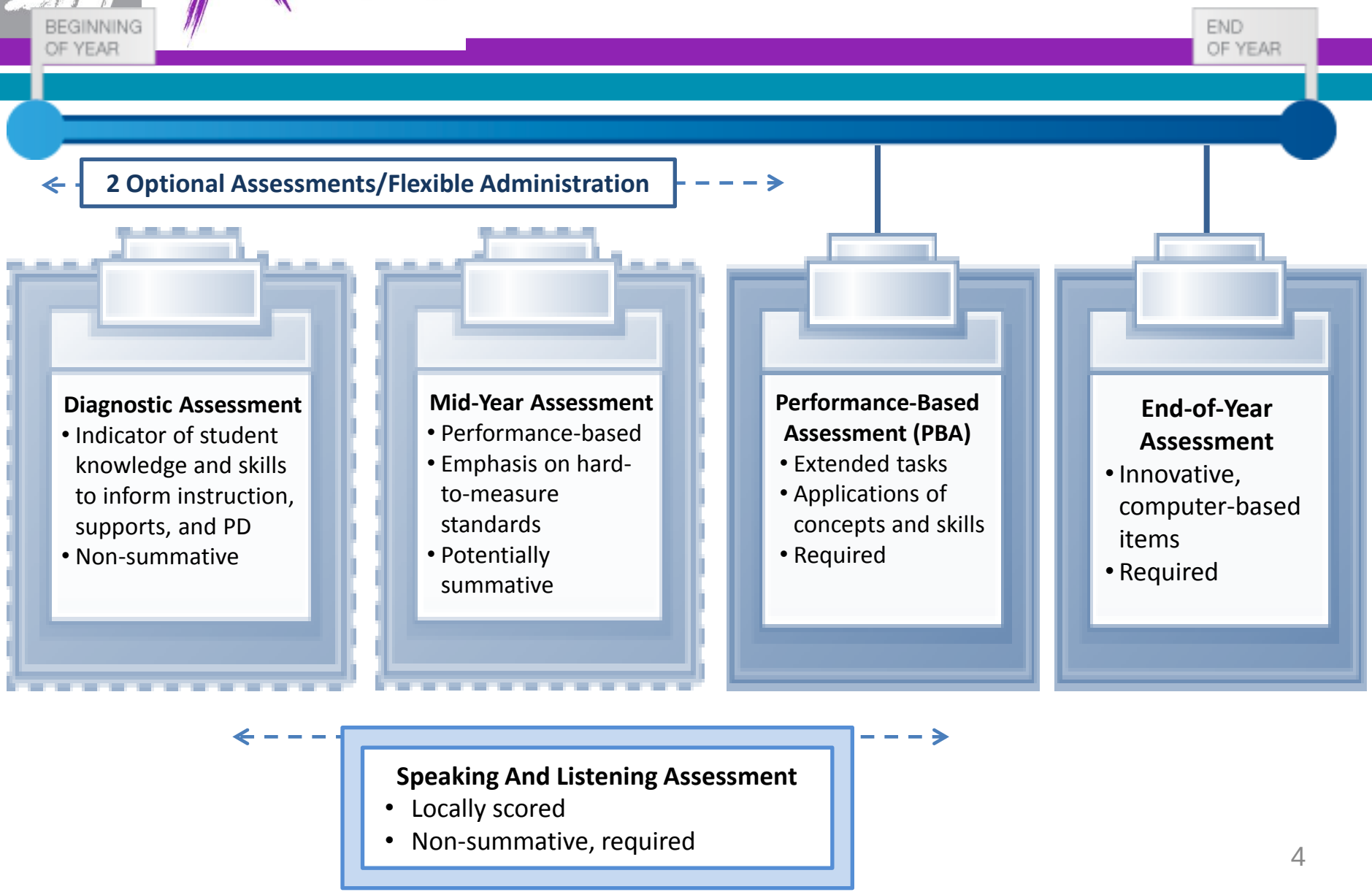
Assessment Design

English Language Arts/Literacy and Mathematics, Grades 3-11



PARCC Claims Structure: Mathematics

Master Claim: On-Track for college and career readiness. The degree to which a student is college and career ready (or "on-track" to being ready) in mathematics. The student solves grade-level /course-level problems in mathematics as set forth in the Standards for Mathematical Content with connections to the Standards for Mathematical Practice.

Total Exam Score Points:
82 (Grades 3-8), 97 or 107(HS)

Sub-Claim A: Major Content[1] with Connections to Practices

The student solves problems involving the Major Content[1] for her grade/course with connections to the Standards for Mathematical Practice.

~37 pts (3-8),
~42 pts (HS)

Sub-Claim B: Additional & Supporting Content[2] with Connections to Practices

The student solves problems involving the Additional and Supporting Content[2] for her grade/course with connections to the Standards for Mathematical Practice.

~14 pts (3-8),
~23 pts (HS)

Sub-Claim C: Highlighted Practices MP.3,6 with Connections to Content[3] (expressing mathematical reasoning)

The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements.

14 pts (3-8),
14 pts (HS)
4 pts (Alg II/Math 3 CCR)

Sub-Claim D: Highlighted Practice MP.4 with Connections to Content (modeling/application)

The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), *engaging particularly in the Modeling practice*, and where helpful making sense of problems and persevering to solve them (MP. 1),reasoning abstractly and quantitatively (MP. 2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

12 pts (3-8),
18 pts (HS)
6 pts (Alg II/Math 3 CCR)

Sub-Claim E: Fluency in applicable grades (3-6)

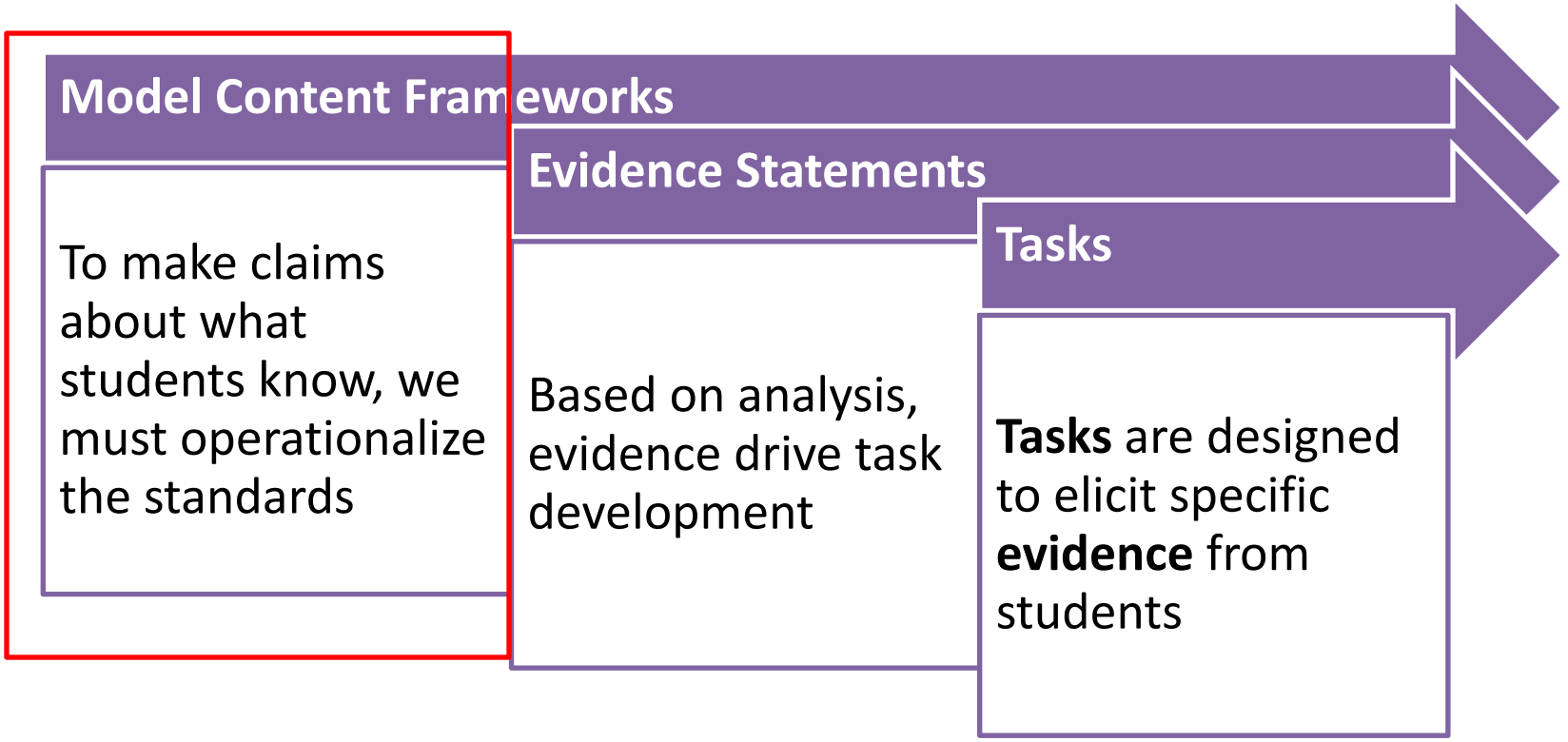
The student demonstrates fluency as set forth in the Standards for Mathematical Content in her grade.

7-9 pts (3-6)

[1] For the purposes of the PARCC Mathematics assessments, the Major Content in a grade/course is determined by that grade level's Major Clusters as identified in the *PARCC Model Content Frameworks v.3.0* for Mathematics. Note that tasks on PARCC assessments providing evidence for this claim will sometimes require the student to apply the knowledge, skills, and understandings from across several Major Clusters.
[2] The Additional and Supporting Content in a grade/course is determined by that grade level's Additional and Supporting Clusters as identified in the *PARCC Model Content Frameworks v.3.0* for Mathematics.
[3] For 3 – 8, Sub-Claim C includes only Major Content. For High School, Sub-Claim C includes Major, Additional and Supporting Content.

The PARCC Assessment System:
An Interconnected Approach

Evidence-Centered Design (ECD) for the PARCC Assessments

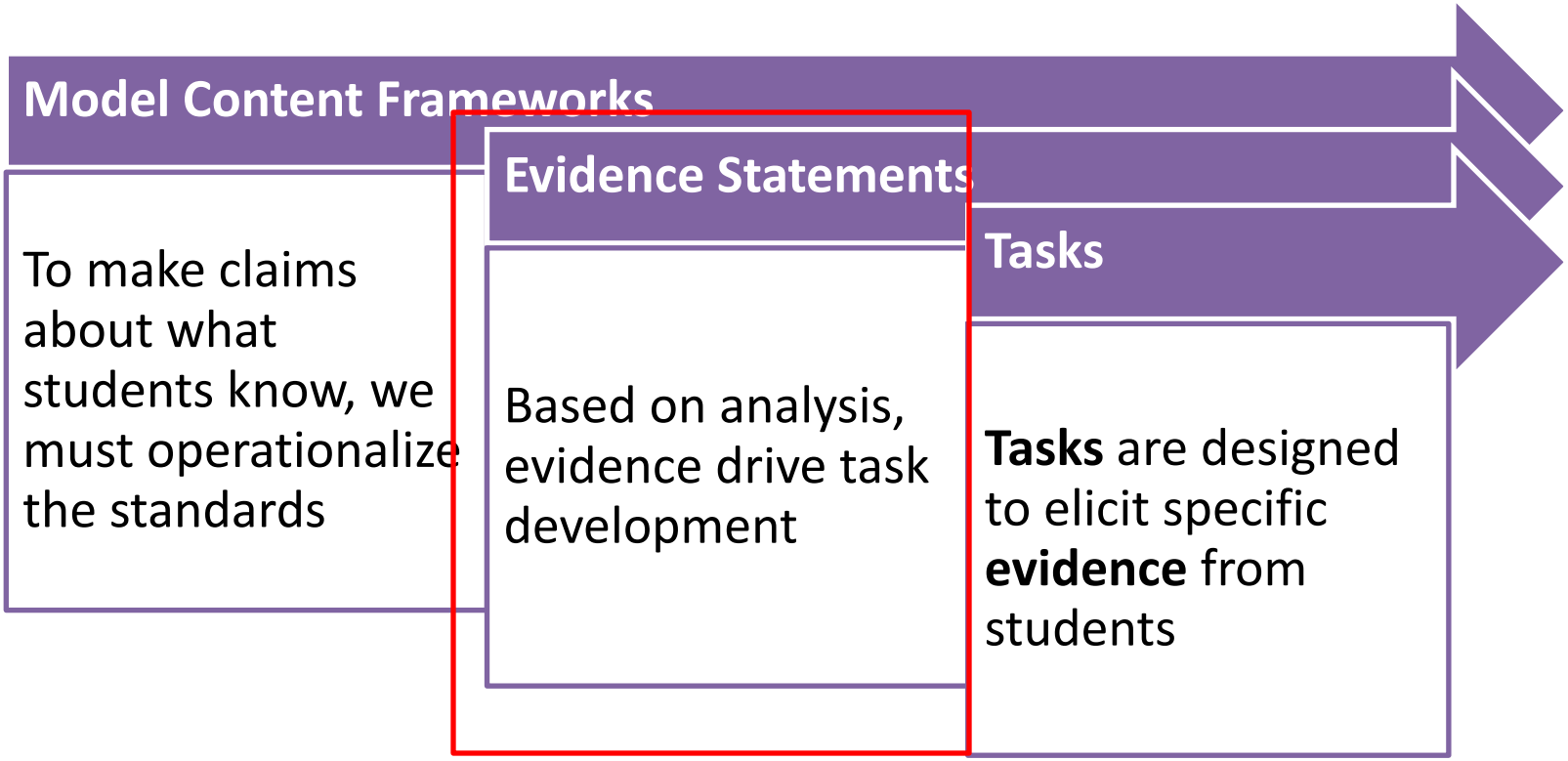


ECD is a deliberate and systematic approach to assessment development that will help to **establish the validity** of the assessments, **increase the comparability** of year-to year results, and **increase efficiencies/reduce costs**.

Approach of the Model Content Frameworks for Mathematics

- PARCC Model Content Frameworks provide a deep analysis of the CCSS, leading to more guidance on how focus, coherence, content and practices all work together.
- They **focus on framing the critical advances in the standards:**
  - Focus
  - Coherence
  - Rigor: Conceptual Understanding, Fluency, Application/Modeling

Evidence-Centered Design (ECD) for the PARCC Assessments



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Overview of Evidence Statements: Types of Evidence Statements

Several types of evidence statements are being used to describe what a task should be assessing, including:

- Those using **exact standards language**
- Those transparently **derived from exact standards** language, e.g., by splitting a content standard
- **Integrative evidence statements** that express plausible direct implications of the standards without going beyond the standards to create new requirements
- **Sub-claim C & D evidence statements**, which put MP.3, 4, 6 as primary with connections to content

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be eliciting from students, including:

1. Those using **exact standards language**

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 1/3^3 = 1/27$.	i) Tasks do not have a context. ii) Tasks center on the properties and equivalence, not on simplification. For example, a task might ask a student to classify expressions according to whether or not they are equivalent to a given expression.	MP.7

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be eliciting from students, including:

2. Those transparently **derived from exact standards** language, e.g., by splitting a content standard

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
8.F.5-1	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).	i) Pool should contain tasks with and without contexts.	MP.2, MP.5
8.F.5-2	Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	i) Pool should contain tasks with and without contexts.	MP.2, MP.5, MP.7

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be eliciting from students, including:

3. **Integrative evidence statements** that express plausible direct implications of the standards without going beyond the standards to create new requirements

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
4.Int.1	Solve one-step word problems involving adding or subtracting two four-digit numbers.	The given numbers are such as to require an efficient/standard algorithm (e.g., 7263 + 4875, 7263 – 4875, 7406 – 4637). The given numbers do not suggest any obvious *ad hoc* or mental strategy (as would be present for example in a case such as16,999 + 3,501 or 7300 – 6301, for example). i) Grade 4 expectations in CCSSM are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should be limited to 4 digits.	MP.1

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be eliciting from students, including:

4. **Sub-claim C & Sub-claim D Evidence Statements**, which put MP.3, 4, 6 as primary with connections to content

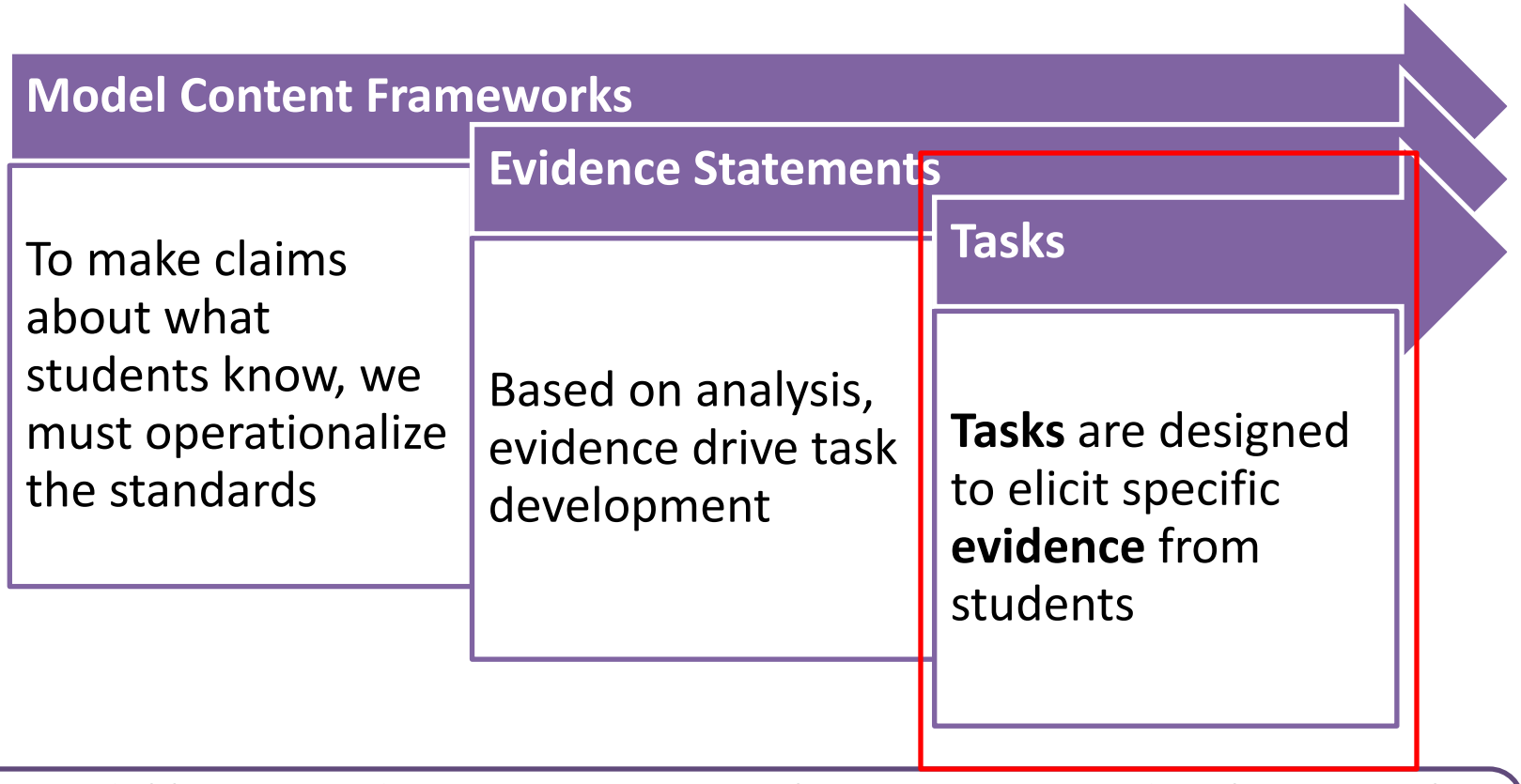
Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
HS.C.5.11	Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.11, involving any of the function types measured in the standards.	i) For example, students might be asked how many positive solutions there are to the equation $ex = x+2$ or the equation $ex = x+1$, explaining how they know. The student might use technology strategically to plot both sides of the equation without prompting.	MP.3

Overview of PARCC Mathematics Task Types

Task Type	Description of Task Type
I. Tasks assessing *concepts, skills and procedures*	• Balance of conceptual understanding, fluency, and application • Can involve any or all mathematical practice standards • Machine scorable including innovative, computer-based formats • Will appear on the End of Year and Performance Based Assessment components • Sub-claims A, B and E
II. Tasks assessing *expressing mathematical reasoning*	• Each task calls for written arguments / justifications, critique of reasoning, or precision in mathematical statements (MP.3, 6). • Can involve other mathematical practice standards • May include a mix of machine scored and hand scored responses • Included on the Performance Based Assessment component • Sub-claim C
III. Tasks assessing *modeling / applications*	• Each task calls for modeling/application in a real-world context or scenario (MP.4) • Can involve other mathematical practice standards • May include a mix of machine scored and hand scored responses • Included on the Performance Based Assessment component • Sub-claim D

For more information see PARCC Task Development ITN Appendix D.

Evidence-Centered Design (ECD) for the PARCC Assessments



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Grade 3 Sample Item

Grade 3 Mathematics (The Field)

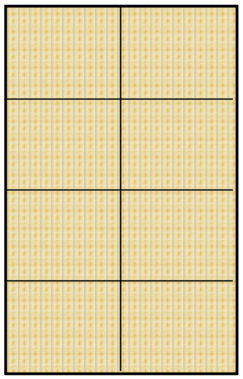
SAMPLE ITEM

Part B

Type a fraction different than 3/4 in the boxes that also represents the fractional part of the farmer's field that is planted with soybeans.

$$\frac{3}{4} = \frac{\square}{\square}$$

Farmer's Fields



Reset

Explain why the two fractions above are equal.

For More Item Specific Information

PARCC Math Sample Problems_GR3_The Field_PartAV2.pdf

- Type II Task – Part B is partial machine-scored and partial hand-scored
- Assessing the standard 3.NF.3b - a major focus in grade 3
- Assessing MP.2 (Reason abstractly and quantitatively) and MP.3 (Construct viable arguments and critique the reasoning of others)
- Unlike traditional multiple choice, there is more than one correct solution and difficult to guess.

High School Sample Type III Task

- MP.4: Model with Mathematics
- MP. 2: Reason Abstractly and Quantitatively
- F-LE.A.2:Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★
- F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

High School Sample Item, part a

Cellular growth

In a cellular regeneration experiment, Jaydon Laboratory found that for cells put in containers with a particular growth medium, the number of cells at the end of each week was double the number of cells at the end of the previous week.

The data for the first 6 weeks of the experiment are shown in the table. Fill in the blanks to complete the table for weeks 7-10.

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480
7	
8	
9	
10	

Submit Answer

High School Sample Item, part b

Cellular growth

About the task | CCSSM Alignment | Part a | **Part b** | Part c | Part d | Scoring

Assume that as the experiment continues, the number of cells at the end of each week continues to be double the number of cells at the end of the previous week. Let w_n represent the number of cells in the growth medium in week n. Drag the tiles to write a recursive definition for the sequence that represents the number of cells in the growth medium at the end of each week.

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

an integer	a real number	2	15	30
+	-	•	÷	w_{n-1}
$n \geq 0$	$n \geq 1$	$n \geq 2$	$n \geq 6$	$n \geq 15$

$w_1 =$ [ ]

$w_n =$ [ ] [ ] [ ] ,

where n is [ ] , such that [ ]

Submit Answer

High School Sample Item, part c

Cellular growth

About the task | CCSSM Alignment | Part a | Part b | **Part c** | Part d | Scoring

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

Let w_n represent the number of cells in the growth medium at the end of week n. Which of these statements are true about the explicit formula for w_n?

Select all that apply.

- $w_n = 15 + 15 \cdot 2(n - 1)$
- $w_n = 15 + 15 \cdot 2(n)$
- $w_n = 15 \cdot 2^{n-1}$
- $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n-1}$
- $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n}$
- $n \geq 1$, where n is an integer
- $n \geq 1$, where n is a real number
- n can be any real number

Submit Answer

High School Sample Item, part d

Cellular growth

About the task | CCSSM Alignment | Part a | Part b | Part c | **Part d** | Scoring

Write your answers to the following problem in your answer booklet.

Consider the table of data about the cellular regeneration experiment.

a. If the number of cells continues to grow according to the pattern shown in the table, at what week number will the number of cells exceed one billion?

b. Explain how the process you used to find the week number relates to either the recursive model or the explicit model you constructed in the previous questions.

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

Additional Sample Items for ELA/Literacy and Mathematics

Additional PARCC Sample Illustrative Items for Mathematics are available at the following link:

http://www.parcconline.org/samples/item-task-prototypes

Successes

- Design
- Content Frameworks
- Specifications
- Prototypes
- Item Review Process
- Performance Level Descriptors
- Type I Items

Challenges

- Type II and Type III Items

Want to help?

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
8.C.1.1	Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane Content Scope: Knowledge and skills articulated in 8.EE.6.	i) Note especially the portion of 8.EE.6 after the semicolon	MP.2, MP.3, MP.7, MP.8

Challenges

Want to help?

Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Calculator	Relationship to Mathematical Practices
HS.D.CCR	Solve problems using modeling: Identify variables in a situation, select those that represent essential features, formulate a mathematical representation of the situation using those variables, analyze the representation and perform operations to obtain a result, interpret the result in terms of the original situation, validate the result by comparing it to the situation, and either improve the model or briefly report the conclusions. Content scope: Knowledge and skills articulated in the Standards as described in previous courses and grades, with a particular emphasis on 7- RP, 8 – EE, 8 – F, N-Q, A-CED, A-REI, F-BF, G-MG, Modeling, and S-ID.	i) Tasks will draw on securely held content from previous grades and courses, include down to Grade 7, but that are at the Algebra II/Mathematics III level of rigor. ii) Task prompts describe a scenario using everyday language. Mathematical language such as "function," "equation," etc. is not used. iii) Tasks require the student to make simplifying assumptions autonomously in order to formulate a mathematical model. For example, the student might autonomously make a simplifying assumption that every tree in a forest has the same trunk diameter, or that water temperature is a linear function of ocean depth. iv) Tasks may require the student to create a quantity of interest in the situation being described.	Y	MP. 4; may also involve MP. 1, MP. 2, MP. 5, MP. 6, MP. 7

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