Martiniello, M. & Wolf, M. K. (2012). Exploring ELLs' understanding of word problems in mathematics assessments: The role of text complexity and student background knowledge.

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In case 3, we discuss possible challenges that ELLs face in tackling word problems in mathematics assessments. ELLs display a broad range of mathematics knowledge and ability in the classroom, but in state and national assessments of mathematics, ELLs are among the lowest scoring groups. For example, on the 2009 National Assessment of Educational Progress (NAEP), 72 percent of eighth-grade ELLs scored below "basic" in mathematics, as compared with 26 percent of non-ELL students (National Center for Education Statistics 2009), and on the 2010 California state accountability assessment in general mathematics, only 12 percent of eighth-grade ELLs scored at the "proficient" level, as compared with 27 percent of English-only eighth graders (California Department of Education 2010).

As researchers strive to ensure that assessments given to ELLs are valid and fair, we seek to understand whether this pattern of low performance by ELLs in mathematics assessments is associated with any of the following student characteristics:

1. Mathematics knowledge and skills (related to opportunity to learn and accessibility of mathematics instruction)
2. Understanding of the test language (related to the interaction between the complexity of the text and the ELLs' level of English language proficiency)
3. Familiarity with sociocultural references (related to the overlap between the students' background knowledge and the sociocultural practices, beliefs, or objects depicted in these assessments).

Disentangling these three factors, we believe, is not only important for assessment specialists, but it can also be very helpful to classroom teachers as they strive to improve ELLs' performance in such assessments. Mathematics teachers learn to focus on the first factor: students' content knowledge. In their practice, mathematics teachers constantly gauge where students are, what students need to learn, and what teaching activities will help students acquire the desired mathematical skills. Less often, mathematics teachers focus on the second and third factors: how English language proficiency (ELP) and background knowledge affect their students' mathematics performance.

In this case we share some of the lessons that we have learned in analyzing large-scale mathematics assessments and conducting think-aloud interviews with ELL students responding to test items. Through think-aloud interviews, we were able to learn more about ELL students' problem-solving processes as well as comprehension processes of the language and cultural references presented in word problems. We will discuss three examples of word problems that require students to apply mathematical concepts or procedures to scenarios depicting real-world situations. These items, from fourth- and eighth-grade mathematics assessments, were identified as posing greater difficulty for ELLs compared to non-ELLs with equivalent mathematics proficiency.

For each example, we will first present the word problem. We will describe potential challenges for ELLs that arise from the text complexity and from the use of cultural references that require background knowledge to understand the item scenario. We will then illustrate some of these challenges with transcriptions of ELLs' think-aloud responses to these word problems. Last, we will suggest ways in which teachers can use the information provided in the think-aloud responses to inform their teaching.

Assessment Item 1: Polysemy and Background Knowledge Issues

Understanding word problems that involve polysemous words and cultural references can be particularly challenging for ELLs taking standardized mathematics assessments in English (Martiniello, 2010, 2008). Polysemous words are words with different meanings or connotations, depending on the context provided by the text or discourse. We find many examples of words with multiple meanings in everyday language and mathematical discourse (e.g., *table* meaning a piece of furniture, *table* meaning a display of data). We showed the following assessment item at the beginning of case 3, but we repeat it below in full, with the accompanying multiple-choice answers, for the reader's convenience as we discuss the potential difficulties related to polysemous vocabulary and background knowledge required to understand the context or scenario depicted in the problem.

The pictograph below shows the amount of money each fourth-grade class raised for an animal shelter.

Amount Raised by Each Class

Class	Amount Raised
Ms. Smith	$ $ $ $ $ $
Mr. Powell	$ $ $ $
Ms. Carly	$ $ $ $ $ $ $ $
Mr. Roper	$ $ $ $ $ $

If Mr. Powell's class raised $20 and Mr. Roper's class raised $30, how much money does one $ represent?

A. $1

B. $4

C. $5

D. $20

Challenges to ELLs

In this word problem, the sociocultural context and complex text language can pose difficulties to ELLs. We will describe these two sources of difficulty briefly, and then we will illustrate the challenge posed to ELLs by words with multiple meanings.

Syntax

The language in the item stem has complex syntactic features that are likely to hamper text comprehension for ELLs as well as for struggling readers. Its two sentences are long and dense, including a reduced relative clause (omitting the relative pronoun: "the amount of money *that* each fourth-grade class raised") and multiple conditional clauses.

Vocabulary

Along with high-frequency words (e.g., "fourth," "grade," "class," "money," "animal"), which are likely to be known by ELLs at beginning-intermediate levels of ELP, this word problem contains words used less frequently. These words, including "pictograph," "represent," "raised," and "shelter," may pose difficulty for ELLs at all levels of ELP. The words "pictograph" and "represent" are discipline-specific and general academic words, respectively. As such, teachers are likely to target the use of these words in classroom discourse.

In the item's scenario, the verb "raised" indicates the [past] action of collecting funds to support an animal shelter. The polysemous word "raise" occurs frequently in conversational and written English and some of its multiple meanings and uses may be known by ELLs at intermediate levels of ELP. In their classrooms, ELLs may be asked to "raise your hands," or "raise the volume." In settings beyond their classrooms, they may hear the expression "raising the rent," or "receiving a raise." Making sense of words like "raise," which carry multiple meanings or connotations depending on the context, presents a double challenge. The reader

must be knowledgeable of the particular meaning or connotation used or must be aware of cues in the context to infer the particular meaning.

Context

Although situated in a school setting, the problem scenario may be unfamiliar to some ELLs, as it involves a particular sociocultural practice of raising funds for a cause—in this case, the protection of animals—which might not be part of all ELLs' cultural or social background. The think-aloud interview below illustrates these challenges.

Interview on assessment item 1

Rosa is a Spanish-speaking fourth grader at intermediate-advanced levels of English language proficiency. Rosa completed first and second grade with honors in her native Puerto Rico. Her family moved to the United States eighteen months earlier, and Rosa is now enrolled in a fourth-grade dual language program.

To gauge her decoding skills and reading fluency, we asked Rosa to read aloud the item text in English and explain what the item required her to do. Rosa had no problem decoding the text of the word problem. She spoke primarily in Spanish during the interview, switching to English sporadically. Below is the transcription of Rosa's responses. Translations from Spanish to English appear in parentheses. When the student uses words in English, they appear in quotation marks.

Student: Este "pictograph" es como una foto que te están enseñando abajo.
(This "pictograph" is like a picture that is shown below.)
Cuánto es la, cuánto es la… uhm… dinero cada cuarto grado [*pause*]
(How much is, how much is the … uhm… money each fourth grade)
Uhm… aumentar, aumentar para el "animal shelter" [*pause*]

(Uhm… increase, increase for the "animal shelter")

Student: Aumentar, "raised," es tener más, (To increase, "raised," is to have more)

Como… yo tengo ahora 5 dólares y después tengo 10 dólares ,

(Like… now I have 5 dollars and later I have 10 dollars.)

Cuánto dinero I "raised"? 5 dólares, puedo aumentar 5 para tener 10.

(How much money did I raise? 5 dollars, I can increase by 5 to have 10.)

"If Mr. Powell class raised 20, how much money does 1 cash sign represent?"

Unlike ELLs at beginner levels of ELP, Rosa could comprehend the item's complex syntactic structures. But not surprisingly, Rosa was not able to infer the particular meaning of the verb "raised" associated with the sociocultural practice of fundraising. Rosa understood "raised" as "increased." Although "increase" is a common meaning of the verb "raise," it is not the intended meaning in the particular context of this word problem Rosa's interpretation of "raised" as "increased" resulted in the misunderstanding of both the word problem scenario and the required task.

Polysemous words can be very challenging for ELLs. These students are likely to know only one of the multiple meanings of a polysemous word, usually the most familiar one, as in Rosa's case.

Polysemy: What can teachers do?

Several approaches offer teachers possibilities for reducing the challenges that polysemous words present to ELLs:

- For a given topic or lesson, identify polysemous words that students will encounter in the mathematical discourse used in the classroom or textbook and provide explicit instruction in the meaning of polysemous words depending on the context.

- When teaching a mathematical concept, ensure that ELLs understand the particular meaning of the polysemous words used, especially if ELLs are likely to be familiar with meanings that are different from those used in their classroom or textbook.
- Use polysemous words that are part of your classroom discourse as opportunities to teach ELLs about language. Progressively expose ELLs to different connotations of the polysemous words, and help them infer a word's meaning on the basis of the context.

Sometimes the context in which polysemous words are used assumes familiarity with social and cultural references, as the next item illustrates.

Assessment Item 2: Background Knowledge Issues

The following example provides an opportunity to consider the importance of gauging and building background knowledge about social, historical, or cultural references for ELL students:

> Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?
>
> A. $2.50
> B. $3.00
> C. $3.75
> D. $4.50

When Jacob, a former ELL student, answered this assessment question, he chose option A, $2.50. Jacob provided his reasoning to support his answer. How do you think Jacob's prior experiences and background knowledge influenced his interpretation of the question and strategy to solve it?

Challenges to ELLs

A challenge for ELLs is understanding the sociocultural references and the polysemous vocabulary in this word problem. These two are very closely related in this example, since the sociocultural referent provides the context needed to infer the intended meanings of the polysemous words.

Vocabulary

To understand this problem scenario containing the polysomous words “check” and “tip”, ELLs must know first that a “restaurant check” is not the same as a “bank check” but is instead the bill for the amount of the meal, and second that the word “tip” in this context is a gratuity given to waiters or waitresses for their service.

Context

In the school context, ELLs at beginning-intermediate levels of ELP might have learned the meaning of the noun “tip” as used in phrases such as “the tip of your nose.” Some ELLs may be familiar with the meaning of “tip” in a restaurant situation if they have received instruction about this word’s multiple meanings in context, and if the experience of dining at a restaurant is relevant in their lives.

Background Knowledge

Tipping behavior is very specific to social and cultural groups. This word problem assumes that readers know that a “tip” is a gratuity offered to those who provide a service and that the amount of the tip is customarily a percentage of the amount of the bill—in this case, 15 percent. Thus, there is an implicit assumption that tipping in restaurants is a common practice and that the tip varies, depending on the total bill.

Interview on assessment item 2

Jacob is a Spanish-speaking eighth-grader with advanced ELP. Born in Mexico, Jacob moved to the United States with his family at age 5. He entered first grade speaking only Spanish. He was classified as an ELL for most of his schooling until the eighth grade, when Jacob was re-designated as proficient in English. His interview was conducted in English.

Student: Of the following, which is the closest approximation of a fifteen percent tip on a restaurant check? Of two four. Twenty-four ninety-nine… And I also remember when we go to restaurants, and the check is like that, and my mom gives two fifty. So, yeah. That's also how I remembered it.

Interviewer: OK. Your mom gives two fifty? And what percent is that?

Student: Fifteen? I don't know.

Interviewer: OK. So what answer did you get?

Student: I put two fifty… I got two fifty because I thought about how much my mom gave.

Interviewer: And so, what do you think the problem is about? Like, if you wanted to describe the problem in your own words.

Student: Like, for people to know, like, how much, like, so they won't give too much or they won't give too little of tips.

Interviewer: Of tips?

Student: Yeah.

Interviewer: OK, so what do you think the problem is asking you to do?

Student: Like how many, fifteen percent tip on a restaurant check you're supposed to give for twenty-four dollars and ninety-nine cents?

Jacob understood the meaning of "tip" and "restaurant check" but could not make sense of them as part of a mathematical problem related to percents. In his experience, his mother gives $2.50 as a gratuity to waiters for their service in restaurants. For Jacob, $2.50 is a fair amount, "not too much, not too little." As we mentioned previously, tipping in restaurants is a highly situational and sociocultural practice. In some countries, tipping waiters and waitresses is not expected. In other countries, small tips are given as tokens of appreciation, but the tip amount is not in direct proportion to the total check amount. In the United States, it is customary to tip waiters and waitresses for their service. The amount of the tip is somewhat discretionary, but it is usually a percentage of the total amount of the bill, and it often depends on the quality of the service.

In this case, Jacob is familiar with the concept of "tip" because he has seen his mother tip in restaurants. But Jacob seems to conceive of the tip as a constant amount and not a variable amount that depends on the total bill. Most of the time, his mother gives $2.50—"not too much, not too little." Jacob disregards the reference to 15 percent and provides his best answer of $2.50, based on his background knowledge.

Background knowledge: What can teachers do?

Textbooks and assessment problems often require particular sociocultural background knowledge. The following approaches can reduce the challenge that ELLs face:

- Check and, if needed, build students' background knowledge about social and cultural references contained in textbooks and assessment problems. Do not assume that ELLs have had the prior experiences needed to understand the problem scenarios.

- Devote time to building background knowledge for ELLs. This process can be time-consuming and frustrating for teachers who want to spend most of their time teaching mathematical content. But ELLs need extra time to build the background knowledge that non-ELLs bring to school by virtue of speaking English at home and growing up in a particular environment.
- Use word problems as opportunities for students to talk about and discuss mathematical ideas in relation to their cultural traditions and practices.
- When introducing a topic, such as percents, for example, determine what your students know. Ask them, for instance, whether or where they have seen percents in their lives. Then you can start working with word problems that are relevant to the backgrounds of your ELLs.
- After your students have the necessary conceptual understanding and the procedural fluency, move to examples with novel situations. Create opportunities to encourage your ELLs to develop connections and communication as described in the Process Standards (NCTM 2000).

Assessment Item 3: Reading Comprehension Issues

When George, a recently arrived ELL from a Spanish-speaking background, answered the following assessment item, he was able to read the text aloud in English, but his answer was incorrect:

A group of students has a total of 29 pencils and everyone has at least one pencil. Six students have 1 pencil each, five students have 3 pencils each, and the rest of the students have 2 pencils each. How many students have only 2 pencils?

A. 4
B. 6
C. 8
D. 9

Because George could read all the English words in the item, his teacher concluded that George lacked the mathematical knowledge required to answer this question. What additional information would you gather to prove or disprove this conclusion?

Challenges to ELLs

In this assessment problem, neither vocabulary nor background knowledge are likely to be a major source of difficulty for ELLs, with the possible exception of students with low ELP or with little or interrupted formal schooling. But the problem text is relatively long and dense in information. This presents two important challenges to ELLs. One is making sense of the text in the item's multiple sentences. This relates to the students' reading comprehension in English. The other is modeling the situation mathematically, translating the text into a mathematical model or writing an equation that allows students to solve the problem (CCSSI 2010). These challenges are not unique to ELLs but are more pronounced for this population.

Context

In this case, the scenario presented in the item is likely to be relevant to most ELLs and connected to their background knowledge, since the problem is situated in the classroom.

Vocabulary

Most of the words in the problem are from a school-based vocabulary or lexicon and are likely to be learned when ELLs enter school (e.g., "students," "pencils").

Syntax, Discourse, and Cognitive Load

The word problem contains multiple sentences, each including relevant mathematical information that needs to be represented and processed before students can solve the problem. Lexical cohesion, which includes a chain of related words to refer to the same thing, requires the students to recognize relationships between words in this item. This increases the item cognitive load and working memory.

Interview on assessment item 3

As we indicated earlier, George is a recently arrived ELL, and he has a beginning-intermediate level of ELP. Born and raised in Mexico, George received most of his formal education in Spanish. George has attended a school in the United States for over a year and is now in eighth grade. His interview was conducted in English.

Student: [*Reads the English text aloud*] A group of students has a total of twenty-nine pencils and everyone has at least one pencil. Six students have one pencil each, five students have three pencils each, and the rest of the students have two pencils each. How many students have only two pencils?

Student: Um, I'm going to add the one where it says six students have one pencil and then the three and I'll probably get four… And then I'll add the two.

Interviewer: OK, why is that? Because the question is asking what?

Student: Um. How many students have only two pencils?

Interviewer: OK, so why don't you go ahead and do that and just explain to me as you're doing it?

Student: I'm going to add 3, 2, and 1 and I'll get 6. That's how I did it.

In his think-aloud interview, George was able to read aloud all the words in the item. He could also identify the sentence posing the item question, distinguishing it amid the long scenario. George scribbled on his booklet all the numbers appearing in this sentence (i.e., 3, 2, 1); he did not include the numerals written in words (i.e., "six", "five"). However, he could neither comprehend the lengthy text he was reading nor could he negotiate its mathematical meaning. George resorted to working with the numbers within the text: 3, 2, and 1. He added these numbers. The addition (6) was given as one of the options (B), and George selected it as his response.

Reading comprehension: What can teachers do?

Teachers can use the following approaches to alleviate the challenges in reading comprehension that textbooks and assessment problems present to ELLs:

- Assess students' comprehension of the text in test items and textbook problems. Do not assume that correctly decoding words means that the student understands and makes meaning of the text. Reading aloud or being able to repeat the phrases in the text does not equal comprehension. Inquire further; ask students to rephrase the scenario in their own words.
- Explain the meaning of phrases such as "a total of," "at least," and "the rest of" as used in this item and more generally as part of mathematical discourse. Ask students to communicate (orally and in writing) by using these phrases.

- Give opportunities for students to work in small groups to make meaning of the problem.
- Use students' native language, if possible, and encourage students to use their native language to help them make sense of the text.
- Guide your ELLs in unpacking the meaning of complex text in each of the item sentences. Whenever possible, guide them in using models or diagrams that interpret the sentences mathematically. Make it a regular practice in the classroom to unpack the problems so that students learn to do this on their own when faced with assessment problems.
- Encourage the use of non-linguistic forms of representation to communicate the mathematical meaning embodied in the text (Martiniello 2009). Help students become fluent in (a) moving across different forms of representation and (b) translating text into mathematical models (see chapter 12).

Summary

In case 3, we have discussed some of the comprehension challenges that ELLs encounter in word problems in mathematics assessments. Our examples illustrate how ELLs at various levels of ELP might struggle with some aspects of—

- text complexity (difficult vocabulary with multiple meanings, or long and complex sentences); and
- unfamiliar sociocultural references (that are not relevant to the students' background knowledge or prior experiences).

We have made some suggestions for teachers to consider in helping their ELL students improve their English language proficiency, comprehension skills, or background knowledge while engaging in mathematical discourse involving word problems. Although being mindful of

teaching both language and content is challenging to mathematics teachers, it is of paramount importance to target both the development of mathematical understanding and language comprehension of ELL students tailoring instruction to their levels of English language proficiency. ELL students should have as many opportunities as possible to communicate their understanding of mathematics word problems verbally. If the students are struggling with expressing themselves in English, teachers might ask them to represent their understanding in different ways (e.g., their native language, symbols, drawing). In our research, we have explored these understandings through think-aloud interviews of ELLs responding to word problems. Through continuous discussions and communication activities, teachers can not only identify and address challenges that ELL students face, but also generate multiple opportunities for students to build their background knowledge, master complex language skills, and engage in rich mathematical discourse.

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