Assessment of Mathematical Proficiencies in the Age of the Common Core

MSRI CIME Conference
April 3-5, 2013

Welcome!

Thanks To:

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- The MSRI Education Advisory Committee
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The Organizers:

- Mark Hoover Thames*
- Kristin Umland*
- Noah Heller
- Alan Schoenfeld

*co-chairs

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- Mark Hoover Thames*
- Kristin Umland*
- Noah Heller

*co-chairs

We’re in for a lot of fun:

- Background
- Live Interview
- Working groups
- Hearing from people on the front (and back) lines.
- Lots of Discussion

Setting the Stage: An introduction to the Fundamental Challenges of Assessment

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The Challenge for this talk:

To make (some) sense of:

- The Common Core Standards
- High Stakes Assessment – what it may be, what its impact may be
- Formative Assessment as a mechanism for making good stuff happen in our classrooms
- Systemic issues we need to address if this is all to “work” . . .

. . . and to raise some questions for us to think about over the span of the workshop.

Let’s start with context.

The Common Core State Standards in Mathematics (CCSSM) now exist.

COMMON CORE STATE STANDARDS FOR

Mathematics

The CCSS-M have two main foci:

Content and Practices

Content:

Key words are “focus and coherence.”

Practices:

The ways in which kids engage with mathematics, building (we hope) productive mathematical dispositions and habits of mind.

The Practices in CCSS-M:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments…
- Model with mathematics
- Use appropriate tools strategically
- Attend to Precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

But what do the words in CCSS-M mean?

Huh?

What do I mean, what do they mean?
The words are there on the page…

Do you know the phrase

W Y T I W YG

?

In a high stakes assessment context, tests drive instruction as much or more than the standards do.

This can be a positive or negative force, vis-à-vis standards and classroom practices.

Some examples . . .

23 What is the y-intercept of the graph of $4x + 2y = 12$?

A -4

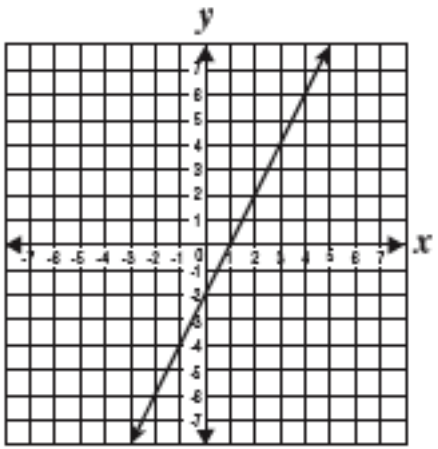
B -2

C 6

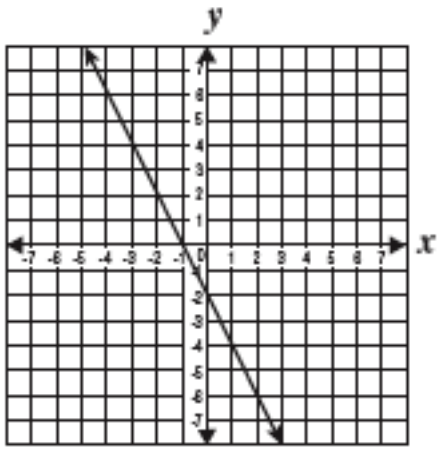
D 12

CSA00299

25 Which *best* represents the graph of $y = 2x - 2$?



A



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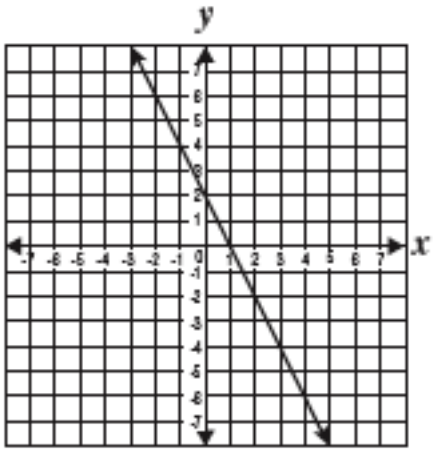
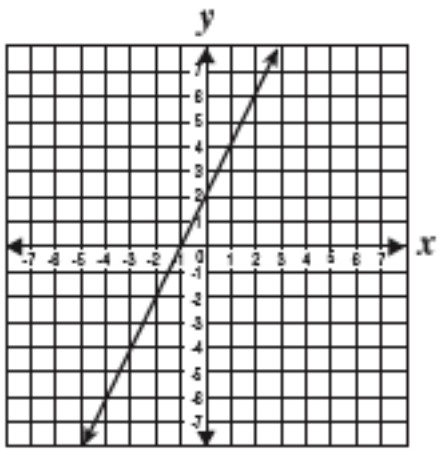
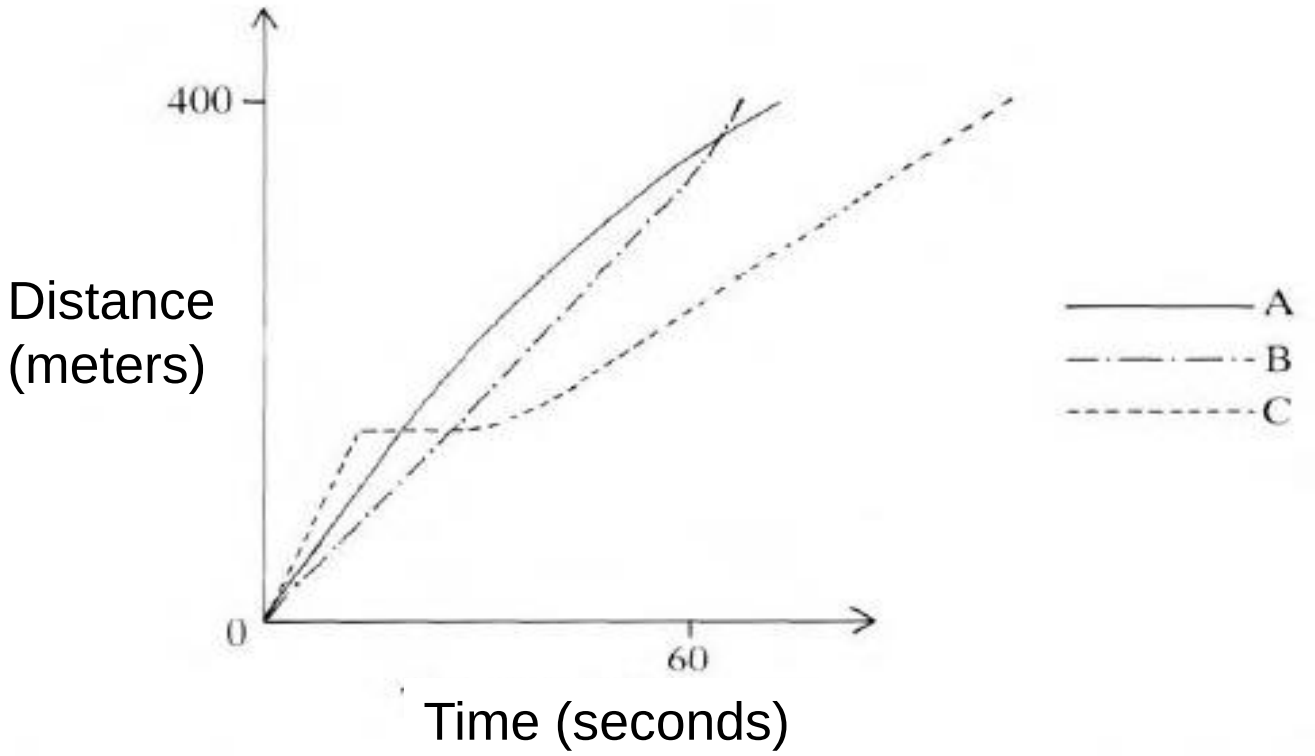


B



D

Hurdles Race



This is a rough sketch of 3 runners' progress in a 400 meter hurdle race. Imagine that you are the race commentator. Describe what's happening as carefully as you can. You do not need to measure anything accurately.

Think of the Content involved:

- Interpreting distance-time graphs in a real-world context
- Realizing “to the left” is faster
- Understanding points of intersection in that context (they’re tied at the moment)
- Interpreting the horizontal line segment
- Putting all this together in an explanation

Think of the Practices involved:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments…
- Model with mathematics…

25% Sale, Part 1

In a sale, all the prices are reduced by 25%.

Julie sees a jacket that cost $32 before the sale. How much does it cost in the sale?

25% Sale, Part 2

In the second week of the sale, the prices are reduced by 25% of the previous week's price.

In the third week of the sale, the prices are again reduced by 25% of the previous week's price.

In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

Alan says that after 4 weeks of these 25% discounts, everything will be free. Is he right? Explain your answer.

Again:
Core content, central practices.

IF the exams tied to CCSS-M stay true to the Common Core Standards, there will be a revolution in testing, and these will compel (or at least demand) changes in classroom practices.

We’re at a branch point:

A. What can we do to help prepare teachers and students for the rich kinds of tasks I’ve just shown?

B. What are the high stakes tests, Produced by PARCC and Smarter Balanced, really going to look like?

A. How do we prepare kids to do well on rich assessment tasks?

There are resources on the web:

- Mathematics Assessment Project
- Silicon Valley Math Initiative
- Illustrative Mathematics
- Inside Mathematics
- Math Forum
- NCTM

web sites

And, we can do more…

By way of formative assessment.

The purpose of formative assessments is not simply to show what students “know and can do” after instruction,
but to reveal their current understandings so you can help them improve.

Important Background Issues

1. Formative assessment is *not* summative assessment given frequently!
2. Scoring formative assessments rather than or in addition to giving feedback destroys their utility (Black & Wiliam, 1998: “inside the black box”)
3. This is HARD to do. Tools help!

A Tool:

The “Formative Assessment Lesson”, or FAL, which offers:

A rich “diagnostic” situation

and

Things to do when you see the results of the diagnosis.

Here are two examples.

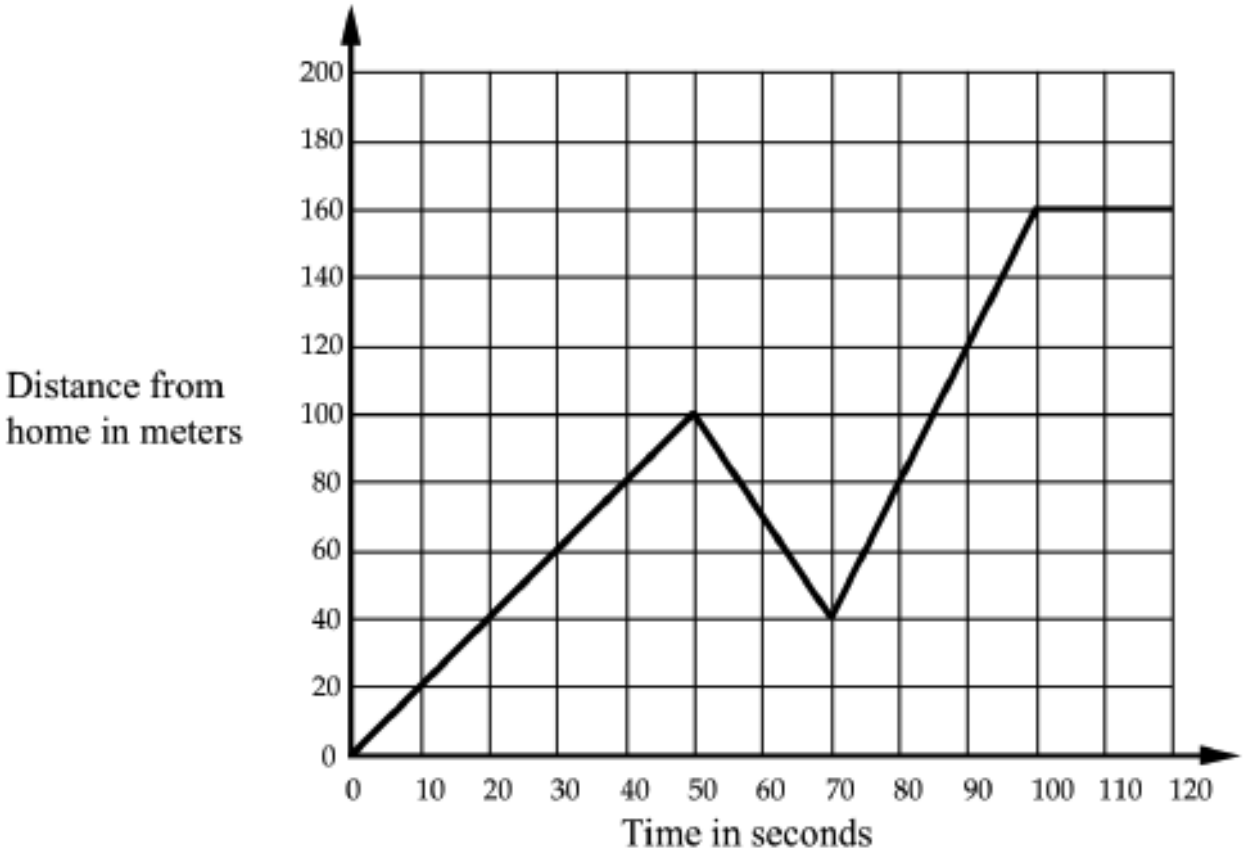
A Challenge:

We know that students have many graphing misconceptions, e.g., confusing a picture of a story with a graph of the story in a distance-time graph.

Here's one way to address the challenge.

Before the lesson devoted to this topic, we give a diagnostic problem as homework:

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



Describe what may have happened. Is the graph realistic? Explain.

We point to typical student misconceptions and offer suggestions about how to address them…

Common Issues	Suggested questions and prompts
Graph interpreted as a picture E.g. The student assumes that as the graph goes up and down, that Tom's path is going up and down. E.g. The student assumes that a straight line on a graph means that the motion is along a straight path. E.g. The student thinks the negative gradient means Tom has taken a detour.	• *If a person walked in a circle around their home, what would the graph look like?* • *If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?* • *In each section of his journey, is Tom's speed steady or is it changing? How do you know?* • *How can you work out Tom's speed in each section of the journey?*
Graph interpreted as speed v time The student has interpreted a positive gradient as speeding up and a negative gradient as slowing down.	• *If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like?* • *How does the distance change during the second section of Tom's journey? What does this mean?* • *How does the distance change during the last section of Tom's journey? What does this mean?* • *How can you tell if Tom is travelling away from or towards home?*

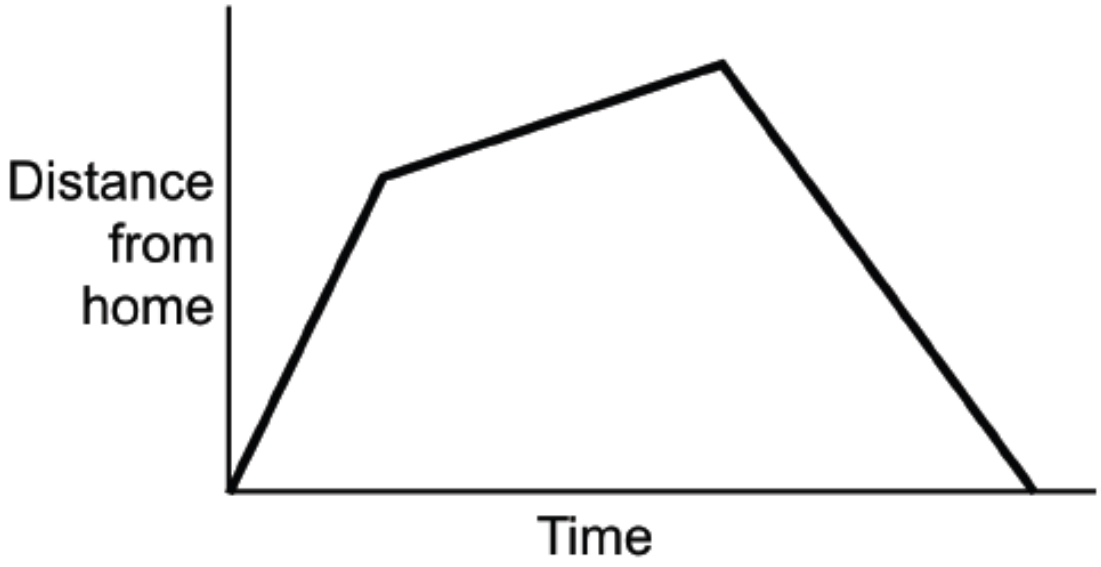
The lesson itself begins with a diagnostic task…

Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

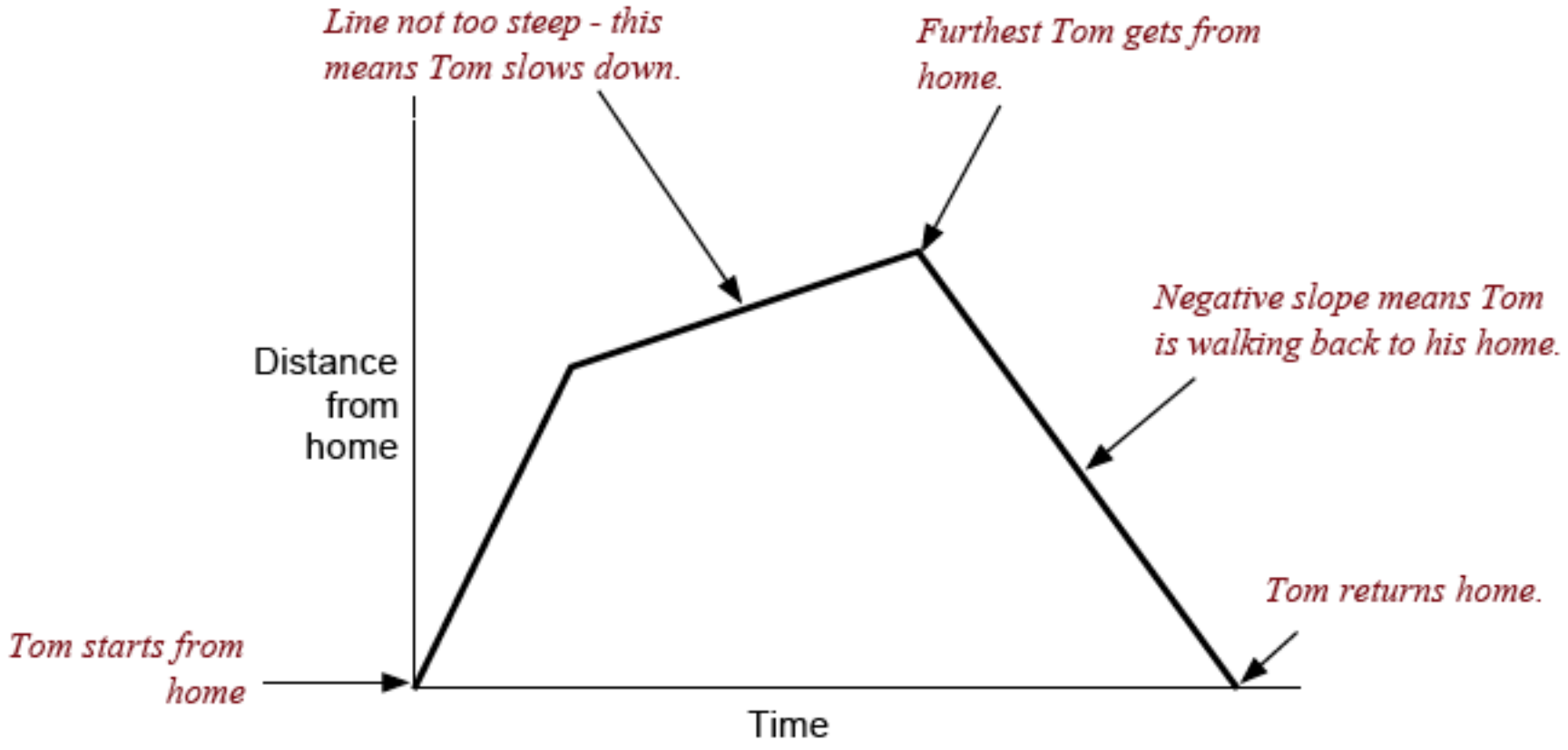
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



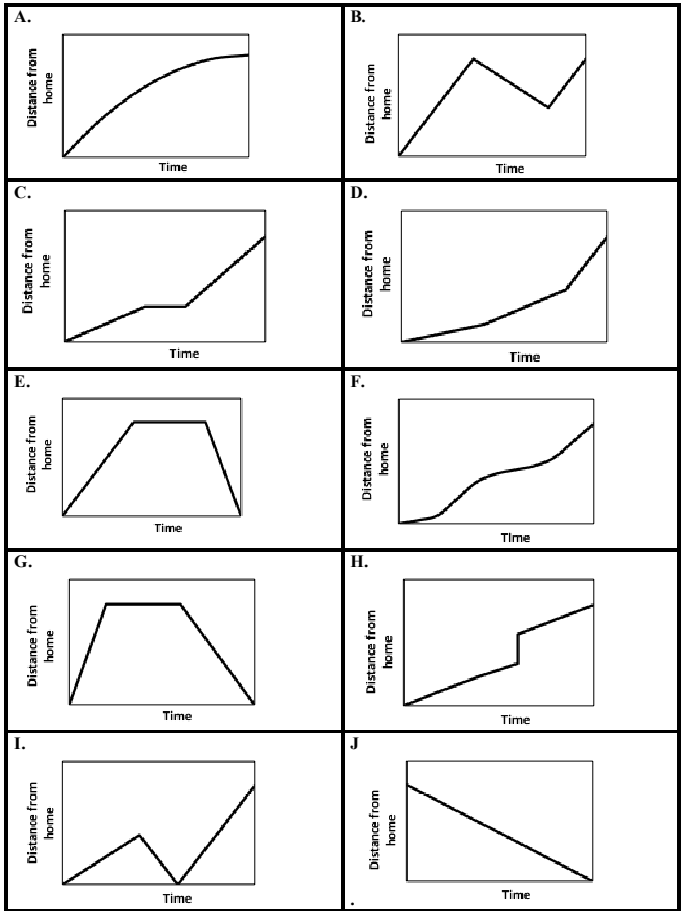
Students are given the chance to annotate and explain…

A graph may end up looking like this:



Follow-up Task: Card Sort
The students make posters.

Card Set A: Distance-Time Graphs



Card Set B: Interpretations

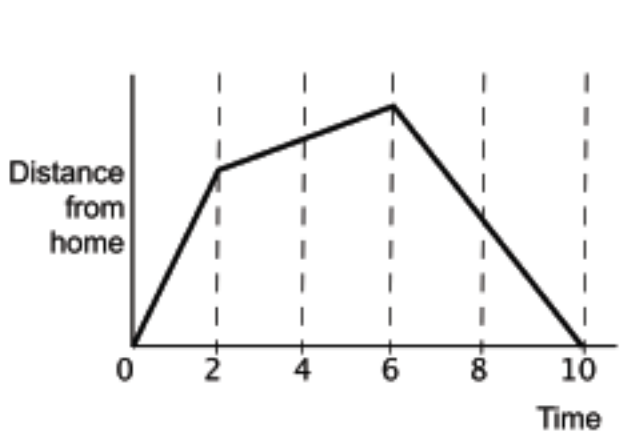
1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	2. Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top and then ran quickly down the other side.
3. Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	4. Tom walked slowly along the road, stopped to look at his watch, realized he was late, then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	6. Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.
7. Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	8. This graph is just plain wrong. How can Tom be in two places at once?
9. After the party, Tom walked slowly all the way home.	10. Make up your own story!

Students work on converting graphs to tables:

Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

Making Up Data for a Graph



Time	Distance
0	
2	
4	
6	
8	
10	

On your whiteboard, create a table that shows possible times and distances for Tom's journey.

Tables are added to the card sort…

Card Set C: Tables of data

A.

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

B.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

C.

Time	Distance
0	0
1	18
2	35
3	50
3	85
5	120

D.

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

E.

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

F.

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

G.

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

H.

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

I

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

J. Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

K.

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

And the class compares solutions together.

Here's another FAL:

Evaluating Statements About Length and Area

Mathematical goals

This lesson unit is intended to help you assess how well students can:

- Understand the concepts of length and area.
- Use the concept of area in proving why two areas are or are not equal.
- Construct their own examples and counterexamples to help justify or refute conjectures.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

G-CO Prove geometric theorems.

This lesson involves a range of *mathematical practices*, with emphasis on:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Shape Statement 1

1. **James says:**

> If you draw two shapes, the shape with the greater area will also have the longer perimeter.

Is James' statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.

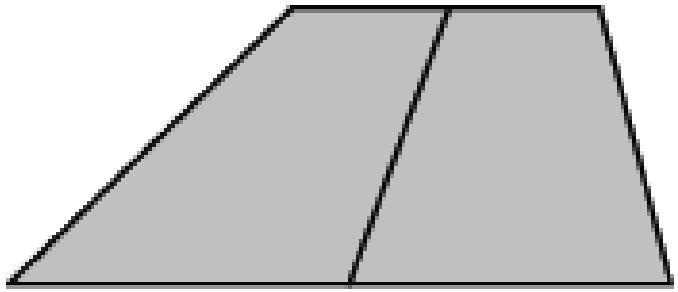
Shape Statement 2

2. Clara says:

> If you join the midpoints of the opposite sides of a trapezoid, you split the trapezoid into two equal areas.

Is Clara's statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.



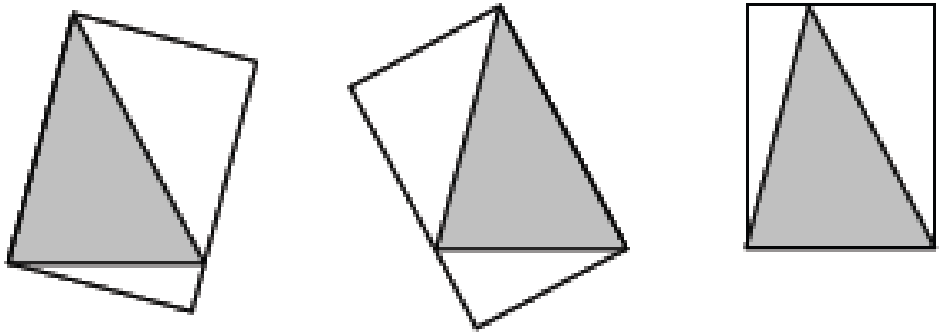
Shape Statement 3

3. Alex says:

> There are three different ways of drawing a rectangle around a triangle, so that it passes through all three vertices and shares an edge. The areas of the rectangles are equal.

Is Alex's statement Always, Sometimes or Never True?

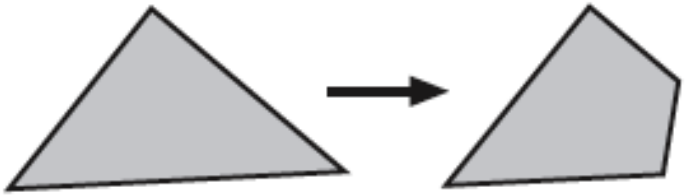
Fully explain and illustrate your answer.



There are more great tasks, e.g.,

Always, Sometimes, or Never True?

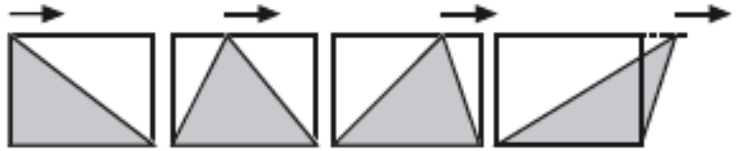
A **Cutting Shapes**



When you cut a piece off a shape you:

(a) Reduce its area.
(b) Reduce its perimeter.

B **Sliding a Triangle**



If you slide the top corner of a triangle from left to right:

(a) Its area stays the same.
(b) Its perimeter changes.

And, the students develop critiquing skills. The task:

Diagonals of a Quadrilateral

If you draw in the two diagonals of a quadrilateral, you divide the quadrilateral into four equal areas.

Is this statement always, sometimes or never true?

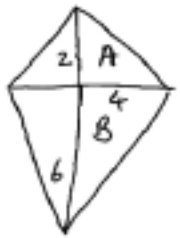
If you think the statement is always true or never true, then how would you convince someone else?

If you think the statement is sometimes true, would you be able to identify all the cases of a quadrilateral where it is true/not true?

They discuss the task, and sort out the mathematics. Then…

They're given other (hypothetical) students' work…

Student Work 1



A

$\frac{1}{2} \cdot 4 \cdot 2$

$\frac{1}{2} \cdot 8$

4

B

$\frac{1}{2} \cdot 4 \cdot 6$

$\frac{1}{2} \cdot 24$

12

Kite = not true

Student Work 2

All triangles are congruent (SSS)

Parallelogram = allways true

And helped to critique it.

These are central skills called for in CCSSM.

You can see more at
http://map.mathshell.org/materials
Or, just google
“Mathematics Assessment Project.”

The site will contain 100 lessons, downloadable free for non-commercial use.

B. What are the high stakes tests, Produced by PARCC and Smarter Balanced, really going to look like?

Some Questions for PARCC & SBAC

1. How long will the tests be?
2. What testing formats will be used?
3. What reporting formats will be used?

(For example, do kids get one score, a content score and a practices score, scores for content, problem solving, reasoning modeling. . .)

4. What governs assessment decisions: mathematics or psychometrics? For example,

- If the assessment uses CAT, how will it maintain attention to and balance of different practices and content?
- If everything is computer-based, how do kids draw pictures?
- Who makes the big decisions: people in math/math ed or those from the testing community?

Systemic Issue 1

How do we provide professional development and material support for the teaching community? At the national level, at the district/building level? What issues do we face in trying to provide PD in a resource-starved environment?

Systemic Issue 2

What are the challenges of implementation at the district and school level?

How do we identify those challenges (e.g., alignment, teacher time for PD, building capacity, coping with change); and how do we work to help district and school leaders leaders support teachers in attaining the goals of CCSS-M?

Whew!

That’s a lot to chew on, over the next few days.

I’m looking forward to hearing what people have to say about these issues.