# MANIFOLDS WITH LOWER SECTIONAL CURVATURE BOUNDS AND ALEXANDROV GEOMETRY

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Abstract. The aim of the talk is to provide a survey of the main tools, results and open problems concerning manifolds with a lower (sectional) curvature bound.

It is well known that *local bounds on sectional curvature* can be described geometrically via *local distance comparison* to constant curvature spaces. For lower curvature bounds this comparison is *global*, as expressed in the Toponogov Comparison Theorem. This together with *critical point theory for distance functions* paved the way for studying manifolds with only a lower sectional curvature bound, resulting in *Finiteness, Structure, and Recognition Theorems*.

There are several equivalent versions of Toponogov's Comparison Theorem, some of which make sense in a general metric space. Moreover, such a metrically expressed lower curvature bound is preserved by the process of taking a Gromov-Hausdorff limit. An Alexandrov space is a *finite Hausdorff dimensional, inner metric space* with a *lower curvature bound*.

It turns out that, despite their general definition, Alexandrov spaces have a surprisingly *rich structure* and are natural objects in their own rite. Their applications and significance to Riemanian geometry stems from the fact that there are several natural *geometric operations* that are *closed* in Alexandrov geometry, but not in Riemanian geometry. These include *taking Gromov-Hausdorff limits, taking quotients* and *taking joins of positively curved spaces*.

All concepts alluded to above will be explained and discussed, as will examples, some of the main results, and fundamental open problems.