MEASURE-METRIC BOUNDARY AND LIOUVILLE THEOREM IN ALEXANDROV GEOMETRY

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ABSTRACT. I will introduce the notion of measure-metric boundary on measure-metric spaces and discuss various motivational examples. I will then describe some recent joint work on measure-metric boundary with Alexander Lytchak and Anton Petrunin.

We show that if an Alexandrov space has a zero measure metric boundary then for almost every point in almost every direction there exists an infinite geodesic and the geodesic flow preserves the Liouville measure. We conjecture that any Alexandrov space without boundary has zero measure-metric boundary and hence the above result should hold for all such spaces. While we can not prove it in general we show that this is true for convex hypersurfaces in smooth manifolds. We also show that finite dimensional Alexandrov spaces have finite measure-metric boundary.