Title: Pro-p-Iwahori-Hecke algebras of p-adic groups.

Abstract: Let (W, S) be a Coxeter system with length ℓ, R a commutative ring, $(q_s, c_s)_{s \in S}$ a family of elements of R, constant on the intersections with S of the conjugacy classes of W. The R-algebra $H_R(W, S, q_s, c_s)$ is the free R-module of basis $(T_w)_{w \in W}$ with product satisfying the relations:

Braid relations: $T_w T_{w'} = T_{ww'}$ for $w, w' \in W$ with $\ell(w) + \ell(w') = \ell(ww')$.

Quadratic relations: $T_s^2 = q_s + c_s T_s$ for $s \in S$.

These algebras are variants of the convolution algebra $H_R(G, I(1))$ of the double cosets of a pro-p-Iwahori subgroup $I(1)$ of a p-adic reductive group G. The algebras $H_R(G, I(1))$ play a key role in the modulo p representation theory of G via the $I(1)$-invariant functor. We will describe the alcove walk bases, the Bernstein relations in $H_R(G, I(1))$, and the simple supersingular modules when R is an algebraically closed field of characteristic p.