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MSRI

Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Edward Frenkel

Talk Title: Gauge Theory and Langlands duality

Date: 9/2/14 **Time:** 11:30 am / pm (circle one)

List 6-12 key words for the talk: Motivations of Langlands duality; moduli stack of flat Langlands dual LG-bundles; Hecke eigensheaves.

Please summarize the lecture in 5 or fewer sentences: The problem of seemingly infinite complexity, i.e., solving an equation for infinitely-many primes and obtaining certain numbers, is solved by one line, whose numbers are encoded in a certain formal Taylor series; this is an example of the Langlands correspondence via a simple example of elliptic curves over the rationals (2-dim reps of absolute Galois group) and modular forms (automorphic reps of GL_2 over the adèles).

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- [x] Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- [x] Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- [x] For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- [x] When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- [x] Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

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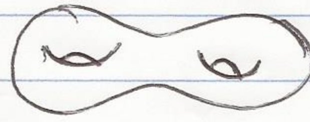
Gauge theory and Langlands duality
Edward Frenkel
Tuesday Sept 2, 2014, 11:30 – 12:30 pm

Number ↔ Algebraic ↔ Riemann Surfaces
Theory curves/$\mathbb{F}_q$



$\mathbb{Q} = \{\frac{p}{q}\}$

$x^2 + 1 = 0$

$F = \mathbb{Q}(i)$

$\mathrm{Gal}(F/\mathbb{Q})$

$\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$

absolute Galois group of the field of rat'l numbers

$X/_{\mathbb{F}_q}$
$q = p^n$,
p prime

Algebraic curves/$\mathbb{C}$

$F = \mathbb{F}_q(X)$

$X = \mathbb{P}^1_{\mathbb{F}_q}$, $F = \left\{ \frac{P(t)}{Q(t)} \right\}$

$\mathrm{Gal}(\overline{F}/F)$

Langlands Program 1967

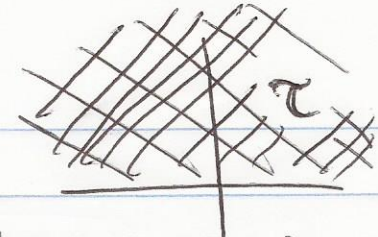
$$\left\{ \begin{matrix} \text{Representations} \\ \text{of } \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \end{matrix} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{Automorphic} \\ \text{Reps of } GL_n(\mathbb{A}_{\mathbb{Q}}) \end{matrix} \right\}$$

adèles

Quantum physics

S-duality
Mirror symmetry

Simple Ex.



Elliptic curves/$\mathbb{Q}$ $\longleftrightarrow$ modular forms

$\mathrm{Im}(\tau) > 0$

n=2

2-dim reps of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ $\longleftrightarrow$ automorphic reps of $GL_2(\mathbb{A})$

adèles

$$y^2 + y = x^3 - x \mod p$$

p prime

$p = 2, 3, 5, 7, 11, \ldots$

$a_p = p - \#\,\mathrm{sol}_p$ (not counting the point at infinity)

$$q(1-q)^2(1-q^{11})^2(1-q^2)^2(1-q^{22})^2 \cdots$$

$$\cdots (1-q^n)^2(1-q^{11n})^2 \cdots$$

formal Taylor series

$$= \sum_{n=1}^{\infty} b_n q^n, \quad b_1 = 1$$

$\boxed{a_p = b_p}$ $\forall\, p$ prime

$$q = e^{2\pi i \tau}, \quad |q| < 1, \quad \tau \mapsto \frac{a\tau + b}{c\tau + d}$$

Shimura-Taniyama-Weil Conj.
"Lectures of L.P. and CFT"
hep-th/0512...

Middle column $\mathbb{Q} \rightsquigarrow F$ function field

G reductive alg. gp $/\mathbb{F}_q$ $F = F(X)$

$F(X)$

$\mathbb{F}_q(\mathbb{P}^1) = \left\{ \frac{P(t)}{Q(t)} \right\}$

X – smooth projective (geom. connected) curve $/\mathbb{F}_q$

$Gal(\bar{F}/F) \supseteq W(F)$

$\{ W(F) \to GL_n \} \overset{\text{Thm}}{\longleftrightarrow} \{ \text{Automorphic Reps of } GL_n(\mathbb{A}_F) \}$

due to Drinfeld, $n=2$
L. Lafforgue, $n>2$

$$\mathbb{A}_{\mathbb{Q}} = \prod_{p \text{ prime}}{}' \mathbb{Q}_p \times \mathbb{R}$$

$$\mathbb{A}_F = \prod_{x \in |X|}{}' F_x, \quad F_x \simeq \mathbb{F}_{q_x}((t_x))$$

residue field of x

$\{ W(F) \to {}^L G \} \longleftrightarrow$ $G(F) \backslash G(\mathbb{A}_F)$

Langlands dual group

general red. alg. gp.

X curve$/\mathbb{C}$, $F = \mathbb{C}(X)$

Analogue of $\mathrm{Gal}_{un}(\bar{F}/F)$ is $\pi_1(X, x)$

$$\{\pi_1(X, x) \to {}^L G\}^{set} \longleftrightarrow \{\mathcal{D}\text{-modules on } \mathrm{Bun}_G\}^{set}$$

$$\{\text{flat } {}^L G\text{-bundles on } X\}$$

Hecke eigensheaves

objects of a category of $\mathcal{D}$-mod.

$$(E^\infty, \nabla = \nabla^{(1,0)} + \nabla^{(0,1)})$$
$$= (E, \underbrace{\nabla_{hol}}_{=\nabla^{(1,0)}})$$

Bun_G – moduli stack of (principal) G-bundles on X
(hol, alg)

Why $\mathcal{D}$-modules?

$$\mathrm{Bun}_G(\mathbb{F}_q) = G(F) \backslash G(\mathbb{A}_F) / G(\mathcal{O}_F)$$

adèles; integer adèles

D^b | D^b

$$\boxed{\mathcal{O}\text{-modules on } \mathrm{Loc}_{{}^L G}} \xleftrightarrow[\text{L.C.}]{\text{categorical}} \boxed{\mathcal{D}\text{-modules on } \mathrm{Bun}_G}$$

Hecke functors

$\mathcal{E} = (E, \nabla)$ in

$\mathcal{O}_\mathcal{E}$ – skyscraper sheaf at $\mathcal{E}$ $\longrightarrow \mathcal{F}_\mathcal{E}$

$\mathrm{Loc}_{{}^L G}$ – moduli stack of flat ${}^L G$-bundles

$\mathcal{E} = (E, \nabla_{hol})$

E: hol, alg.

∇_{hol}: hol, alg. conn. on E

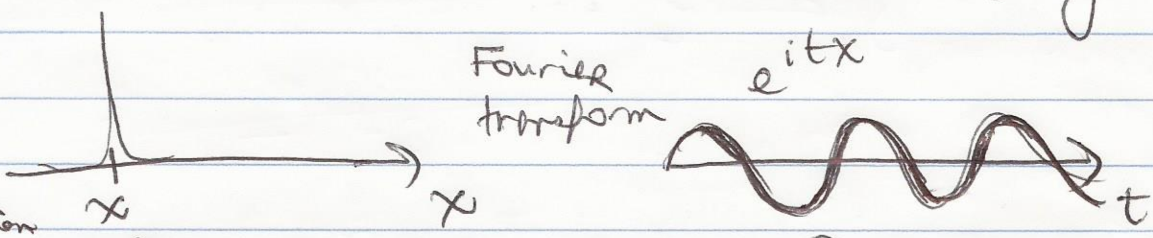
L.C. = Langlands correspondence

$\mathcal{F}_\mathcal{E}$ must be a Hecke eigensheaf.

$G = GL_1$: Thm. Laumon + Rothstein

G – non-abelian : Arinkin – Gaitsgory

δ-function



$$\left(\frac{\partial}{\partial t} - ix\right)\psi = 0$$

$$\mathcal{O}_{\mathcal{E}} \longrightarrow \mathcal{F}_{\mathcal{E}}$$

nonabelian Fourier – Mukai transform!