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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Edward Frenkel

Talk Title: Gauge Theory and Langlands duality

Date: 9/4/14 **Time:** 11:30 **am** / pm (circle one)

List 6-12 key words for the talk: Categorical Langlands correspondence, Hecke eigensheaves, moduli stacks.

Please summarize the lecture in 5 or fewer sentences: The categorical Langlands correspondence between $\mathcal{O}$-modules on the moduli stack of flat ^LG-bundles and $\mathcal{D}$-modules on Bun_G was discussed in the previous lecture. Frenkel will discuss how this equivalence is related to S-duality (electro-magnetic duality) in 4-dim'l super Yang-Mills theory.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

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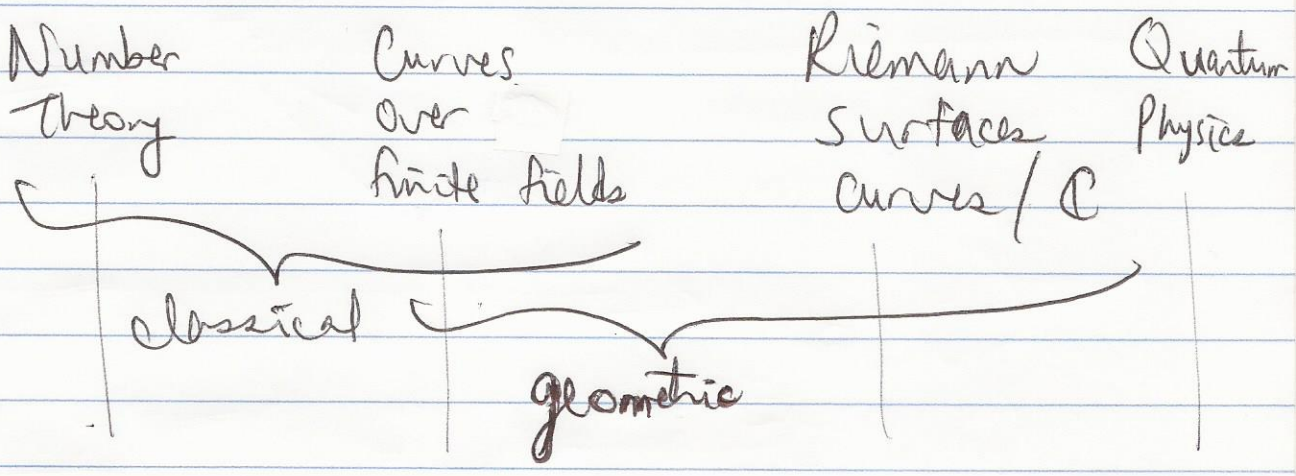
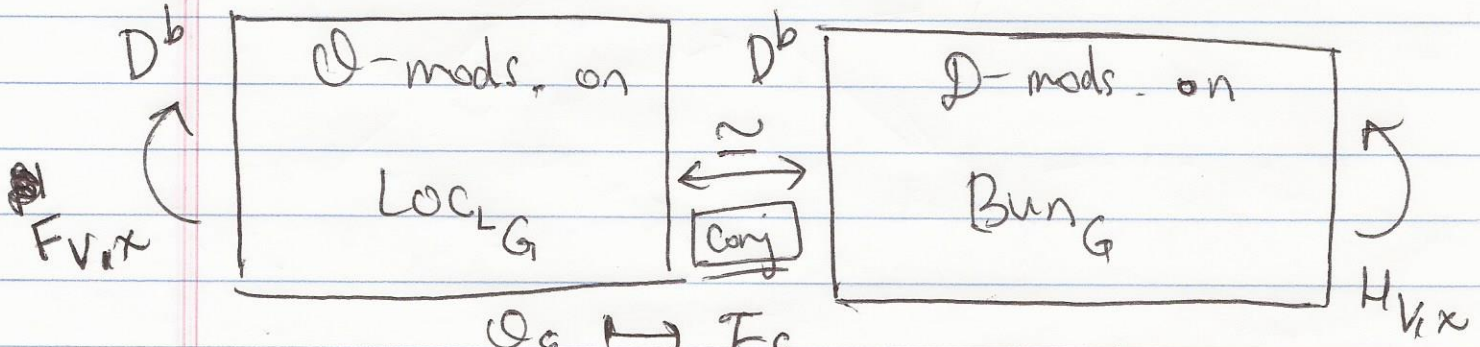
Gauge Theory & Langlands Duality

Edward Frenkel

Thurs, Sept 4, 2014, 11:30 - 12:30 pm



Categorical Langlands Correspondance

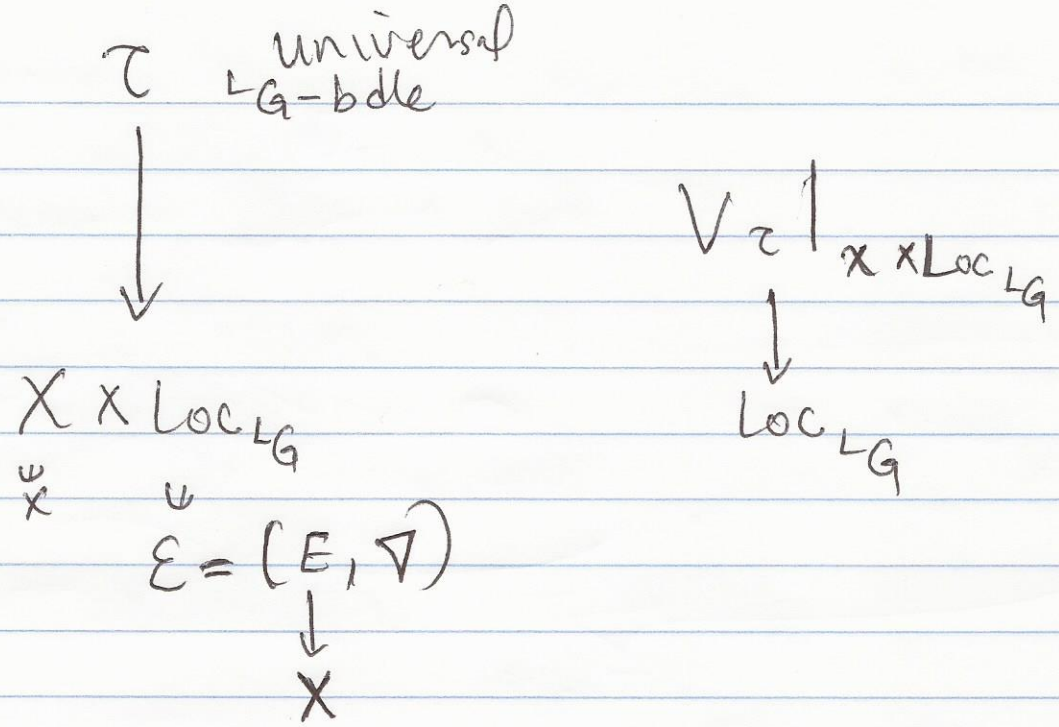


X = curve / $\mathbb{C}$

G = reductive alg. gp / $\mathbb{C}$

${}^L G = G^\vee$ = Langlands dual of G

$G = GL_1$ - know abelian Fourier-Mukai transform.



$x \in X$, $V \in {}^L G$, $V_\tau := \tau \times_{{}^L G} V$ — assoc. v.b.

$\mathcal{V}_\tau$ - $\mathcal{O}$-mod on $Loc_{{}^L G}$ of sections of

$V_\tau|_{X \times Loc_{{}^L G}}$

Frobenius (Wilson) functor:

$$F_{V,x}(\mathcal{F}) = \mathcal{V}_{\tau,x} \otimes \mathcal{F}$$

Let $\mathcal{F} = \mathcal{O}_\mathcal{E}$, $\mathcal{E}$ - smooth pt

$\mathcal{O}_\mathcal{E}$ - skyscraper sheaf.

$$F_{V,x}(\mathcal{O}_\mathcal{E}) \simeq \underline{V} \otimes \mathcal{O}_\mathcal{E}$$

← eigensheaf

$\forall x \in X$, $\forall V \in Rep\, {}^L G$.

Electromagnetic duality

$\vec{E}$, $\vec{B}$, $\vec{E} \to \vec{B}$

$\vec{B} \to -\vec{E}$

Quantum level: max'l ($N=4$) supersymmetric electromagnetism.

e - "electric charge".

$e \longrightarrow \frac{1}{e}$

Perturbative vs. non-perturbative

$\sum_{n=0}^{\infty} \frac{1}{n!} x^n$ vs. $\exp(\pi i) = -1$

(x ← variable)

$$\phi(q) = \sum_{n=0}^{\infty} p(n)\, q^n = \prod_{i=1}^{\infty} (1-q^i)^{-1}$$

(q ← variable; $p(n)$: all partitions of n)

$$\eta(q) = q^{\frac{1}{24}} \cdot \phi(q)^{-1}$$

$$q = e^{2\pi i \tau}$$

$$\tau \longrightarrow \frac{a\tau+b}{c\tau+d}$$

$$\tau \longmapsto -\frac{1}{\tau}$$

N=4 SUSY

General gauge thy: Gauge (Y-M) 4-D w/ gauge group $U(1)$

" " " $G_{\mathbb{C}}, g$

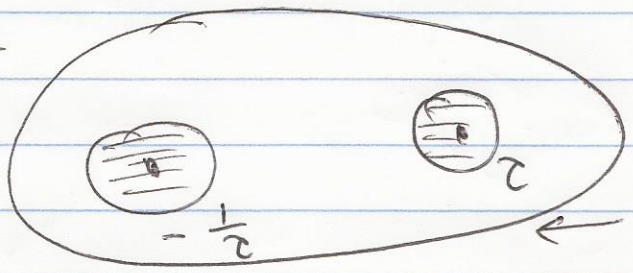
Bourbaki talk, arXiv: 0906.2747,

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

Gauge group, parameter: $({}^L G_{\mathbb{C}}, -\frac{1}{\tau})$ $\longleftrightarrow$ $(G_{\mathbb{C}}, \tau)$ Gauge group, parameter — S-duality

$^L G$: max'l cmpt of the dual of the Lie group G.

if $\theta = 0$: $\tau \sim \frac{1}{g^2}$, $-\frac{1}{\tau} \sim g^2$



different ways to look at the same theory

Kapustin-Witten 2006

$M_4 = X \times \Sigma$ Dimensional reductions $|X| \ll |\Sigma|$

Effective 2D thy on Σ:
supersymmetric sigma model with target mfd

$$\mathcal{M}_H(G) = \mathcal{M}_H(G, X)$$

$$\Sigma \longrightarrow \mathcal{M}_H(G, X)$$

Hitchin moduli space

super Poincaré

$P = P_{\bar{0}} \oplus P_{\bar{1}}$, $Q^2 = 0$

$\circlearrowleft Q$

Topological field theory. In 2D,

B-model $\xleftrightarrow{\text{Mirror symmetry}}$ A-model

$\mathcal{M}_H({}^L G)_J$ $\qquad$ $\mathcal{M}_H(G)$ w

J cmplx structure.

quantum cohom.

$\{\mathcal{O} : Q \cdot \mathcal{O} = 0\} / \{Q \cdot \mathcal{O}'\}$

$\mathcal{M}_H(G)_I = \{(E, w)\}$ Higgs pair.

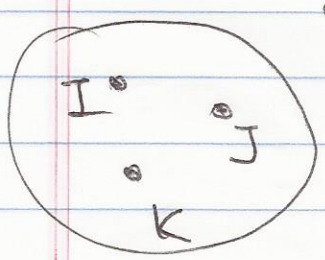
Hyperkähler mfd. $\qquad$ principal hol. G-bdle on X;

$w \in \Gamma(X, \mathfrak{g}_E \otimes K_X) \cong H^1(X, \mathfrak{g}_E)^*$

$\mathfrak{g}_E := E \times_G \mathfrak{g}$

$T^*_E(\mathrm{Bun}_G)$

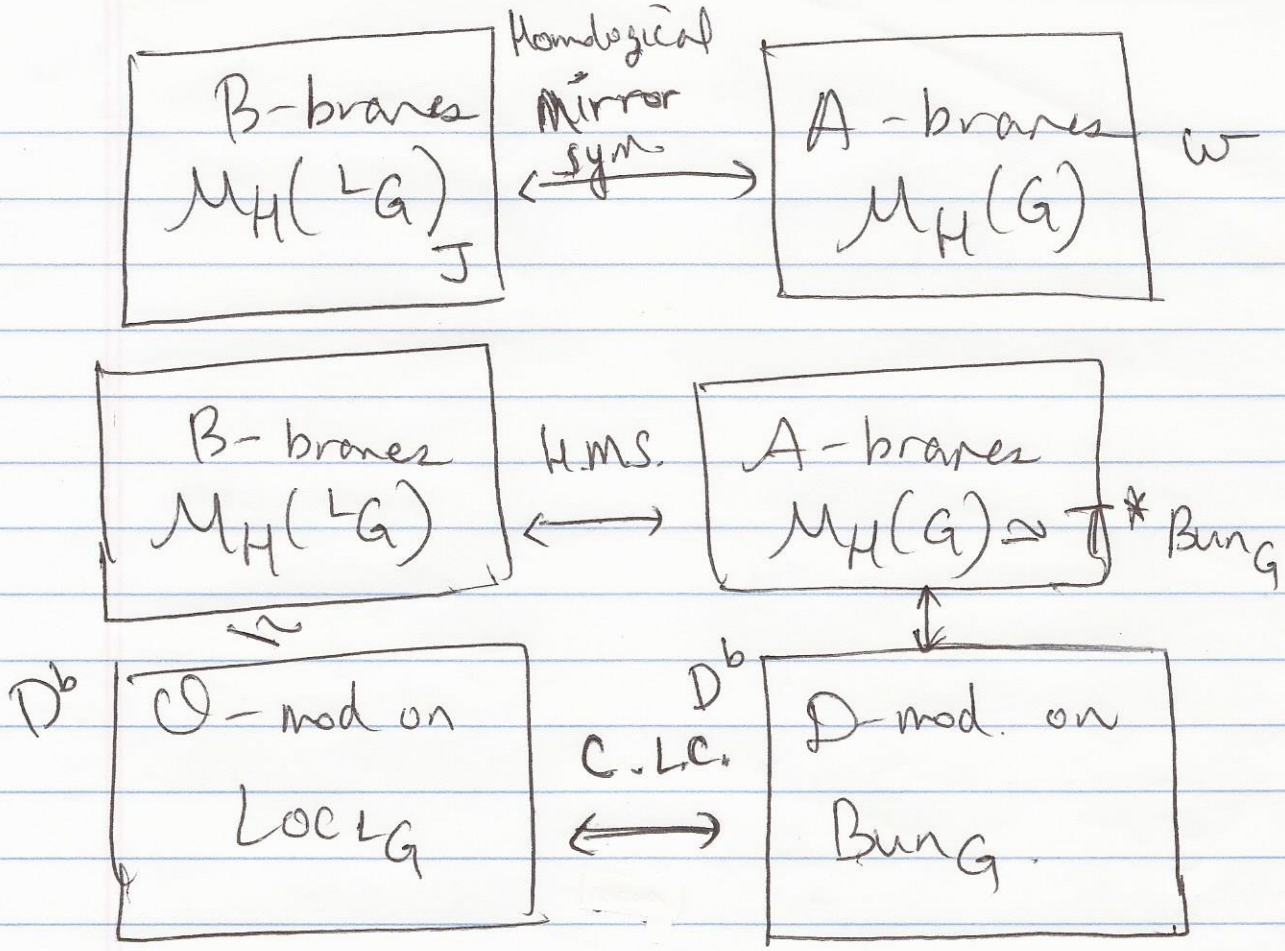
E



$\mathcal{M}_H({}^L G) = \{(E, \nabla)\}$

${}^L G$-bdle, ∇ conn.

$\mathrm{Loc}\,{}^L G = \{(E, \nabla)\}$



Curves over finite fields

Curves / $\mathbb{C}$

Automorphic functions

"comfns"

X

— ! —

sheaves

sheaves