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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im Email/Phone: mim2@illinois.edu

Speaker's Name: Dima Arinkin

Talk Title: The Geometric Langlands Correspondence

Date: 9/2/14 Time: 10:30 am pm (circle one)

List 6-12 key words for the talk: Compact Riemann surface, general linear group setting of the Geometric Langlands correspondence

Please summarize the lecture in 5 or fewer sentences: There are two sides to the geometric Langlands conjecture: the automorphic side and the Galois side, where the conjecture is precisely the correspondence between two completely different objects. We will explore this correspondence in the setting for $rk N$ vector bundles on X ; i.e., $Bun := Bun_{GL(N)} X$.

CHECK LIST

(This is NOT optional, we will not pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue PEN. We will NOT accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

②

The Geometric Langlands Correspondence

Dima Arinkin

Tuesday Sept 2, 2014, 10:30-11:30 am

Introduction to Geometric Langlands Conjecture

Day 1. $GL(N)$

Day 2. Properties

Day 3. Toward proof

① $X =$ compact Riemann surface
 X/\mathbb{K} smooth projective curve/ \mathbb{K}
 $\text{char}(\mathbb{K}) = 0$

Automorphic

Galois

$\text{Bun} = \text{Bun}_{GL(N)} X$
 $= \left\{ \begin{array}{l} \text{rk } N \text{ v. bundles} \\ \text{on } X \end{array} \right\}$

Classically: X/\mathbb{F}_q , $\mathbb{F}_q(X)$ function field of X

Function: $\text{Bun}(\mathbb{F}_q) \rightarrow \mathbb{C}$

Geometrically:
Cat. of $\boxed{\text{D-modules on Bun}}$

alg systems of PDEs
= quasicoh. sheaves with flat connections

has to be viewed as a stack.

Rough goal: indexed by local systems on X
 Find a family of "special"
 D-modules on Bun that forms
 a "basis."

Defn Galois side
 Rank N local systems on X ,
 is a $(\text{rk } N)$ v. bdl. on X
 together with a flat connection.
 Classically, $\pi_1(X) \rightarrow GL(N)$.

$$LS = \{ \text{loc. sys} \}$$

② Example (Geometric class field theory)
 $N=1$ (rk 1 v. bdl. on X)

$\text{Bun} = J = \text{Jacobian of } X$

Starting from a rk 1 loc. system ℓ
 on $X \longleftrightarrow \text{Aut}_\ell : \text{D-module on } J$.

ℓ is given by $\text{Mon}(\ell) : \pi_1(X) \rightarrow \mathbb{C}^*$

$$\pi_1(J) = H_1(X) = \pi_1(X)^{\text{ab}}$$

Define Aut_ℓ to be
 rk 1 loc. system on J s.t. $\text{Mon}(\text{Aut}_\ell)$

(3) How are these $\text{Aut}_\mathcal{L}$ special?

$\mathcal{L} \in \mathcal{L}_S$

↓

$\text{Aut}_\mathcal{L}$ is a \mathcal{D} -module on Bun .

Hecke eigenproperty.

Ex $N=1$.

Fix $x \in X$, it gives $t_x: \text{Bun} \rightarrow \text{Bun}$
 $E \mapsto E(x)$

Classically,

$H := \text{Bun}(\mathbb{F}_q)$, operators

↻

t_x

t_x^* act on {functions $H \rightarrow \mathbb{C}$ },
($x \in X$)

find eigenvectors.

Geometrically:

$t: X \times \text{Bun} \rightarrow \text{Bun}$

$(x, E) \mapsto E(x)$

Hecke eigenproperty:

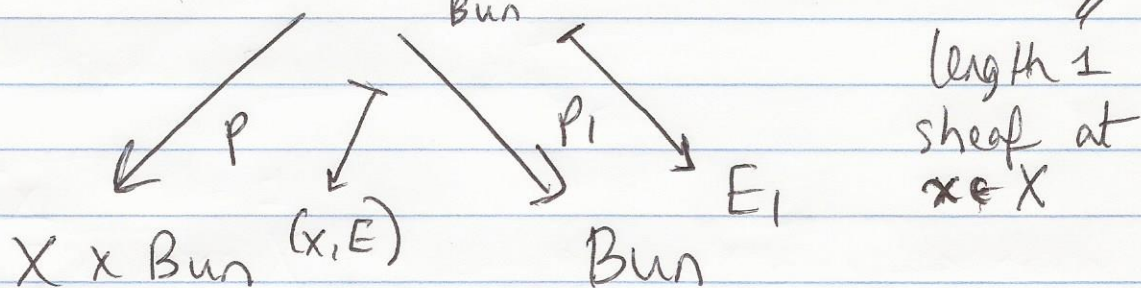
$$\boxed{l \otimes \text{Aut}_\mathcal{L} = t^*(\text{Aut}_\mathcal{L})}$$

canonical isom.

$$lx \otimes \text{Aut}_\mathcal{L} = t_x^*(\text{Aut}_\mathcal{L})$$

What happens if $N > 1$?
Universal Hecke correspondence

$$\text{Hecke} = \left\{ (E \rightarrow E_1 \text{ s.t. } 0 \rightarrow E \rightarrow E_1 \rightarrow E_1/E) \right\}$$



Hecke eigenproperty $\mathcal{L} \in \mathcal{L}_S$

$$\mathcal{L} \otimes \text{Aut } \mathcal{L} = p_* (\pi_1^* (\text{Aut } \mathcal{L}))$$

↑
eigenvalues

