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### NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im Email/Phone: mim2@illinois.edu

Speaker's Name: Dima Arinkin

Talk Title: The Geometric Langlands Correspondence

Date: 9/3/14 Time: 10:30 am/pm (circle one)

List 6-12 key words for the talk: properties of the Geometric Langlands correspondence, geometric Satake correspondence.

Please summarize the lecture in 5 or fewer sentences: Arinkin defines  $r_k N$  vector bundles on a Kilmann surface and he also defines  $r_k N$  local systems on  $X$ . He describes ~~the~~ Hecke eigensheaves, Hecke category and geometric Satake equivalence and describes when the category of quasicoherent sheaves on local systems of  $G$  dual is equivalent to  $\mathcal{P}$ -modules on  $G$ -vector bundles.

### CHECK LIST

(This is NOT optional, we will not pay for incomplete forms)


- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3<sup>rd</sup> floor.
  - **Computer Presentations:** Obtain a copy of their presentation
  - **Overhead:** Obtain a copy or use the originals and scan them
  - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.  
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to [notes@msri.org](mailto:notes@msri.org) with the workshop name and your name in the subject line.

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# The Geometric Langlands Correspondence

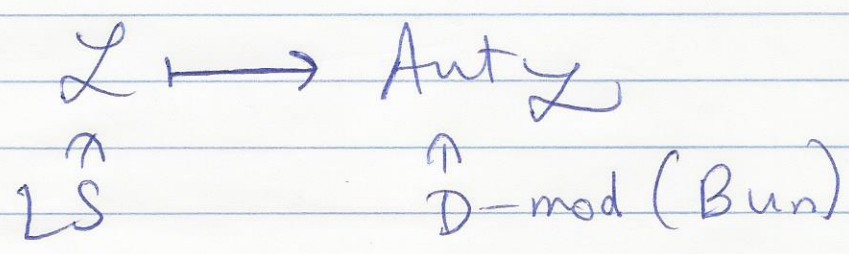
Dima Arinkin

Wed, Sept 3, 2014, 10:30-11:30 am

$X =$  
 unramified global Langlands correspondence  
 (function field)  
 for  $GL(N)$ .

$Bun = \{ \text{rk } N \text{ v. bundles on } X \}$

$LS = \{ \text{rk } N \text{ l. systems on } X \}$



s.t.

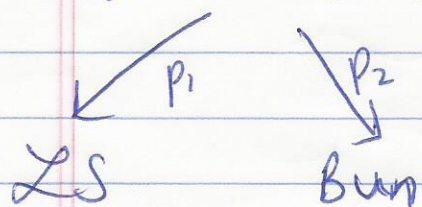
- Hecke <sup>eigen</sup> property
- $\{ \text{Aut } Z \}$  form a basis.

(• Parabolic induction)

$\{ \text{Aut } Z \}_{ Z \in LS }$  should form a family <sup>LS</sup>

of  $\mathcal{D}\text{-mod}_{Bun}$   $\text{Aut}$  on  $LS \times Bun$   
 is an  $\mathcal{O}\text{-D}$ -module.

Get a functor  
 $\mathcal{Q}\text{Coh}(\mathcal{L}\mathcal{S}) \longrightarrow \text{D-mod}(\text{Bun})$   
 $p_{2,*}(\text{Aut} \otimes p_1^*(-))$



Derive everything!

(Naive) GLC: This is an equivalence.

Other groups

$G =$  reductive group /  $\mathbb{C}$   
 $G^\vee =$  Langlands dual of  $G$   
 $\text{Bun}_G = \{ G\text{-bundles on } X \}$   
 $\mathcal{L}\mathcal{S}_{G^\vee} = \{ G^\vee\text{-local systems on } X \}$

$\text{Aut}$  is  $\mathcal{L}\mathcal{S}_{G^\vee}$ -family of  $\text{D}_{\text{Bun}_G}$ -mods.

giving an equivalence

$\mathcal{Q}\text{Coh}(\mathcal{L}\mathcal{S}_{G^\vee}) \longrightarrow \text{D-modules}(\text{Bun}_G)$

How to modify  $G$ -bundles at  $x \in X$ ?

Modification:  $F_1, F_2 \in \text{Bun}_G$ , and

$$F_1|_{X - \{x\}} \cong F_2|_{X - \{x\}}$$

Classified by

$$G(\mathcal{O}) \backslash G(K) / G(\mathcal{O})$$

$$K = \mathbb{C}((z))$$

$$\mathcal{O} = \mathbb{C}[[z]],$$

$z$  is a word at  $x$ .

Ex.  $G = GL(N)$ , minimal modification.  
 $\begin{pmatrix} z & & & \\ & \circ & & \\ & & \ddots & \\ & & & \circ \end{pmatrix}$

Classically:  $K = \mathbb{F}_q((z))$ .

Hecke alg } Functions:  $G(\mathcal{O}) \backslash G(K) / G(\mathcal{O})$  }

algebra under  
convolution.

$\downarrow$   
 $\mathbb{C}$

Hecke category :

$$Gr = G(K) / G(\mathcal{O}) \curvearrowright G(K) \cong G(\mathcal{O})$$

affine Grassmannian

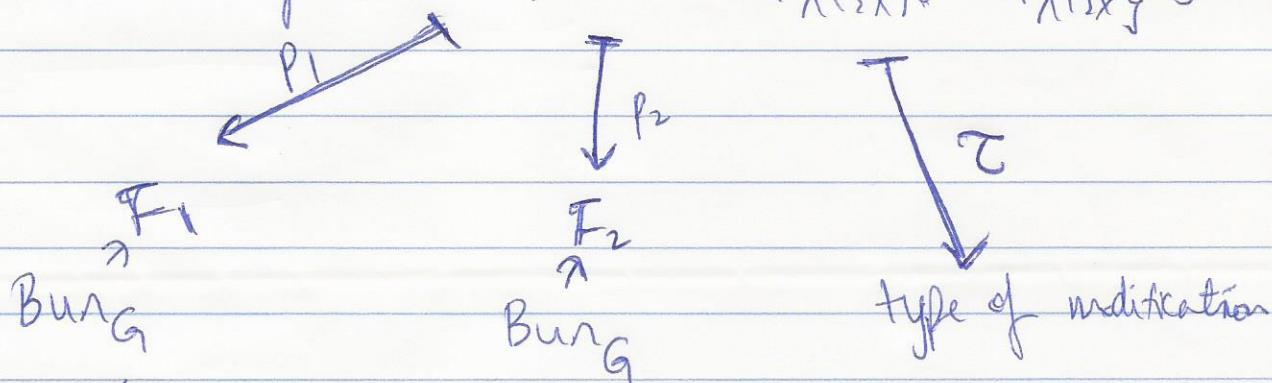
$$\mathcal{H} := D\text{-mod}(Gr)^{G(\mathcal{O})}$$

$$\cong D\text{-mod}(G(\mathcal{O}) \backslash G(K) / G(\mathcal{O}))$$

$\mathcal{H}$  is a monoidal category under convolution  $*$  :  $\mathcal{H} \times \mathcal{H} \rightarrow \mathcal{H}$

$\mathcal{H}$  acts on  $D\text{-mod}(Bun_G)$

Informally :  $\{ (F_1, F_2, F_1|_{X_1 \times X_2} \cong F_2|_{X_1 \times X_2}) \}$



$M \in \mathcal{H}$  acts as

$$G(\mathcal{O}) \backslash G(K) / G(\mathcal{O})$$

$$p_{1,*} (p_2^* (-) \otimes \tau^* (M))$$

## Geometric Satake equivalence

Thm  $\mathcal{H} \simeq \text{Rep}(G^\vee)$

representations of  $G^\vee$ .  
as monoidal cats.

Rem. For abelian categories.

This is ~~an~~ used in Hecke eigenproperty.

$\mathcal{L} \in \mathcal{L}S_{G^\vee} \mapsto \text{Aut}_x \in \mathcal{D}\text{-mod}(\text{Bun}_G)$ .

Given  $M \in \mathcal{H}$ ,

$M * \text{Aut}_x = \lambda_M \otimes \text{Aut}_x$  for some

v. space  $\lambda_M$  (eigenvalue of  $M$  on  $\text{Aut}_x$ )

As  $x$  varies,  $\lambda_M$  becomes a  
l. system (of v. spaces) on  $X$ .

$M \in \mathcal{H}$

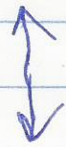
$\downarrow$   
 $\rho \in \text{Rep}(G^\vee)$

so  $\lambda_M = \rho(\mathcal{L})$

Example :  $G = GL(N) = G^\vee$

minimal modification  $\mathbb{P} \in G(\mathbb{O}) \backslash G(\mathbb{K}) / G(\mathbb{O})$

$\delta_{m,m} \in \mathcal{H}$



standard  $\in \text{Rep}(G^\vee)$   
rep

•  $\mathcal{QCoh}(LS_{G^\vee}) \simeq \text{D-mod}(Bun_G)$