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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Thomas Hales

Talk Title: Introduction to the Langlands program and the fundamental lemma

Date: 9 / 4 / 14 **Time:** 9 : 00 am / pm (circle one)

List 6-12 key words for the talk: automorphic representation theory, trace formula, motivic integration.

Please summarize the lecture in 5 or fewer sentences: Hales gives an introduction to motivic integration for p-adic fields, where integrals take values in rings. Viewing algebraic geometry as a model theory of 1st order language of rings is discussed, as well as extended alg. geom and definable assignment.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
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Introduction to the Langlands program and the Fundamental Lemma
Thomas Hales
Thurs, Sept 4, 2014, 9–10 am

Last time, fundamental lemma (FL)
pos char.

<u>Thm</u> The fundamental lemma holds for all p-adic fields of char 0.

$$O^{\kappa}_{va}(1_{g(O_F)}) = q^{r_\kappa} SO_a(1_{h(O_F)})$$

- lift to char 0 in suff. large res. char.
  (motivic integration)
- extend to all res. char.
  (trace formula)

(Waldspurger, Cluckers–H–Loeser)

Motivic integration is a "universal" integration for p-adic fields.

Integrals take values in rings

$$\varphi(X) \xrightarrow{\int} \varphi(pt)$$

$$X \to pt.$$

For each p-adic field, F,

$f \in \mathcal{C}(X)$,

$f_F \in \mathcal{C}(X)_F \quad f_F : X_F \to \mathbb{Q}$

$$\left(\int f\right)_F \in \mathbb{Q}.$$

For all F of suff. large res. char, want

$$\int f_F = \left(\int f\right)_F$$

$f \mapsto f_F$ will be defined in such a way that if

$$k_{F_1} = k_{F_2}$$

<u>Thm</u> [Cluckers–Loeser], $\forall f \in \mathcal{C}(\Lambda)$, $\exists N \in \mathbb{Z}$

$$f_{F_1} = 0 \iff f_{F_2} = 0 \quad \text{all } F_i \text{ res. char} > N \text{ if } k_{F_1} = k_{F_2}.$$

We will apply this thm to show that if the F.L. holds for F_1, $k_{F_1} = k_{F_2}$, then F.L. holds for F_2 also.

Algebraic Geometry "=" model theory of 1st order language of rings.

R ring.

$V(R)$

$(\times, +, 0, 1)$

We can make a 1st order language of rings.

All systematically correct formulas

$=, \wedge, \vee, \neg, \exists, \forall, \ldots$

R structure for $\times_R, +_R, 0_R, 1_R$.

Let's add two more functions "extended algebraic geometry."

	$\mathbb{C}((t))$
	$\Downarrow$
ord	$\mathrm{ord}\left(\sum_{i\geq N} a_i t^i\right) = N$
$\overline{ac}$	$a_N \neq 0$.
	$\overline{ac}\left(\sum_{i\geq N} a_i t^i\right) = a_N$

$\text{ord} : (\mathbb{Q}_p \to \mathbb{Z})$ usual valuation

$p^i \longmapsto i$

$\overline{ac} : \mathbb{Q}_p^\times \longrightarrow \mathbb{F}_p^\times$

$x p^i \longmapsto u \bmod p$, x unit.

The Denef-Pas language is the 1st order language with $\overline{ac}$, ord.
This is a 3-sorted language.
valued field
residue field
value group, 3 types of quantifiers.

There are 3 sorts of variables

Formula φ "type" $(m, n, r) \in \mathbb{N}^3$
# of free variables of each type.
Fix a field k of char 0, $K \supseteq k$.
$V(K)$ = solutions of φ in
$K((t))^m \times K^n \times \mathbb{Z}^r$.

V is a "definable assignment."

φ . $\exists y$ s.t. $y^2 = x \wedge x \neq 0$ $(1,0,0)$

$V(K) = (K((t))^{\times})^2$

ψ $\qquad$ $\mathrm{ord}(x) \geq 0$

$V(K) = K[[t]]$

$V(K) =$ set of elmts in K that are not a square

$k \subseteq K \subseteq \overline{K}$

$$\cancel{V(K) \longrightarrow} V(\overline{K}) = \phi$$

Category : objects def. subassignments
morphisms functions whose graph is def. subass.

Def_k $\qquad$ $\begin{array}{ccc} X & \rightarrow & Y \\ & \searrow \quad \swarrow & \\ & S & \end{array}$ $\qquad$ Def_S $\qquad$ $\varphi(X)$

Some p-adic integrals

$$\int_{\mathbb{Z}_p} dx = 1$$

$$\int_{p^n\mathbb{Z}_p} dx = \frac{1}{p^n}, \text{ motivically } \frac{1}{\mathbb{L}^n},$$

$\mathbb{L}$ symbol.

<u>Ex.</u> $\displaystyle\int dxdy = \sum_{(\bar{x},\bar{y})\in E} \int_{(p\mathbb{Z}_p)^2} dxdy = \#E \cdot \frac{1}{p^2}$

$\{(x,y) : y^2 = x^3+x \bmod p\}$

$$E = \{(\bar{x},\bar{y}) \in \mathbb{F}_p^2 : y^2 = x^3+x\}$$

motivic $\dfrac{[E]}{\mathbb{L}^2}$.

<u>Ex.</u> $\displaystyle\int_{\mathbb{Z}_p} |x|^n dx = \frac{1-\frac{1}{p}}{1-\frac{1}{p^{n+1}}}$ $\qquad \dfrac{1-\frac{1}{\mathbb{L}}}{1-\frac{1}{\mathbb{L}^{n+1}}}$

$\varphi(X)$ is a ring

gens of ring

X is a def. subass. of type (m,n,r)

K_0 Groth. group of subobjects $(m, n+n', r)$

$$[Y]+[Y'] = [Y\cup Y'] + [Y\cap Y']$$

$$\left\{\begin{array}{l} \mathbb{L},\ \mathbb{L}^{-1},\ (1-\mathbb{L}^{-i})^{-1},\ i>0, \\ \mathbb{L}^{\alpha},\ \alpha : X \to \mathbb{Z}. \end{array}\right.$$

To transfer the fundamental lemma
to char 0,
we must express it in terms of

ord

$\overline{ac}$

$\mathbb{L}$, $[E]$.

E/F

p poly.,

$E = F^r$.