(T6)

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im   Email/Phone: mim2@illinois.edu

Speaker's Name: Dima Arinkin

Talk Title: The Geometric Langlands Correspondence

Date: 9/5/14   Time: 9:00 am / pm (circle one)

List 6-12 key words for the talk: Towards a proof of the geometric Langlands correspondence

Please summarize the lecture in 5 or fewer sentences: Arinkin discusses triangulated categories and when geometric Langlands correspondence fails in certain naive constructions. He then restricts to compact objects in D-mod($Bun_G X$) and describes its equivalence in QCoh($LS_{{}^LG} X^{(g)}$).

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

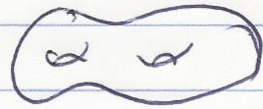
- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - <u>**Computer Presentations**</u>: Obtain a copy of their presentation
    - <u>**Overhead**</u>: Obtain a copy or use the originals and scan them
    - <u>**Blackboard**</u>: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - <u>**Handouts**</u>: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

The geometric Langlands Correspondence

Dima Arinkin

Friday, Sept 5, 2014, 9–10 am

<u>Recall</u>

$X =$  , G reductive gp

$G^\vee / \mathbb{C}$.

$\mathrm{Bun}_G X = \{ G\text{-bundles on } X \}$

$LS_{G^\vee} X = \{ G^\vee\text{-l. sys. on } X \}$

(Naive)
$$\mathrm{D\text{-}mod}(\mathrm{Bun}_G X) \overset{?}{\cancel{\simeq}} \mathrm{QCoh}(LS_{G^\vee} X)$$

GLC

① **Def.** Let $\mathcal{C}$ is a triangulated cat. with $\oplus$.

$M \in \mathcal{C}$ <u>compact</u>

$\oplus$ $\quad \mathrm{Hom}(M, N_i) \xrightarrow{\sim} \mathrm{Hom}(M, \oplus N_i)$

Examples: (reasonable)

① S scheme or (reasonable) stack

$M \in QCoh(S)$ is compact $\iff$
M is perfect (in D^b_{coh}, finite Tor-dimension)

$\Updownarrow$

(locally, has a finite resoln by finite free modules).

② S = smooth variety

$M \in$ D-mod(S) is compact
iff M locally has a finite resolution by finite free D-modules.

$\parallel$

D^b_{coh}(D-modules)

②' S = smooth stack
Compact objs in D-mod(S) can be described.

Evidence of failure of naive GLC.

a) Eisenstein series construction.

$L \subseteq G$ is levi.

$$\text{D-mod}(Bun_L X) \xrightarrow[\text{Eisenstein}]{} \text{D-mod}(Bun_G)$$

$$\begin{array}{ccc} \text{D-mod}(Bun_L X) & \xrightarrow[(A)]{\text{Eisenstein}} & \text{D-mod}(Bun_G) \\ \wr\!\wr \; ⓘ\,?\; GLC & & \wr\!\wr \; ?\; GLC \\ QCoh(LS_{L^\vee}) & \xrightarrow[\substack{\text{Eisenstein} \\ (G)}]{} & QCoh(LS_{G^\vee}) \end{array}$$

(A) Automorphic E—
(G) Geometric E—.

<u>Problem</u> Eisenstein(A) preserves compactness
Eisenstein(G) preserves coherence
but not compactness = perfection.

(b) Drinfeld's calculation, $G = GL(2)$, genus > 1.
Computed Aut on $LS_{G^\vee} \times Bun_G$
Turns out it is of ∞-Tor dim over $LS_{G^\vee}$.

$\Downarrow$

Correspondence functor D-mod $\to$ QCoh does not preserve compactness.

Problem: $LS_{G^\vee}$ is not smooth!
So coherent $\neq$ perfect.

$K_0 \neq K^0$

$$\begin{array}{ccc} & & \text{perfect complexes} \\ & & \cap \\ \text{Compact objects} & \dashrightarrow & \boxed{\;?\;} \\ & & \cap \\ \cap & & \text{Coherent} = D^b_{coh} \\ \text{D-mod}(Bun_G X) & \not\simeq & QCoh(LS_{G^\vee} X) \end{array}$$

Singular support in $D^b_{coh}(LS_{G^\vee})$

(or on any l.c.i.)

Re: S = smooth,
holonomic D-mods on S.

$\downarrow$

sing supp $\subseteq T^*S$.

S = l.c.i.
There is "$H^{-1}T^*S$"

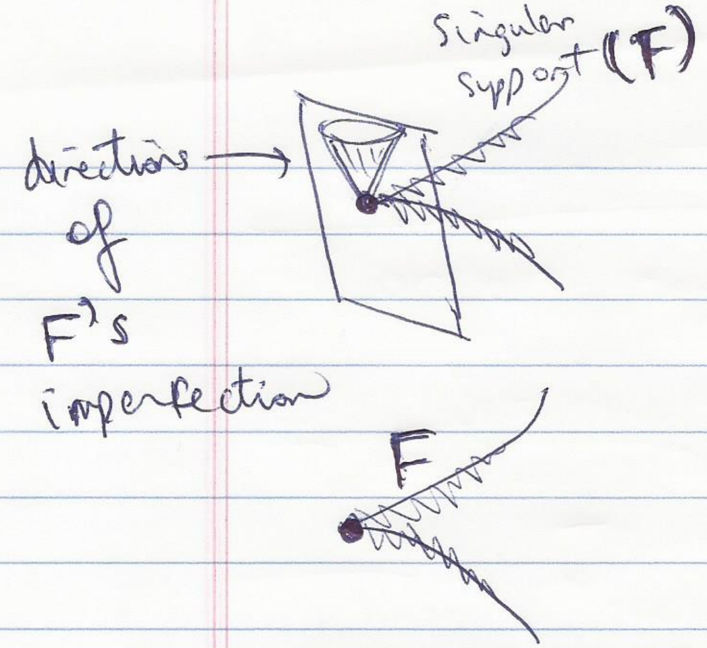
$\downarrow$

S sing. supp

measures non-smoothness

$\mathcal{F} \in Coh(S) \longmapsto$ closed conical
sing. supp$(\mathcal{F}) \subseteq H^{-1}T^*S$.

S = l.c.i.



$$\begin{array}{c} H^{-1}T^*S \\ \big\downarrow \\ S \end{array}$$

shifted cotangent bdle

<u>Calculation</u>: $H^{-1}T^*LS_{G^\vee} =$

$= \{(\mathcal{L}, A) : \mathcal{L} \in LS_{G^\vee},\ A \in \Gamma(\mathfrak{g}_{\mathcal{L}}),$ A horizontal $\}$

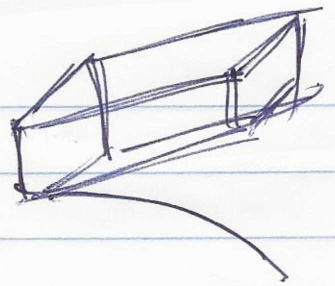
$$\mathcal{N}ilp \subseteq H^{-1}T^*LS_{G^\vee}$$
$$\| $$
$$\{(\mathcal{L}, A) : A \text{ nilpotent}\}$$

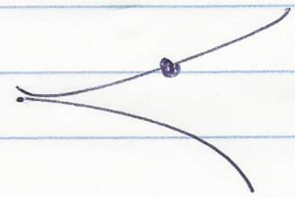
$$\{M \in Coh : \text{Sing supp}(M) \subseteq \mathcal{N}ilp\}$$

$$\boxed{\text{Cmpt objs} \overset{\simeq}{\dashrightarrow} Coh_{\mathcal{N}ilp}(LS_{G^\vee}X)}$$

$$\cap\!\!\!\!\not\,\,\,\|$$

$$D\text{-mod}(Bun_G X) \not\simeq QCoh(LS_{G^\vee}X)$$





<u>Thm</u> (A. G.) $\mathrm{Coh}_{\mathrm{Nilp}}$ is the smallest Δ subcat. of $\mathrm{Coh}(\ \ , LS_{G^\vee})$

$\mathrm{Perf}(LS_{L^\vee})$ Eisenstein (G) $\mathrm{Coh}_{\mathrm{Nilp}}(LS_{G^\vee})$

$$\mathrm{Coh}(LS_{L^\vee}) \longrightarrow \mathrm{Coh}(LS_{G^\vee})$$

<u>GLC</u>

$$D\text{-mod}(\mathrm{Bun}_G X) \xrightarrow{\sim} \mathrm{Ind}\,\mathrm{Coh}_{\mathrm{Nilp}}(LS_{G^\vee})$$

ind-coherent sheaves