(17)

MSRI

Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Pramod Achar

Talk Title: The Springer Correspondence

Date: 9/5/14 **Time:** 10:30 am / pm (circle one)

List 6-12 key words for the talk: perverse sheaf, intersection cohomology, modular Springer correspondence

Please summarize the lecture in 5 or fewer sentences: Achar finishes the lecture series of the Springer correspondence for the Lie algebra and modular settings. He discusses that the endomorphism ring of the Springer sheaf is isomorphic to the group algebra of the Weyl group, and that Irr(kW) is bijective with the set of simple quotients of the Springer sheaf.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - <u>Computer Presentations</u>: Obtain a copy of their presentation
  - <u>Overhead</u>: Obtain a copy or use the originals and scan them
  - <u>Blackboard</u>: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - <u>Handouts</u>: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack <u>with this completed sheet on top</u> and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to <u>notes@msri.org</u> with the workshop name and your name in the subject line.

The Springer Correspondence
Pramod Achar
Fri, Sept 5, 2014, 10:30 – 11:30 am

All this week:
Perv. sheaves w/ coeff in
$\underbrace{\mathbb{C} \text{ or } \overline{\mathbb{Q}}_\ell}_{\text{char. } 0}$.

Varieties / $\mathbb{C}$ or $\mathbb{F}_q$, $\overline{\mathbb{F}}_q$

Powerful tools: BBD
- Decomp. thm.
- weights, purity

This talk:
- Perv. sh. w/ coeff. in a field k of pos. char.
- Varieties: over $\mathbb{C}$

Difficulties
- Things fail to be semisimple
- Things are hard to compute

stalks of IC's $\longleftarrow$ Key: stalks vanish in odd degrees.
(reason: purity argument)

Warm-up: $G = GL_2$

$$\begin{array}{ccc} \tilde{N} & \xrightarrow{\pi} & N \\ \| & & \| \\ T^*(G/B) & & \mathbb{C}^2/\{\pm 1\} \end{array}$$

$T^*(G/B) = T^*\mathbb{P}^1$

Let k be a field.
Let $A_k = R\pi_* \underline{k}[2]$.

$C_{reg} = \text{orbit of } \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = N \setminus \{0\}$

$C_0 = \{0\}$.

<u>Obs</u>. $A_k|_{C_{reg}} = \underline{k}[2]$,

$A_k|_{\{0\}} = H^\bullet(\mathbb{P}^1)[2]$
$= \begin{cases} \text{1-dim'l in deg } 0, -2 \\ 0 \quad \text{elsewhere} \end{cases}$

<u>Exercise</u> (Medium)
Compute $IC(C_{reg}, k)|_0$ char $k \neq 2$
Answer:

$H^i(IC(C_{reg}, k)|_0 = \begin{cases} k & \text{if } i = -2 \\ 0 & \text{——} \end{cases}$

char $k=2$

$$H^i(IC(C_{reg}, k))|_0 = \begin{cases} k & \text{if } i=-2,-1 \\ 0 & \text{otherwise} \end{cases}$$

ODD

Hint: $C_{reg} = (\mathbb{C}^2 \setminus \{0\}) / \{\pm 1\} \underset{htpy}{\simeq} \mathbb{R}\mathbb{P}^3$

<u>Cor.</u>

$IC(C_{reg}, k)$ is a composition factor of A_k, but <u>not</u> a direct summand.

A_k is <u>not</u> s.s.

<u>Exercise.</u> (Medium +)

Show that $A_k \simeq IC(C_{reg}, k) \oplus IC(C_0, k)$ if char $k \neq 2$

is indecomposable if char $k=2$.

In fact, it has the following comp. series:

$$\boxed{IC(C_0, k)} \leftarrow \text{cosocle}$$

$$\boxed{IC(C_{reg}, k)}$$

$$\boxed{IC(C_0, k)} \leftarrow \text{socle}$$

char $k > 0$

Highlights:

~2000 Soergel: $\mathrm{Perverse}_{(B)}(\mathcal{B}, k)$

$\wr$

"modular cat $\mathcal{O}$"

for an alg. gp G/k.

~2007 Mirković–Vilonen:
geometric Satake
$\mathrm{Perv}_{G(\mathcal{O})}(\mathrm{Gr}, k) \simeq \mathrm{Rep}(\check{G})$ ↖ k

~2007 Juteau's thesis: Modular Springer correspondence.
Fiebig, Mautner, Williamson, ... others.

Main diagram

$$\begin{tikzcd}
\tilde{\mathcal{N}} \arrow[r, hook] \arrow[d] & \tilde{\mathfrak{g}} \arrow[d] & \tilde{\mathfrak{g}}_{rs} \arrow[l, hook'] \arrow[d] \\
\mathcal{N} \arrow[r, hook] & \mathfrak{g} & \mathfrak{g}_{rs} \arrow[l, hook']
\end{tikzcd}$$

$A_k = R\pi_* \underline{k}[\dim \mathcal{N}]|_{\mathcal{N}}$ (Springer sheaf)

Thm (Juteau)
(a.) $\mathrm{End}(A_{\Bbbk}) \simeq \Bbbk[W]$

BUT this can't tell you everything about A.

Prog uses $\mathbb{T}$ (Fourier transform)

(b.) $\boxed{\mathrm{Irr}(\Bbbk W) \xleftrightarrow{\sim} \{\text{simple quot's of } A_{\Bbbk}\}}$

Modular Springer corresp.

Pf. Study $j_{!*}$.

let $X_{G,\Bbbk} = \{ (\mathcal{C}, \mathcal{E}) \} = \mathrm{Irr}(\mathrm{Perv}_G(\mathcal{N}, \Bbbk))$

nilpotent orbit

irred. G-equiv. $\Bbbk$-loc. sys. on $\mathcal{C}$.

Rephrase thm a bit:
$\mathrm{Irr}(\Bbbk W) \hookrightarrow X_{G,\Bbbk}$
image simple obj. that are quotients of $A_{\Bbbk}$.

Juteau: "decomposition numbers match"

Let E be an irred char 0 W-rep.
Choose a $\mathbb{Z}_p$-form of it,
then $\otimes \mathbb{F}_p$.
$\rightsquigarrow$ char p W-rep.

Decomp. numbers: $E \in \mathrm{Irr}(\mathbb{Q}_p W)$
$d_{E,F}$ $F \in \mathrm{Irr}(\mathbb{F}_p W)$

Can do same thing for perv sh.

<u>Thm</u>. $d_{E,F} = d_{\text{perv. sh. corresp. to } E,\ \text{perv. sh. corr. to } F}$

$\nearrow$

Plays a role in determination of the image of modular Springer corresp.

GL_n: $IC(C_{p\text{-restricted}})$
Other classical gps: Juteau-Lacouvey-Sarlin

<u>Side Note</u>

$H_{2d}^{BM}(Z) = \mathbb{Q}[W]$

Steinberg $\nearrow$ $\searrow$ use this to construct Springer corresp.

Chriss-Ginzburg

Riche (unpublished):
BM homology
modular Springer

<u>Problem</u>. Explain the missing pairs.

<u>Thm</u> (A–Henderson–Juteau–Riche)
[If G has a factor of type E_8, assume char $k \neq 2$]

(a.) $\forall\ (C, \mathcal{E}) \in X_{G,k}$,
$\exists\ (L, C_0, \mathcal{E}_0)$ unique up to conj,
s.t. $(C_0, \mathcal{E}_0) \in X_{L,k}$ cuspidal and
$IC(C, \mathcal{E}) \twoheadleftarrow \mathrm{ind}_{L \subseteq P}^{G}\ IC(C_0, \mathcal{E}_0)$
(Note: ind + res still defined, exact.)

So
$$X_{G,k} = \coprod_{(L, C_0, \mathcal{E}_0)} X_{G,k}^{(L, C_0, \mathcal{E}_0)}$$

(b.) $X_{G,k}^{(L, C_0, \mathcal{E}_0)} \longleftrightarrow \mathrm{Irr}\left(k\left[N_G(L)/L\right]\right)$

hardest part (maybe)

So $X_{G,k} \xleftrightarrow{\sim} \coprod_{(L, C_0, \mathcal{E}_0)} \mathrm{Irr}\left(k\left[N_G(L)/L\right]\right)$

For GL_n, $\chi_{G,k}$ indep't of k

But $|\mathrm{Irr}(kW)| < |\mathrm{Irr}\,\mathbb{C}W|$

So except: more cuspidals in pos. char.
Char 0: No cuspidals on any Levi ~~except~~ T.
Char p: The Levis of the form $GL_{p^{i_1}} \times GL_{p^{i_2}} \times \dots \times GL_{p^{i_k}}$ have a cuspidal pair.

Char 0:

$$\mathrm{Perv}_G(\mathcal{N}_G, \mathbb{C}) \simeq \bigoplus_{(L, C_0, \mathcal{E}_0)} \mathrm{Rep}(\mathbb{C}[N_G(L)/L])$$

Char p:

__Thm__ (AHJR) For GL_n, $\mathrm{Perv}_G(\mathcal{N}_G, k)$ has a filt by Serre subcat.

$$0 = \mathcal{A}_0 \subseteq \mathcal{A}_1 \subseteq \dots \subseteq \mathcal{A}_k = \mathrm{Perv}_G(\mathcal{N}_G, k)$$

s.t.

$$\mathcal{A}_i/\mathcal{A}_{i-1} \simeq \mathrm{Rep}(k[N_G(L)/L])$$

these run over triples $(L, C_0, \mathcal{E}_0)$

+ there is a recollement structure.