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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Julia Gordon

Talk Title: Transfer principles and uniform estimates for orbital integrals

Date: 11 / 17 / 2014 **Time:** 11 : 00 am / pm (circle one)

List 6-12 key words for the talk: Model Theory, Motivic Integration, Orbital Integral, Transfer principle, Uniform Estimate

Please summarize the lecture in 5 or fewer sentences:
Gordon gave an introduction to formal languages and model structures culminating in the language of a valued field. Then she gave a survey of the proof of transfer for the fundamental lemma. Finally she discussed uniform bounds for orbital integrals.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Julia Gordon – Transfer principles and uniform estimates for orbital integrals

(Model theory in Harmonic Analysis)

To Hales 1999

in '95 Kontsevich introduced "Motivic integration"

Denef – Loeser motivic measures on arc spaces valued in $K_0(Mot)$

~2004 Cluckers – Loeser model theory

analysis ⊇ definable ⊇ algebraic

"measurable"

field independent

Languages

1. Language of rings

constants : 0, 1

operations : +, ×

$\varphi(x_1, \dots, x_n)$ defines a subset of K^n for any structure K of L_{ring}

free variable = not bound by a quantifier

can interpret L_{ring} in K, i.e., K is a ring

of K^n

Def A subset is definable if it is cut out by some $\varphi(x_1, \dots, x_n)$

<u>Note</u> Quantifier free sets (where φ has no quantifiers) correspond to constructible sets.

<u>Note</u> Quantifier elimination theorem from model theory is equivalent to Chevalley's theorem on constructible sets.

<u>Presburger Language</u>

$\mathbb{Z}$ $0, 1$ constants

$+$ operations

$=, \geq, \equiv_n \ \forall n \in \mathbb{Z}$ relations

Godel's incompleteness $\Longrightarrow$ no quantifier elimination if you include $\times$

The definable subsets of $\mathbb{Z}$ are finite sets, arithmetic progressions.

In $\mathbb{Z}^n$ you have liner constraints

and arithmetic sets inside

Denef – Pas for valued fields

3 sorts of variables

$x_1, \ldots, x_n$ VF variables

$y_1, \ldots, y_n$ RF variables

$n_i, k_i \in \mathbb{Z}$ variable

On $\mathbb{Z}$ get presburger language L_{pres}

On VF + RF get L_{ring}

Then get relations between languages

ord : $VF \to \mathbb{Z}$ valuation

ac : $VF \to RF$ — can be interpreted gives a choice of uniformizer π

ac(x) = the first $\neq 0$ component of π-adic expansion

non-archimedean valued field

Example Definable subset of F

$$\left\{ x \in VF \;\middle|\; \exists y : \begin{array}{l} ac(x) = y^2 \wedge \\ ord(x) \equiv_{10} 3 \end{array} \right\}$$

field independent

Def A definable function is a function whose graph is a definable set.
(Note: these definable sets of $VF^n \times RF^m \times \mathbb{Z}^r$)

Ex $p(x_1, \ldots, x_n)$ p polynomial

$|p(x_1, \ldots, x_n)|$ ← not yet

$ord(f(x_1, \ldots, x_n))$

Motivic functions on X definable set

$$H_F(x) = \sum_{i=1}^{N} \# Y_{i,x} \, q^{\alpha(x)} \left(\prod_{j=1}^{N} \beta_{ij}(x) \right) \left(\prod_{p} \frac{1}{1-q^{a_{ip}}} \right)$$

$\alpha : VF \to \mathbb{Z}$

$\beta_{ij} : VF \to \mathbb{Z}$

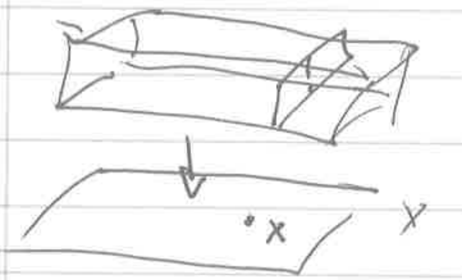
definable

card of residue field

$a_{il} \in \mathbb{Z}$

needed to make closed under integration

$$Y_i \subseteq X \times (RF)^m$$



$\# Y_{i,x}$ = cardinality of fiber of Y_i over x

want $\int_{P^n} H_P(x,y)\, dx = H_P(y)$

so you get an F independent notion of integration.

This was proved by Cluckers & Loeser.

motivic exponential function made from motivic functions + additive characters ψ of VF.

Get integration + theory of Fourier transform by Cluckers-Loeser.

No working theory of multiplicative characters.

Transfer principles

f motivic (exponential) function defined on $X \subset VF^n \times RF^m \times \mathbb{Z}^r$

$f_F \equiv 0$ on X for all F of large characteristic and char 0 or pos char $\gg 0$.

Get the other ~~case~~ by transfer.

Cluckers-Hales-Loeser

- orbital integrals / transfer factors are motivic

1) G split

$$\begin{array}{c} G \\ \downarrow \text{definable} \\ GL_n \subseteq VF^{n^2} \end{array}$$

In general encode Gal gp of extension that splits ~~[illegible]~~ G and a cocycle.

everything that goes into orbital integrals is definable so we can apply transfer to them.

Gordon-Cluckers-Halupczok

- analytic transfer
  - $H_{F,\psi}$ integrable (L^1) or bounded for all ψ

This statement ~~transfers~~!!

Lets you transfer Harish-Chandra theorem.

Uniform bounds

f fixed, $|D(x)|^{1/2}\,|O_x(f)|$ - bdd on g

How does bound depend on v as $F = F_v$ varies.

$\exists\ a \in \mathbb{N}$ such that

$$q_{F_v}^{a} \geq |D(x)|^{1/2}\,|O_x(f)|$$

for char $RF \gg 0$