

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn Email/Phone: jhilburn@uoregon.edu

Speaker's Name: Helene Esnault

Talk Title: Relation between the etale fundamental group and stratifications

Date: 11 / 19 / 2014 Time: 9 : 30 am / pm (circle one)

List 6-12 key words for the talk: Etale fundamental group, Tannakian fundamental group, crystals, iso-crystals, d-modules

Please summarize the lecture in 5 or fewer sentences: Esnault surveyed various generalizations of the Grothendieck-Malcev theorem which relates the etale fundamental group to the Tannakian fundamental group of the category of local systems. In positive characteristic there is no analogue of the topological fundamental group so things are much more difficult.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Hélène Esnault - Relation between the étale fundamental group and stratifications

\mathbb{C} smooth variety

(finite local systems) \subseteq (local systems)

Tannakian fundamental GP.

$\pi_1^{ét}(X)$

$\pi^{loc sys}(X)$

\leftarrow

\leftarrow

?

Arithmetic existence thm

$\pi_1^{loc sys}(X)$

profinite completion

Q: what information is in kernel?

Theorem (Milne - Grothendieck) If the étale fundamental

group $\pi_1^{ét}(X) = \{1\}$ then $\pi^{loc sys}(X) = \{1\}$

PA

$\pi_1^{top}(X(\mathbb{C}))$

\xrightarrow{if}

$\pi_1^{ét}(X)$

group of finite type

\xrightarrow{alg}

$\pi^{loc sys}(X)$

So any rep $\rho: \pi_1^{top}(X) \rightarrow GL(r, \mathbb{C})$ factors

\uparrow
 $GL(r, A)$

think type

$\rho \in \text{Hom}(\pi_1^{top}(X), GL(r, \mathbb{C}))$

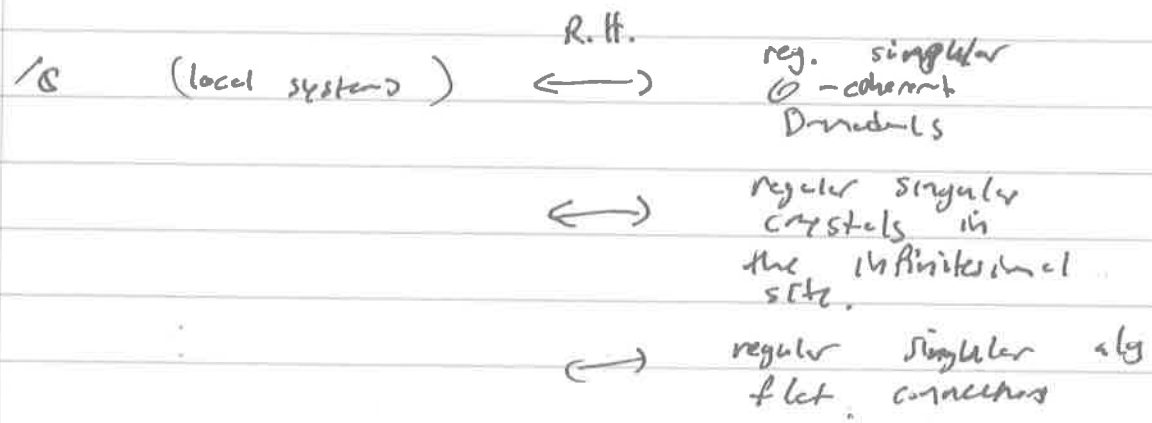
\xrightarrow{alg} A

20's)

Mumford - ~~Gieseker~~ - Katz

Char $\neq 0$ analogs





Should assume X smooth

$K \subset \bar{K}$ char $K > 0$ connections are $\mathcal{O}_{X^{(p)}}$ -linear.

2 ways to go:
 \mathbb{C} -coh D -modules (r.s.)
 K -linear & t - η -linear

K -linear with vectors
 $K = \text{Frac}(W(K))$

Frobenius divided sheaves

\mathbb{C} -coh crystals on infinitesimal site

$(X$ projective) \mathbb{C} -coh crystal in crystalline site.

$\pi^{\text{iso crys}} \rightarrow \pi^{\text{ét}(X)} = (\pi^{\text{iso crys}})^{\wedge p}$
 ↗
 dos Santos in setting of D -modules

$\pi^{\text{iso crys}} \rightarrow \pi^{\text{ét}(X)} \cong \mathbb{C} \otimes_K \bar{K}$

- Conjecture X projective
- 1) $\pi^{\text{ét}(X)} = 1 \Rightarrow \pi^{\text{iso crys}}(X) = 1$? (Gusev)
- 2) $\pi_1^{\text{ét}(X)} = 1 \Rightarrow \pi^{\text{iso crys}}(X) = 0$? (de Jong)

L. Kindler

X quasi-projective (K. Schmidt)

very sing \oplus -coh D-modules (no assumption of res of sing)

$$\pi^{sing} \rightarrow \pi_{\text{ét}}$$

$$\downarrow \quad \downarrow$$

$$(\pi^{sing})^{AP} \rightarrow \pi_{\text{ét}, \text{tors}}$$

remark: ~~X not proper~~
 If $\pi_{\text{ét}} = 1$, $\pi^{sing} = 1$
 If $\pi_{\text{ét}, \text{tors}} = 1$, $\pi^{sing} = 1$

Q: what is analogue of topological fundamental group in char p.

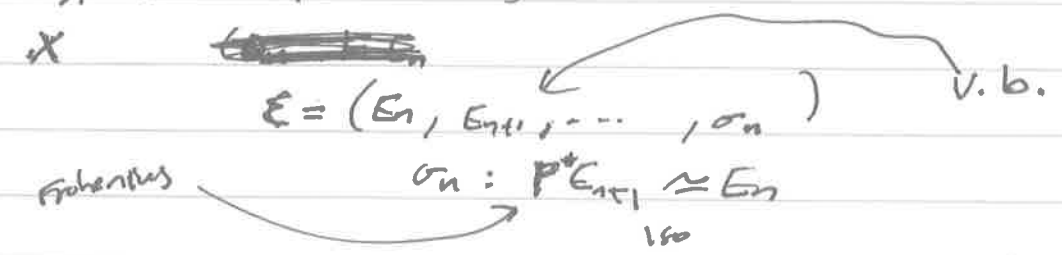
Thm

X projective

$$\pi_{\text{ét}} = 1 \Rightarrow \pi^{sing} = \{1\}$$

X not proper and for $\begin{cases} \pi_{\text{ét}, \text{tors}} \\ \pi_{\text{ét}} \end{cases}$ no result.
 (Aⁿ ok) Some results in \mathbb{F}_p

analogy π_{top} being of fit .



$$E(n) = (E_n, E_{n+1}, \dots, E_{n+1}, \dots)$$

extension of ω -costr-1's slope stable
 $n > 0$ all E_n ~~of fixed rank~~

slope stable of Hilbert ~~polynomial~~ P_0 .
 polynomial

- Langer . existence of quasi-projective moduli spaces of \mathcal{H} -stable V.P. w/ fixed $\rho_{\mathcal{H}}$.
- Specialization of coefficients of monodromy $\Rightarrow \mathbb{A}^1/\mathbb{F}_p \Leftrightarrow$ Hrushovski (Varshavsky) existence of closed periodic points.

X non-proper smooth
 $X \hookrightarrow \bar{X} \hookrightarrow$ normal open embedding

Deligne (relying on π_1^{top} of fct.)

(E, ∇) n.s. extend to (E, ∇) on \bar{X}

Analogous to Deligne

\exists finitely many

$$\mathcal{X}_i(n) \in \mathbb{Q}[n]$$

st $\forall (c_0, c_1, \dots)$

on X there

exists no such that

for all $n > n_0$

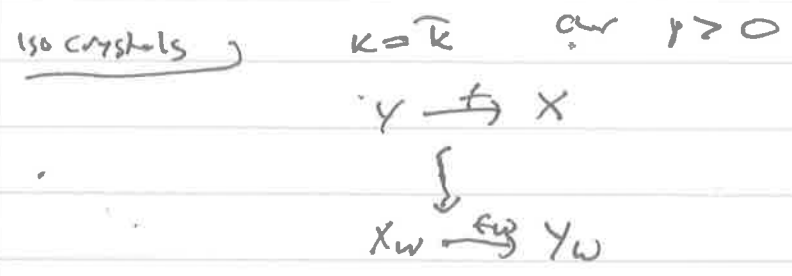
$$\chi(j_*, \mathcal{E}_m(m+1))$$

$$\in \mathbb{Q}[n]$$

is one of the \mathcal{X}_i

$$\{c_n(\bar{E})\} \in H^{2n}(\bar{X})$$

\uparrow finite set



$R = k_W \rightarrow \Omega_{Y_W/X_W}$
 as iso crystal "often" stable, but as shown ~~unstable~~ unstable by work of Griffiths.
 positive \swarrow

~~prop~~ prop de Song's conjecture is true for rank 1 iso crystals ($p \geq 3$)
 $H_{cris}^1(G, \mathcal{O}_X/X_W) \cong \mathbb{Z}$
 $\text{Ext}_{cris}^1(H, H) = 0$

proof reduced to case of irreducible iso crystals.

Theorem (Shiho) X smooth projective / $k = \bar{k}$
 assume $\pi_1^{et}(X) \geq 1$. Then any iso crystal E st $\exists E$ crystal with $E \otimes \mathbb{Q} = \mathbb{Z}$, st $E(X)$ torsion free and ~~strongly~~ strongly semi-stable of Hilbert polynomial P_0 .

2 Ingredients

- $\text{Spin}(E, \text{metric}) \Rightarrow$ the only strongly
 semistable torsion free coherent sheaves
 with Hilbert polynomials P_0 or the trivial
 ones $\oplus \mathbb{C}$.
- Birkhoff: level increasing Fuchsian pullback
 of isocrystals.