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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Yannis Sakellaridis

Talk Title: Non-categorical structures in harmonic analysis

Date: 11 / 21 / 14 **Time:** 11 : 00 am / pm (circle one)

List 6-12 key words for the talk: Asymptotics of matrix coefficients, Spherical varieties, Scattering, Compactification,

Please summarize the lecture in 5 or fewer sentences: Sakellaridis defined the asymptotics map from functions on a dense open subset of a spherical variety to a certain degeneration of that spherical variety. Then he described several conjectures involving the image of this map. Some of this work can be seen as a function theoretic analogue of Gaitsgory's talk.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Sakellaridis — non-categorical structures in harmonic analysis
(Dennis for dummies)

$X \circlearrowleft G$ — X = affine spherical variety
$\check{G}_X$ — Gaitsgory Nadler dual group

/ F non archimedean

G – split

$X \supset X^\circ$ open G-orbit $\supset \mathring{X}$ – open B-orbit

Basic Example $X = X^\circ = H \circlearrowleft G = H \times H$

$G \circlearrowright \text{Fns}(X)$ $X = X(F)$ $G = G(F)$

Prototype Bernstein asymptotics of matrix coefficients

$X = X^\circ = H$

τ smooth rep of H $H \supset P = LU$

$\tau \otimes \tilde{\tau} \xrightarrow[\langle , \rangle]{H\text{-diag}} \mathbb{C}$ $P^- = LU^-$

$\exists!$ $\tau_U \otimes \widetilde{\tau_{U^-}} \xrightarrow[\langle , \rangle]{} \mathbb{C}$

coinvariants

L-invariant pairing

such that

$$\langle \tau(\ell) v, \tilde{v} \rangle = \langle \tau_U(\ell) v_U, \tilde{v}_{U^-} \rangle$$

for $\ell \in L^{>>0}$

Asymp

By Frob reciprocity — universal mapping from taking $Z = C^\infty(H)$

$$\begin{array}{ccc} C^\infty(H) & \dashrightarrow & \\ \uparrow_{H\times H} & & \\ Z\otimes Z & \xrightarrow[H\times H]{} & C^\infty(H_\emptyset) \end{array}$$

where $H_\emptyset = \angle \backslash (\Delta H \times N^{-}\backslash H) = L \overset{P\times P^-}{\times} (H\times H)$

"boundary degeneration of H"

[BK, SV] $H_\emptyset$ has two realizations
(1) Degeneration to normal bundle
(2) Affine degeneration to assoc graded.

$H_\emptyset = X^\circ_\emptyset \subset X_\emptyset$ (another spherical variety)

(1) $X^\circ \hookrightarrow \overline{X}$ wonderful compactification
or
smooth toroidal compactification

$Z \subseteq \overline{X}$ closure of G orbit
$N_Z \overline{X}$ is spherical $\supset X^\circ_Z$ open orbit.

Brion
Knop
Luna
⋮

$\forall$ Isomorphism classes of these are in bijection with Θ = subsets of the simple roots of $G_X^\vee$.

e.g. group case $G = H\times H$ $\check{G} = H^\vee \times H^\vee$
$X = H$ but $G_X^\vee = H^\vee \xrightarrow[(\Delta, c)]{} H^\vee \times H^\vee$
Chevalley involution

Ex SO_3/SO_2 $\quad x^2+y^2-z^2=-1$

$\cap$

X°

$X^\circ_\emptyset = SO_3/N$

2) (Knop, "Automorphisms, root systems, — ")

$\mathfrak{X} \supset X^\circ$

char 0 $\quad k[X] = \bigoplus V_\lambda$ ← highest weight module

$\lambda \in \mathfrak{X}^{++}_X$ ← antidominant + $\mathfrak{X}$-regular weights

This is only a decomposition as a module. Recovering a filtration determined by cone of "positive roots" for X, R_X.

Root system encodes pattern of coordinate ring to be graded

⇝ degenerate to associated graded

$\mathfrak{X}$ ← general fiber X

$\downarrow$ $\quad$ special fiber

$B = t_X$ $\quad X_\emptyset = \mathrm{gr}\, k[X]$

$k[X_\emptyset]$ is graded ⟺ $X_\emptyset$ is horospherical

stab contains N = max unipotent

$N\backslash G \to X_\emptyset$

<u>Thm</u> (Bernstein / BK, Kato–Takano / Lagier, JV)
$\underbrace{\text{Bernstein}}_{\text{gr}}$ $\underbrace{\text{BK, Kato-Takano}}_{\text{symmetric}}$ $\underset{\text{wavefront}}{\uparrow}$

$\exists$ canonical morphism

$$\mathrm{Asymp}_\Theta : C^\infty(X^\circ) \to C^\infty(X^\circ_\Theta)$$

<u>Thm</u> (BK, SV) If $\mathrm{Supp}(f)$ has compact closure in X then $\mathrm{supp}(f_\Theta)$ has compact closure in X_Θ.

<u>Natural question</u> $S(X)$ (If X smooth $S(X) = C_c^\infty(X)$) what is $\mathrm{Asymp}_\Theta(S(X))$.

<u>Geometric models</u> $F = \mathbb{F}_q((t)) \supseteq \Theta = \mathbb{F}_q[[t]]$

$S(X) \ni \mathbb{1}^\circ$, if X is smooth $\mathbb{1}_{X(\Theta)}$

$X(\Theta) = \mathrm{Maps}(\underset{\substack{\| \\ \mathrm{Spec}\, \mathbb{F}_p[[t]]}}{D} \to X)$ ind scheme over $\mathbb{F}_q$

Want a finite dimensional model. Choose $\mathrm{Maps}(C \to X)^\heartsuit$ generically in X°/G

Global model for $X(\Theta)$

$(E, \phi) \in \mathcal{X}(\mathbb{F}_q)$
$\underset{\text{G-bundle}}{\nearrow}$ $\nwarrow$ $\phi : C \to X \overset{G}{\times} E$ generically in $X^\circ \overset{G}{\times} E$

Assume the degree points are

$c_1, \ldots, c_n \in C(\mathbb{F}_q)$.

$D_{\mathcal{E}} \simeq D$

$E_{D\,C} \simeq G$

$$(\mathcal{E}, \phi) \xrightarrow[\mathrm{val}_c(\mathcal{E}, \phi)]{} X(\mathcal{O}) / G(\mathcal{O})$$

<u>philosophy</u> $\Phi^0 \in S(X) \leftrightsquigarrow IC_{\mathfrak{X}}$

More precisely

$IC_{\mathfrak{X}} \rightsquigarrow \Phi$ on $\mathfrak{X}(\mathbb{F}_q)$

sheaf function dictionary

Expectation: $\Phi(\mathcal{E}, \phi) = \prod_i \Phi^0(\mathrm{val}_{c_i}(\mathcal{E}, \phi))$

$\in C^\infty(X^\circ)^{G(\mathcal{O})}$ supported on $X(\mathcal{O})$.

Affine degeneration:

$\mathcal{X}$ for H s.s. this is Vinberg monoid

$\downarrow$

B

$$\mathfrak{X} = \mathrm{Maps}(C, \mathcal{X}/G)^{\heartsuit}$$

$\downarrow$

B

showed up in Dennis's talk

S. Schieder computed nearby cycles.

$IC_{\mathcal{X}} \rightsquigarrow \Psi(IC_{\mathcal{X}}) \in \mathrm{Perv}(\mathcal{X}_\phi)$

<u>Conjecture</u>

$\mathrm{Asym}(\Phi^0) = \Psi(\Phi^0)$

convert Φ_0 to sheaf, take nearby cycles, convert back to a function

similar result in
Bezrukavnikov,
Finkelberg,
Ostrik.

Thm (BK, OV) $\quad \mathrm{Asymp} = R_\Theta^{-1} \circ R$

(Gaitsgory - Nadler - Vilonen)

In gp case $\quad X = H \qquad \mathcal{B} = TN$

$\Phi^0 = \mathbb{1}_{H(\mathcal{O})}$

$X_\phi^\circ = {}_T(N^H_N \times_N^\phi)$

$X_\phi^\circ / G(\mathcal{O}) = X_H$

$\hat{\lambda} \rightsquigarrow e^\lambda q^{\langle \lambda, \rho \rangle} \mathbb{1}_{\lambda K}$

$\mathrm{Asym}(\Phi^0) = \prod \frac{(1-e^{\alpha})}{(1-q^{-1}e^{\alpha})} \quad (\star)$

Rmk: This normalization

$N \backslash SL_2 \subset F^2 - \{0\}$

$1_{\theta^2} = \frac{1}{1-q^{-1}e^{\alpha}}$

for SL_2 $(\#)$ needs

1   1+q   1+q   1+q

·   ·   ·   ·   . . .

0   1   2   3

$\Downarrow$ S. Schelder $\psi(\mathbb{S})$

gen of $(\#)$ to $X = X^\circ$ affine homogeneous

$$\frac{\prod_{\alpha \in \Phi_X^+} (1-e^{\alpha})}{\prod (1-q^{-\nu}e^{\omega})}$$

Original question

Asym : $S(X) \to C^\infty(X_\phi^\circ)$

Sw scattering maps