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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jrhil47@gmail.com

Speaker's Name: Dennis Gaitsgory

Talk Title: Picard-Lefschetz oscillators for the Drinfeld-Lafforgue compactification

Date: 11 / 20 / 2014 **Time:** 2 : 00 **am pm (circle one)**

List 6-12 key words for the talk: Eisenstein series, Drinfeld-Lafforgue compactification, Pseudo-identity, Miraculous space, Vanishing cycles

Please summarize the lecture in 5 or fewer sentences:
The diagonal map from Bun_G to Bun_G X Bun_G has a natural compactification \bar{Bun_G}. Recently Simon Scheider computed the intersection cohomology sheaf of \bar{Bun_G} in terms of the vanishing cycles of a certain one family degeneration.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Gaitsgory — Picard Lefschetz oscillators for Drinfeld–Lafforgue compactification

really need IndCoh$_N$

Schneider

$$\begin{array}{ccc} \text{Dmod}(\text{Bun}_G) & \overset{L_G}{\simeq} & \text{QCoh}(\text{LocSys}_{\check{G}}) \\ \circlearrowright \text{ID}^{\text{verdier}} & ? \times & \circlearrowright \text{ID}^{\text{serre}} \end{array}$$

Need inversion for forms or cartan involution $\tau_G V$ for an arbitrary reductive group.

still doesn't work !!

$M \twoheadleftarrow P \to G$

$$\begin{array}{ccccc} & & \text{Bun}_P & & \\ & \swarrow^{q} & & \searrow^{p} & \\ \text{Bun}_M & & & & \text{Bun}_G \end{array}$$

$\text{Eis} : \text{Dmod}(\text{Bun}_M) \to \text{Dmod}(\text{Bun}_G)$

$\quad \| $

$p_! q^*$

$\tau_G \circ \text{Eis} \circ \tau_M = \text{Eis}^-$ ← for P^- opposite parabolic

Need another idea:

$$Y_1 \qquad\qquad Y_2$$

$$\text{Dmod}(Y_1) \xrightarrow{F} \text{Dmod}(Y_2) \qquad \mathcal{F} \longmapsto \pi_{2*}(\pi_1^!(\mathcal{F}) \overset{!}{\otimes} Q_F)$$

Functors correspond to kernels Q_F in $\text{Dmod}(Y_1 \times Y_2)$

Ex $Y = Y_1 = Y_2$

$$\mathrm{Id} = \Delta_*(\omega_Y) \qquad \Delta : Y \to Y \times Y$$

Drinfeld: $\Delta_!(k_Y)$ corresponds to a functor $Ps-\mathrm{Id} : \mathrm{Dmod}(Y) \to \mathrm{Dmod}(Y)$

pseudo-identity

Def Y is miraculous if $Ps-\mathrm{Id}$ is a self equivalence.

Ex $V/\mathbb{G}_m$.

$$Ps-\mathrm{Id}(\delta_0) = k_Y[?]$$
$$Ps-\mathrm{Id}(k_Y) = \delta_0[?]$$

almost looks like Fourier transform but that goes from $V \mapsto V^*$

Also $Ps-\mathrm{Id}^2 \neq 1$

Thm Bun_G is miraculous

Thm $(Ps\mathrm{Id})_{\mathrm{Bun}_G} \circ \mathrm{Eis}_* = \mathrm{Eis}^- \circ (Ps-\mathrm{Id})_{\mathrm{Bun}_M}$

Normal Eis should be $\mathrm{Eis}_!$. We'll use this notation from now on

$$\mathrm{Eis}_* = \mathbb{D}^{\mathrm{Verd}}_{\mathrm{Bun}_G} \circ \mathrm{Eis}_! \circ \mathbb{D}^{\mathrm{Verd}}_{\mathrm{Bun}_M} = p_* \circ q^!$$

Thm $Ps-\mathrm{Id}$ acts as Id on cuspidal guys.

$$\cup$$
$$D(\mathrm{Bun}_G)_{\mathrm{cusp}}$$

Comes from looking at interaction of Braden's Thm and $Ps-\mathrm{Id}$

$$\mathrm{Bun}_G \overset{j}{\hookrightarrow} \overline{\mathrm{Bun}}_G \xrightarrow[\text{proper}]{} \cancel{}\ \mathrm{Bun}_G \times \mathrm{Bun}_G$$

wonderful compactifications

Want to understand $j_!(k_{\mathrm{Bun}_G})$

$$G\times G \circlearrowright \mathrm{Vinb}_G \circlearrowleft T$$
$$\downarrow$$
$$t_{adj}$$

$\mathrm{Vinb}_G \supseteq \mathrm{Vinb}_G^0$

Ex $G = SL_2$

$\mathrm{Vinb}_G = \mathrm{Mat}_{2\times 2}$

$\overline{G} = \mathrm{Vinb}^0_G / T$

DrLaf_G classifies maps from X to Vinb_G that generically land in $G\times G \backslash \mathrm{Vinb}_G^0$

$\overline{\mathrm{Bun}}_G = \mathrm{DrLaf}/T$

$$\begin{array}{ccc} \mathrm{Bun}_G & \longrightarrow & \mathrm{DrLaf} \\ \downarrow & & \downarrow \\ 1 & \longrightarrow & t_{adj} \end{array}$$

$G = SL_2$

$t_{adj} = \mathbb{A}^1$

$$\begin{array}{ccc} \mathrm{DrLaf}_G & \supseteq & (\mathrm{DrLaf}_G)_0 \\ \downarrow & & \downarrow \\ \mathbb{A}^1 & \leftarrow & 0 \end{array}$$

11

want $\Psi(K_{\mathrm{DrLat}}) \in \mathrm{Dmod}((\mathrm{DrLat}_G)_0)$

Bosic

P-L

oscillator

$$\begin{array}{cc} \mathbb{A}^2 & x,y \\ \downarrow & \downarrow \\ \mathbb{A}^1 & xy \end{array}$$

$$gr\, \Psi = (\delta \otimes V) \oplus (K_1 \oplus K_{-})$$

V standard 2 dim rep of SL_2

$\mathrm{DrLat}\, G = (M_1, M_2, M_1 \xrightarrow{\alpha} M_2, \alpha \neq 0)$

M_1, M_2 are SL_2-bundles

$(\mathrm{DrLat}_G)_0 = (M_1, M_2, M_1 \xrightarrow{\alpha} M_2, \alpha$ has rank 1$)$

$d \geq 0$ Z^d be locally closed substack of $(\mathrm{DrLat})_0$ consisting of

$$M_1 \to \mathcal{L}_1 \hookrightarrow \mathcal{L}_1(D) = \mathcal{L}_2 \hookrightarrow M_2$$

where $\deg(D) = d$.

$$\begin{tikzcd} \overline{Z}^d \arrow[d, "f^d"] \arrow[dr, "g_d"] & \\ (\mathrm{Dr\,Lat})_0 & X^{(d)} \end{tikzcd}$$

classifies

$(M_1, M_1 \to \mathcal{L}_1, D;\ \mathcal{L}_1(D) \to M_2)$

As a product w/ SL_2 $gr(\Psi)$ will be $\bigoplus_d f^d_! \circ (g^d)^* (\mathcal{F}^d)$

$\mathcal{F}^d \in \mathrm{Perv}(X^{(d)})$ equipped with action of Lefschetz $\mathfrak{sl}_2$.

$\mathcal{E}$ local sys on X $\rightsquigarrow$ $\mathcal{E}^{(d)} \in \mathrm{Perv}(X^{(d)})$

$\rightsquigarrow \Lambda^{(d)}(\mathcal{E}) \in \mathrm{Perv}(X^{(d)})$

$D = \sum d_i x_i \in X^{(d)}$

$\Lambda^{(d)}(\mathcal{E})_D = \otimes \Lambda^{d_i}(\mathcal{E}_{x_i})$

$\mathcal{F}^d$ is the ~~external~~ exterior power of the constant ~~[illegible]~~ local system on X w fibers V.