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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZIOL **Email/Phone:** (315) 569-6968

Speaker's Name: ELLEN EISCHEN

Talk Title: p-ADIC EISENSTEIN SERIES AND APPLICATIONS

Date: 8 / 15 / 14 **Time:** 11 : 00 am / pm (circle one)

List 6-12 key words for the talk: EISENSTEIN SERIES, UNITARY GROUPS, p-ADIC INTERPOLATION

Please summarize the lecture in 5 or fewer sentences: IN THIS TALK, CERTAIN VALUES OF EISENSTEIN SERIES ON UNITARY GROUPS ARE INTERPOLATED INTO FAMILIES.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- [x] Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- [x] Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- [x] For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- [x] When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- [x] Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

P-ADIC FAMILIES OF EISENSTEIN SERIES AND APPLICATIONS

– E. EISCHEN

GOAL: DISCUSS CONSTRUCTION OF A FAMILY OF EISENSTEIN SERIES, AND HOW TO p-ADICALLY INTERPOLATE PARTICULAR VALUES OF CERTAIN EISENSTEIN SERIES ON UNITARY GRPS (SIGN. (n,n)) IN BOTH HOL'C AND NON HOL'C (C^∞) CASES

MOTIVATION

(CONJECTURAL) APPLICATIONS	INSPIRATION
1) NUMBER THEORY: CONSTRUCT p-ADIC L-FNS ATTACHED TO AUT FORMS ON UNITARY GRPS (E.-HARRIS-LI-SKINNER)	KATZ'S p-ADIC L-FNS FOR CM FIELDS
2) HOMOTOPY THEORY: WHAT HIGHER DIM'L ANALOG OF "WITTEN GENUS" (IN TOP. ~~[illegible]~~ AUT FORMS) (BEHRENS-HOPKINS-NAUMANN)	KATZ'S p-ADIC FAMILIES OF E. SERIES (FOR TMF, DONE BY ANDO-HOPKINS-REZK)

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THE IDEA

1) CHOOSE "NICE" FAMILY OF E_i SERIES
2) COMPUTE FOURIER COEFFS (IE, q-EXP COEFFS) WHERE HOL'C
3) APPLY WT-RAISING C^∞ (AND p-ADIC) DIFF. OPS TO OBTAIN C^∞ (AND p-ADIC) AUT FORMS + COMPARE

NOTE $(*) D_\infty E(\underset{\text{A}}{\text{CM PT}}) = (*) D_{p\text{-ADIC}} E(\underset{\text{A}}{\text{ORD CM PT}})$

PERIODS

THESE p-ADIC OPERATORS HAVE NICE ACTION ON q-EXPS

4) USE q-EXP PRINCIPLE + INTERPOLATION OF q-EXP COEFFS TO CONSTRUCT p-ADIC FAMILIES

SETUP

- V = n-DIM'L VECTOR SPACE / K CM FIELD, $E \subset K$ TOT REAL FIELD
- SIMPLIFY NOTATION: $E = \mathbb{Q}$, K QUAD IM
- FIX $\sigma: K \hookrightarrow \overline{\mathbb{Q}}$
- FIX p ST p SPLITS IN K
- $\langle\ ,\ \rangle_V$ NON DEG HERMITIAN PAIRING

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- $W = V \times V$ w/ PAIRING

$$\langle \underset{\in W}{(u,v)}, \underset{\in W}{(u',v')} \rangle_W = \langle u,u' \rangle_V - \langle v,v' \rangle_V$$

HAS SIG (n,n)

- $$\underbrace{U(V, \langle\,,\,\rangle_V)}_{\text{SIG } (a,b)} \times \underbrace{U(V, -\langle\,,\,\rangle_V)}_{\text{SIG } (b,a)} \hookrightarrow \underbrace{U(W, \langle\,,\,\rangle_W)}_{\text{SIG } (n,n)} =: G$$

- FIX A SIEGEL PARABOLIC IN G, CALL IT P

- $f \in \mathrm{IND}_{P(\mathbb{A})}^{G(\mathbb{A})}(\underbrace{\chi|\,|^{-s}}_{\text{REALLY } \chi|\,|^{-s}\circ\det})$, $\quad \chi : K^\times\backslash\mathbb{A}_K^\times \longrightarrow \mathbb{C}^\times$ HECKE CHAR

- $$E_f(g) = \sum_{\gamma \in P(E)\backslash G(E)} f(\gamma g)$$

- FIX $\overline{\mathbb{Q}} \hookrightarrow \mathbb{C}$, $\overline{\mathbb{Q}} \hookrightarrow \mathbb{C}_p$

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FIX $k, \nu \in \mathbb{Z}$, $k \geq n$. HAVE A RECIPE

$$\rightsquigarrow \quad G_{k,\nu,F} = (*)\, E_{f_{(k,\nu,F)}}$$

p-ADIC PARAMETER

PROP (E) LET R BE AN $\mathcal{O}_K$-ALG. LET

$$F : (\mathcal{O}_K \otimes \mathbb{Z}_p) \times M_n(\mathcal{O}_E \otimes \mathbb{Z}_p) \longrightarrow R$$

BE LOC CONST, SUPP'D ON $(\mathcal{O}_K \otimes \mathbb{Z}_p)^\times \times GL_n(\mathcal{O}_E \otimes \mathbb{Z}_p)$ SATISFYING

$$F(ex, N_{K/E}(e)^{-1} y) = N_{k,\nu}(e) F(x,y)$$

WHERE $e \in \mathcal{O}_K^\times$, $x \in \mathcal{O}_K \otimes \mathbb{Z}_p$, $y \in M_n(\mathcal{O}_E \otimes \mathbb{Z}_p)$, $N_{k,\nu}(e) = e^{k+2\nu}(e\bar{e})^{-\nu}$.

THEN $\exists$ $G_{k,\nu,F}$ ON $U(n,n)$ OF WT (k,ν) DEF'D $/R$ WHOSE q-EXP'N AT CERTAIN CUSPS IS OF THE FORM

$$\sum_{\beta \in L} c(\beta) q^\beta$$

LATTICE

W/ $c(\beta)$ A FINITE LINEAR COMBO OF TERMS OF THE FORM

$$F(a, N_{K/E}(a)^{-1}\beta)\, N_{k,\nu}(a^{-1}\det(\beta))\, N_{E/\mathbb{Q}}(\det(\beta))^{-n}$$

WHEN $R = \mathbb{C}$, THIS IS FOURIER COEFF ($@\ s = \frac{k}{2}$) OF C^∞-AUT FORM $G_{k,\nu,F}$ OF WT (k,ν) HOL'C @ $s = \frac{k}{2}$

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CONSEQUENCE: USING p-ADIC q-EXPN PRINCIPLE AND p-ADICALLY INTERPOLATING COEFFS, CAN PUT $G_{k,\nu,F}$ INTO A p-ADIC FAMILY

CHOICE OF SIEGEL SECTIONS f

- CHOOSE $f = \bigotimes_v f_v$ ← OVER ALL PLACES v OF E

($\Rightarrow$ FOURIER COEFFS OF E_f FACTOR OVER v)

FOR NOW, FIX ∞-TYPE OF χ TO BE $\sigma^{-k-2\nu}(\sigma\bar{\sigma})^{\frac{k}{2}+\nu}$

CASES:

- $v|\infty$: BUILT DIRECTLY FROM CANONICAL AUTOMORPHY FACTORS ($\underline{\text{EX}}$: $\det\left((cz+d)^{+k+\nu}(\bar{c}z^T+\bar{d})^{-\nu}\right)$)

$\rightsquigarrow$ FOURIER COEFF IS $(*)\det(\beta)^{k-n}$

- $v\nmid p\infty$: BUILT FROM CHAR FNS OF CERTAIN LATTICES (RECIPE OF SHIMURA)

$\rightsquigarrow$ FOURIER COEFF IS $(*')$ (PRODUCT OF POLYS DEP'T ON β AND v, EVALUATED AT CERTAIN p-INTEGRAL ELTS OF E)

- $v|p$: GIVEN $\tilde{F}: M_n(\mathcal{O}_{E_v}) \times M_n(\mathcal{O}_{E_v}) \longrightarrow R$ SUBJECT TO "CERTAIN CONDITIONS" (TRANSFORMATION PROPERTIES), $\exists$ $f_{\tilde{F}} \in \mathrm{IND}_{P(E_v)}^{G(E_v)}(\chi_v |\ |_v^{-2s})$, W/ FOURIER COEFF $(*)\tilde{F}(1,\beta^T)$