Title: Analytic Continuation of p-adic Modular Forms and Applications to Modularity

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$$f(q)\left(\frac{dq}{q}\right)^{\otimes k}$$

$$(E,P) \longmapsto (E, P, \text{can sub})$$

<u>THM</u>: (Lubin, Katz) $s: X^{ord}_{\mathbb{Q}_p} \longrightarrow Y^{an}_{\mathbb{Q}_p}$

extends to $s: X^{an}_{\mathbb{Q}_p}[0, \frac{p}{p+1}) \longrightarrow Y^{an}_{\mathbb{Q}_p}$

Furthermore, if $v(E,P) < \frac{p}{p+1}$, $H = \text{can sub}$

then $C \neq H$ $v(E/C, \bar{P}) = \frac{1}{p} v(E,P)$

<u>Cor</u>: $U_p(X^{an}_{\mathbb{Q}_p}[0,v]) \subset X^{an}_{\mathbb{Q}_p}[0, \frac{v}{p}]$

$0 \leq v < \frac{p}{p+1}$

$U_p \circlearrowright M_k^{+}$ + compact ✓.

Analytic cont'n (take 1).

If $\mathcal{V} \subset \mathcal{U} \subset X^{an}_{\mathbb{Q}_p}[0, \frac{p}{p+1})$

s.t. $U_p(\mathcal{U}) \subset \mathcal{V}$

$f \in H^0(\mathcal{V}, \omega^k) \Rightarrow U_p f \in H^0(\mathcal{U}, \omega^k)$

$U_p f = a_p f \quad , \quad a_p \neq 0$

$f = \frac{1}{a_p} U_p f \longrightarrow$ extends f from $\mathcal{V}$ to $\mathcal{U}$

<u>Cor</u>: $f \in H^0(X^{an}_{\mathbb{Q}_p}[0, \frac{v}{p}]) \qquad v < \frac{p}{p+1}$

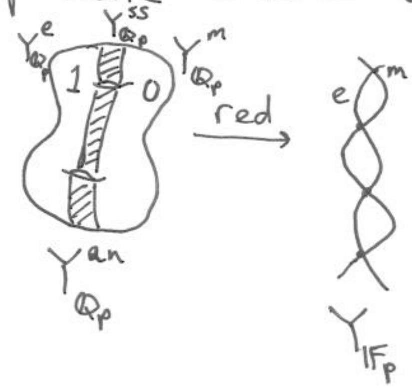
$\Rightarrow f$ extends to $X^{an}_{\mathbb{Q}_p}[0, v]$.

$\Rightarrow f \in M_k^{\dagger} \qquad U_p f = a_p f, \ a_p \neq 0 \Rightarrow$ f automatically extends to the can. locus $X^{an}_{\mathbb{Q}_p}[0, \frac{p}{p+1})$

$\frac{1}{p} |x| < 1$

$0 < v(x) < 1$

p-adic Geometry of $Y^{an}_{\mathbb{Q}_p}$



$$v(R) = \begin{cases} 0 & \text{mult.} \\ v(\text{param}) & \text{ss.} \\ 1 & \text{etale} \end{cases}$$

$w: Y^{an}_{\mathbb{Q}_p} \longrightarrow Y^{an}_{\mathbb{Q}_p}$

$w(E, P, H) = (E/H, \bar{P}, \frac{E[p]}{H})$

$v(wR) = 1 - v(R)$

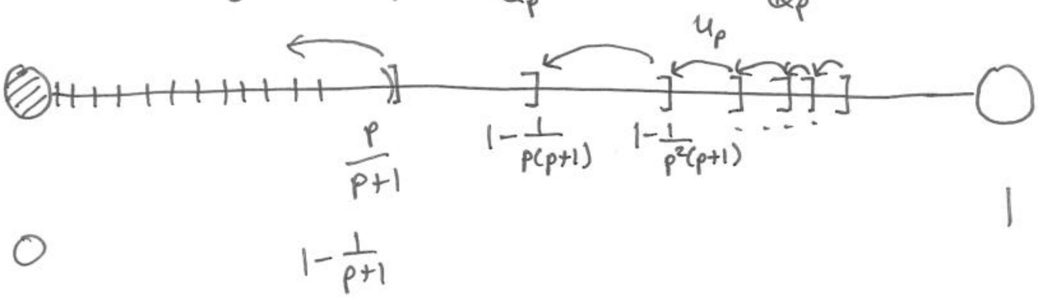
Prop $0 \le v < \frac{p}{p+1}$ then the image of $s : X^{an}_{\mathbb{Q}_p}[0,v] \longrightarrow Y^{an}_{\mathbb{Q}_p}$ is $Y^{an}_{\mathbb{Q}_p}[0,v]$

$$\mathcal{M}_k^{p\text{-adic}} = H^0(Y^{an}_{\mathbb{Q}_p}[0,0], \omega^k)$$

$$\mathcal{M}_k^{\dagger} = \varinjlim_{v>0} H^0(Y^{an}_{\mathbb{Q}_p}[0,v], \omega^k)$$

$$\begin{cases} U_p \\ T_\ell, U_\ell \end{cases} \quad U_p \text{ on level } \Gamma_1(N) \cap \Gamma_0(p) \quad (E,P,H) \longrightarrow \sum_{C \neq H} (E/C, \bar{P}, \bar{H})$$

advantage $U_p : Y^{an}_{\mathbb{Q}_p} \xrightarrow{Cor} Y^{an}_{\mathbb{Q}_p}$



Buzzard $f \in \mathcal{M}_k^{\dagger}$ $\quad U_p f = a_p f$
$a_p \neq 0$

$\Rightarrow f$ extends to

$$Y^{an}_{\mathbb{Q}_p}[0,1) = Y^{m}_{\mathbb{Q}_p} \cup Y^{ss}_{\mathbb{Q}_p}$$

Pilloni reinterpreted

- $v(E,P,H) = 1 - \deg(H)$
- Fargues $\Rightarrow U_p$ increases the degree (strictly) unless $\deg \in \mathbb{Z}$

Thm (Buzzard–Taylor)

$[K:\mathbb{Q}_p] < \infty$, $\mathcal{O}$, $\mathfrak{m}$

$\rho : G_{\mathbb{Q}} \longrightarrow GL_2(\mathcal{O})$

1. ρ ramifies at fin. many primes

2. $\rho|_{D_p} \sim \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$ $\quad \alpha, \beta : D_p \to \mathcal{O}^\times \quad$ α, β unramif. distinct mod $\mathfrak{m}$

3. $\bar{\rho}$ modular and abs. irred.

$\Rightarrow \rho = \rho_h \qquad h \in \mathcal{M}_1(\Gamma_1(N)) \quad \exists N \quad p \nmid N$

$f_\alpha, f_\beta \in \mathcal{M}_1^{\dagger}(\Gamma_1(N))$ $\qquad \boxed{\begin{array}{l} \alpha = \alpha(\mathrm{Frob}_p) \\ \beta = \beta(\mathrm{Frob}_p) \end{array}}$

$U_p f_\alpha = \alpha f_\alpha$

$U_p f_\beta = \beta f_\beta$ $\qquad$ s.t. $\rho \sim \rho_{f_\alpha} \sim \rho_{f_\beta}$

$f_\alpha, f_\beta \qquad \Gamma_0(p) \cap \Gamma_1(N)$ in

$h \qquad \Gamma_1(N)$

$$\begin{tikzcd} Y_{\mathbb{Q}_p} \arrow[d, bend right, "\pi_2"'] \arrow[d, bend left, "\pi_1"] \\ X_{\mathbb{Q}_p} \end{tikzcd} \qquad \pi_2 = \pi_1 \circ w$$

$\mathrm{Span}\{\pi_1^* h, \pi_2^* h\} = \mathrm{span}\{f_\alpha, f_\beta\}$

$\begin{cases} h & T_p\text{-eigenform} \\ f_\alpha, f_\beta & U_p\text{-eigenforms} \end{cases}$

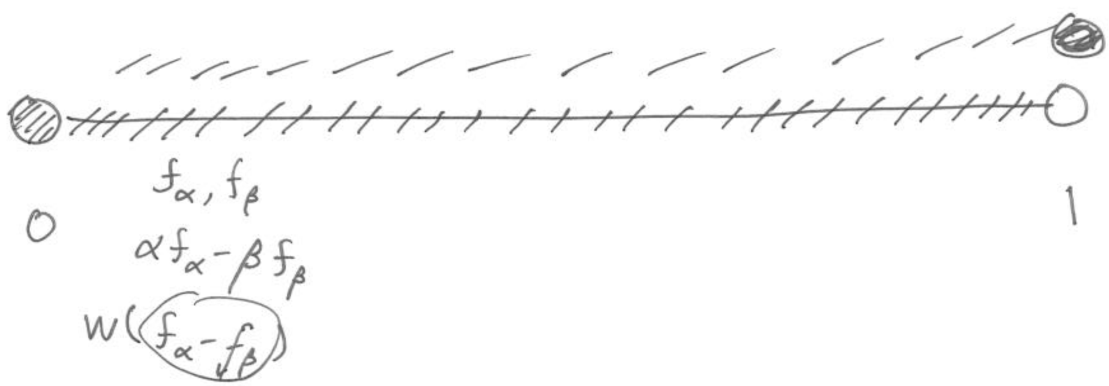
Exercise $\quad f_\alpha = \pi_1^* h - \frac{1}{p\alpha} \pi_2^* h$

$$f_\beta = \pi_1^* h - \frac{1}{p\beta} \pi_2^* h$$

Cor $\quad \pi_1^* h = \frac{\alpha f_\alpha - \beta f_\beta}{\alpha - \beta} \quad (*)$

$$\pi_2^* h \overset{*}{=} w(\pi_1^* h) = \frac{f_\alpha - \cancel{\beta} f_\beta}{\alpha - \beta} \quad (**)$$

- $\alpha f_\alpha - \beta f_\beta$ is classical of level $\Gamma_1(N) \cap \Gamma_0(p)$
- it descends to level $\Gamma_1(N)$



HMS

$[F : \mathbb{Q}] = 2 \quad \mathcal{O}$

p inert in F

$Y_{\mathbb{Q}_p} \quad (\underline{A}, H) \quad \mathcal{O} \curvearrowright H \subset A[p]$

order p^2

isotropic

$X_{\mathbb{Q}_p} \quad \underline{A}$

$$(v_1, v_2) : Y^{an}_{\mathbb{Q}_p} \longrightarrow \left([0,1] \cap \mathbb{Q}\right)^2$$