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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Sean Howe Email/Phone: seanpkh@gmail.com

Speaker's Name: Chris Skinner

Talk Title: p-adic L-functions, Iwasawa theory, and elliptic curves

Date: 12 / 4 / 2014 Time: 3 : 30 am / pm (circle one)

List 6-12 key words for the talk: p-adic L functions, special values, Birch-Swinnerton-Dyer, Iwasawa main conjecture

Please summarize the lecture in 5 or fewer sentences: Recalls the BSD conjecture and describes some new results towards the p-part of the special value assertion for rank ≤ 1. Uses results on Iwasawa main conjecture as key tool.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

$E/\mathbb{Q}$ an elliptic curve

$$\underset{\substack{\| \\ r}}{\mathrm{rk}\, E(\mathbb{Q})} \underset{\text{BSD}}{=} \mathrm{ord}_{s=1} L(E,s)$$

$$\frac{L^{(r)}(E,1)}{r!} \overset{?}{=} \Omega_{E/\mathbb{Q}} \frac{\# Ш(E/\mathbb{Q})}{(\# E(\mathbb{Q})_{tor})^2} \prod_{\ell} c_\ell(E/\mathbb{Q})$$

$$0 \to E(\mathbb{Q}) \otimes \mathbb{Q}_p/\mathbb{Z}_p \to \mathrm{Sel}_{p^\infty}(E/\mathbb{Q}) \to Ш_{p^\infty}(E/\mathbb{Q}) \to 0$$

GZ-K
Gross-Zagier Kolyvagin

$$\mathrm{ord}_{s=1} L(E,s) = r \le 1 \underset{\overset{?}{\Leftarrow}}{\Rightarrow} \mathrm{rk}\, E(\mathbb{Q}) = r, \quad \# Ш(E/\mathbb{Q}) < \infty$$

$$\searrow\!\!\!\nwarrow^{?} \qquad \nearrow_{\forall p} \ \swarrow$$

$\mathrm{Sel}_{p^\infty}(E/\mathbb{Q})$ has corank $= r$

<u>Theorem A</u> Let $E/\mathbb{Q}$ be semistable, $p \ge 5$ a prime of good reduction. Suppose

(a) $E[p]$ is irreducible

(b) If $N_E = 2\ell$ then $p \nmid c_2(E/\mathbb{Q})$

(c) If $\ell \mid N_E$, $\ell \equiv \pm 1 \bmod p$, then $p \nmid c_\ell(E/\mathbb{Q})$

If corank of $\mathrm{Sel}_{p^\infty}(E/\mathbb{Q}) = 1$ then rank $E(\mathbb{Q}) = 1$, $\mathrm{ord}_{s=1} L(E/s) = 1$, $\# Ш(E/\mathbb{Q}) < \infty$

<u>Theorem B</u> For E, p as in Theorem A, satisfying a) and b),
If $\mathrm{ord}_{s=1} L(E,s) = r \le 1$ then

$$\mathrm{ord}_p\left(\frac{L^{(r)}(E,1)}{\Omega_{E/\mathbb{Q}} R_{E/\mathbb{Q}}}\right) = \mathrm{ord}_p\left(\# Ш(E/\mathbb{Q}) \prod_\ell c_\ell\right)$$

<u>Theorem A</u> S-U, Wei Zhang, X. Wan

<u>Theorem B</u> " " + D. Jetchev

$V = T_p E \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ $\quad H^1_f(\mathbb{Q}, V)$ = Bloch-Kato cohomology

$\dim_{\mathbb{Q}_p} = \mathrm{corank}_{\mathbb{Z}_p} \mathrm{Sel}_{p^\infty}(E/\mathbb{Q})$

- parity conjecture $\Rightarrow \varepsilon(E) = -1$
- Choose $K/\mathbb{Q}$ imag. quadratic field s.t. $\dim H^1_f(K, V) = 1$ $(L(E_K, 1) \neq 0)$
  Expect to show Heegner point $P_K \in E(K)$ is non-torsion

- p splits in K $\quad p = v\bar{v}$

$L_v(E/K, \psi)$ interpolate special values

anti-cyclo character

$$L(V^*(1)\otimes\psi, 0)$$

K_∞

$\Gamma \mid \mathbb{Z}_p$-ext.

K

BDP formula

$$L_v(E/K, \mathbb{1}) = (*)\cdot(\log_{E(K_v)} P_K)^2$$

certain factors not written

Iwasawa theory suggests

$$L_v(E/K, \mathbb{1}) \neq 0 \iff H^1_v(K, V) = 0$$

$(\mathrm{res}_w\, c = 0 \quad \forall\, w \neq \bar{v})$

X. Wan proved enough of main conjecture to conclude $\Leftarrow$ holds

for $L_v(E/K, \psi)$

$$L_v(E/K, \psi) \mid \mathrm{char}_{\mathbb{Z}_p[[\Gamma]]}(\text{Greenberg Selmer group})$$

$$\left.\begin{array}{l} \dim_{\mathbb{Q}_p} H^1_f(K, V) = 1 \\ \underline{H^1_f(K,V) \hookrightarrow H^1_f(K_v, V)} \end{array}\right\} \xRightarrow{\text{duality}} H^1_v(K, V) = 0$$

$$\Longrightarrow L_v(E/K, \mathbb{1}) \neq 0$$

Would like to remove this assumption

$$\Longrightarrow P_K \text{ is non-torsion}$$

From the divisibility of the char. ideal,

$$\#\,\mathbb{Z}_p/(*)(\log_{E(K_v)} P_K)^2 \leq \underbrace{\#\,\mathrm{Sel}_v(E/K)}_{\#Ш_{p^\infty}(E/\mathbb{Q})\cdot [E(K_v):\widehat{E(K)}]^2} \cdot \prod_{\substack{\ell \mid N_E \\ \text{split in } K \\ (2 \text{ split in } K)}} c_\ell(E/\mathbb{Q}_\ell)^2$$

$$\#\,\mathbb{Z}_p/(*)(\log_{E(K_v)} P_K)^2 = [E(K_v) : \mathbb{Z}_p P_K]^2$$

In Thm A choose K so that a unit.

In Thm B choose K so that at most one such ℓ

$$[E(K):\mathbb{Z}P_K]_p \le \#Ш(E/\cancel{K}) \cdot \prod_\ell c_\ell(E/\mathbb{Q}_\ell)^2$$

$$\frac{L'(E/K,1)}{\Omega_{E/K}} = [E(K):\mathbb{Z}P_K]^2 R_{E/K} \underbrace{\frac{\langle f_E, f_E\rangle}{\langle f_E', f_E'\rangle}}_{\prod_{\ell \text{ inert}} c_\ell}$$

$$\mathrm{ord}_p(\quad) \le \mathrm{ord}_p\Big(\#Ш(E/K)\cdot \prod_{\ell \text{ split}} c_\ell^2 \prod_{\ell \text{ inert}} c_\ell\Big)$$

Kolyvagin gives lower bound $\ge \#Ш(E/K)$

Jetcher allows to add $\cdot \max c_\ell(E/\mathbb{Q}_\ell)^2$

So if choose K correctly, get $=$

Still need to separate $L(E_K,1)$ out

- IF E ordinary at p, use special value formula for $\frac{L(E_K,1)}{\Omega_{E/K}}$ coming from MC for E_K

- IF E is supersingular at p, combine w/work of Kobiyashi to deduce supersingular MC for E for E_K – give/the desired formula for $L(E_K,1)$