MSRI

Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu Email/Phone: pengcheng.xu@okstate.edu

Speaker's Name: Jason Manning

Talk Title: Dehn filling of groups and spaces

Date: 03 / 18 / 2013 Time: 9 : 30 am / pm (circle one)

List 6-12 key words for the talk: Dehn filling; relatively hyperbolic groups; CAT(0) geometry; Gromov-Thurston 2π Theorem;

Please summarize the lecture in 5 or fewer sentances: The speaker talked about the Hyperbolic Dehn filling which is originally constructed by Thurston. He discussed the Gromov-Thurston 2π theorem and Fujiwara's question. Then he discussed the work in the context of relatively hyperbolic groups which is due to Osin (and independently Groves and himself).

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Monday
1^{st} talk
9:30 a.m — 10:30 a.m

Dehn filling of groups and spaces

speaker
Jason Manning

Classical construction with Dehn surgery.

$K \hookrightarrow M^3$
$\cong$
S^1

$N \cong D^2 \times S^1$ $\qquad (M^3 \setminus N) \cup_\varphi N$ where $\varphi : \partial N \to \partial N$
$\varphi \not\simeq id.$

we get a new 3-mfld M

Two stages: ① Drill out K.
② Fill it back in. $\to$ Dehn filling

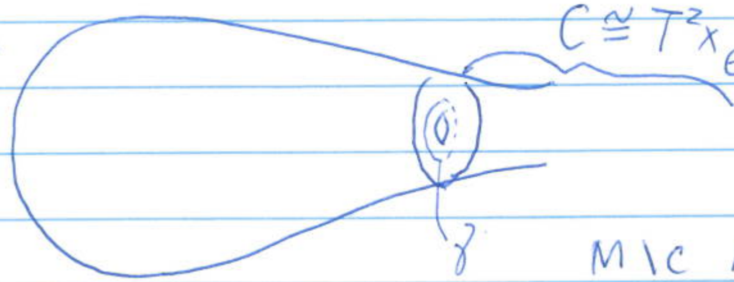
M_1, M_2 closed 3-mflds.
Then $\exists$ sequence of Drilling and Fillings connecting them.

Hyperbolic Dehn Filling (Thurston)

Suppose $M \setminus K$ has a hyperbolic metric.

Cartoon:

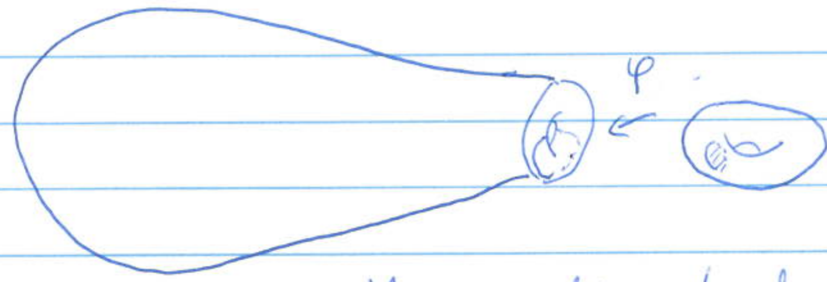
$C \cong T^2 \times_{e^{-t}} (0, \infty)$



$M \setminus C$ has flat boundary (bdry)

Filling: Choose geodesic loop γ on $\partial(M \setminus C)$. Glue in $D^2 \times S^1$ by φ so $\varphi(\partial D^2 \times \{1\}) = \gamma$

(see picture on next page).



M_γ = resulting closed mfld

Thurston: For most γ, M_γ has a hyperbolic metric.
(Deformation theory proof).

Version (w/) with more control, but weaker conclusion:
Gromov - Thurston "2π" Thm: If $|\gamma| > 2\pi$, can build negatively curved metric on M_γ, isometric to M outside C.

If you ~~m~~ want
① Lots of "hyperbolic" fillings
choose C "BIG"
② To preserve more geometry
choose C "SMALL".

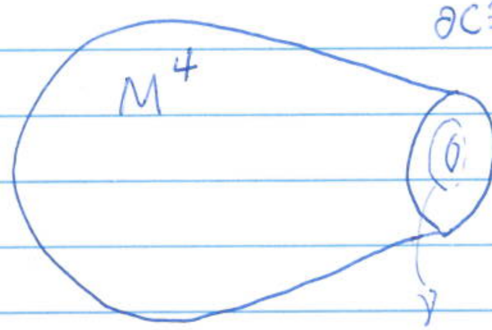
Also $\exists$ more combinatorially ~~pl~~ version:
Agol - Lackenby "6 thm"

Try to generalize 2π Thm:
① To higher dim's hyperbolic mflds?
② To relatively hyperbolic pairs?

Prove 2π by explicit construction on Riemanian metric on $D^2 \times S^1$.

Fujiwara – M: Can construct similar warped-product metrics on $C(T^k) \times T^{n-1-k}$
cone

use this to produce nonpositively curved pseudo-mfld as "Dehn fillings" of high-dim's hyperbolic mflds



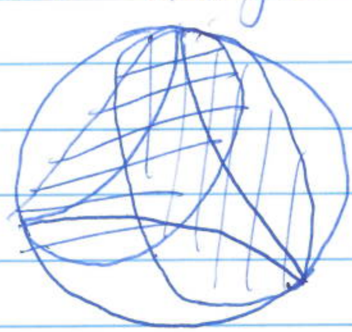
$\partial C \cong T^3 \supset \gamma \cong T^2$
with large injectivity radius
$(>2\pi)$

Example of (FM)
M_γ has a locally CAT$(-\varepsilon)$ metric, and is never $\simeq$ closed Asph manifold

etc.

Q: Are there similar metric constructions for $\mathbb{C}\mathbb{H}^n$-mflds.

Def: A f.g. group is hyperbolic if its Cayley graph has thin triangles.



3^{rd} side $\subset$ δ-nbhd of union of other two edges.
δ independent of the $\triangle$.

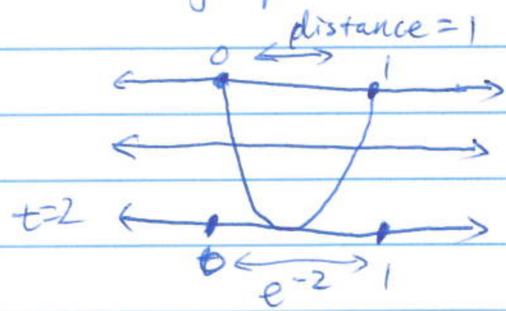
π_1(closed hyperbolic M) is hyperbolic.

Fact: G hyperbolic $\Rightarrow$ $\mathbb{Z}\oplus\mathbb{Z} \hookrightarrow G$ can't happen.

So π_1(cusped hyperbolic) is not hyperbolic.

Horoballs: Hide a (possibly non-hyp) space in a hyperbolic one:

Γ A graph. $\mathcal{H}(\Gamma) = \Gamma \times_{e^{-t}} (0,\infty)$



$\mathbb{R} \times_{e^{-t}} (0,\infty) \cong$ horodisk in $\mathbb{H}^2$

Let (G, H) be a group pair (f.g, f.g)

C a Cayley graph for G.

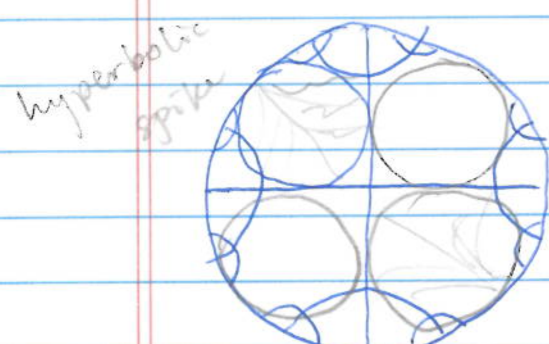
$X(G,H) = C \cup \bigsqcup_{g \in G/H} \mathcal{H}(gH)$ (Assume Cayley graph of $H \subset$ Cayley graph of G.)

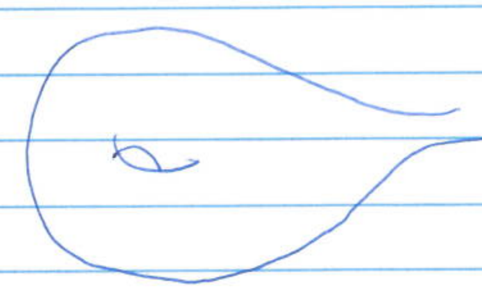
$\cup$: glue on at $t=0$.

$\mathcal{H}(gH)$: $\boxed{\text{horoball bases on } C(H)}$

Example: $G = F_2$, $H = \langle xyx^{-1}y^{-1} \rangle$

$X(G,H) \overset{\text{quasi-isometric}}{\simeq} \mathbb{H}^2$





Examples of relatively hyp (G, H)

① $G = \pi_1$ (one-cusped finite volume hyperbolic mfld)
$H = \pi_1(\text{cusp})$

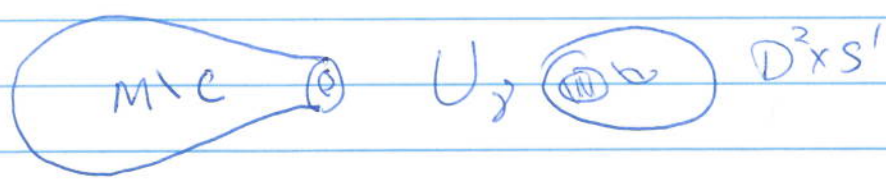
② G = Hypebolic group
H = Quasi convex malnormal subgroup.

$\{$Malnormal: $g \notin H \Rightarrow H \cap H^g = \{1\}\}$

$\{$quasiconvex$\}$ QC. M geodesic metric $\lambda > 0$, $A \subset M$ is λ-quasiconvex if every geodesic w/ endpoints in A lies in $N_\lambda(A)$

$H < G$ hyperbolic is QC if $H \subset$ Cayley graph is λ-QC for some λ.

Dehn filling from point of view of π_1.



$$\pi_1(M_\gamma) = \pi_1(M) / \langle\langle \gamma \rangle\rangle.$$

$(\pi_1(M), \pi_1(C)) \rightarrow$ relatively hypebolic

Fill.

$(\pi_1(M_\gamma), \mathbb{Z}) \nearrow$

Relatively hyperbolic filling (Osin, Groves-M, Dehmani-___-Osin)
Guirardel

(see next page)

Dehn filling of (G, H)

Choice of $N \triangleleft H$

$\rightsquigarrow G \xrightarrow{\pi} G/\langle\langle N \rangle\rangle_G = G_N$

$$\begin{tikzcd}
(G, H) \arrow[r] \arrow[dr] & (G_N, \pi(H)) \\
& H/N \arrow[u]
\end{tikzcd}$$

Relatively hyp Dehn filling:

Suppose (G, H) is R.H. There is a FINITE set B so that if $(N \cap B) \setminus \{1\} = \emptyset$, and $N \triangleleft H$, then

① $(G_N, \pi(H))$ is relatively hyp

② $\pi(H) \cong H/N$

Moreover, given F finite, in G, we can rechoose B s.t

③ $\pi|_F$ is injective.

Moreover (Agol-Groves-M) can choose B so fillings "behave well" on finitely many specified (Rel) QC subgroups of G

$\square$