MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu **Email/Phone:** pengcheng.xu@okstate.edu

Speaker's Name: Daniel Groves

Talk Title: Wise's Malnormal Special Quotient Theorem

Date: 03/18/2013 **Time:** 11 : 00 am / pm (circle one)

List 6-12 key words for the talk: separability; quasiconvex subgroup; hierarchy virtually special cube complex; hyperbolic group

Please summarize the lecture in 5 or fewer sentances: Wise's Malnormal Special Quotient Theorem is the key technical result in his work on groups with a quasiconvex hierarchy, and an important tool in Agol's proof of the Virtual Haken Conjecture. The speaker explained the context of this theorem and discussed an alternate proof of MSQT.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

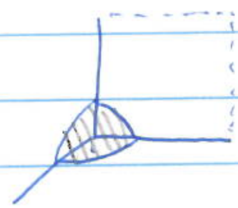
2nd talk Monday:

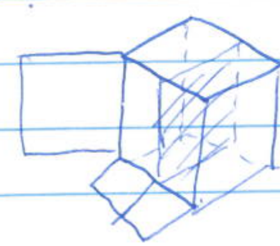
Wise's Malnormal Special Quotient Theorem

speaker: Daniel Groves

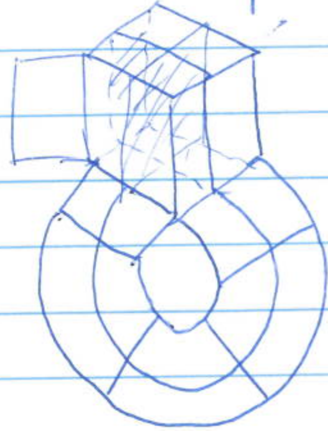
How can we see if a (hyperbolic) group acts properly and cocompactly (and virtually specially) on a CAT(0) cube complex? (And why should we care?)

Def: A cube complex is built from Euclidean cubes by identifying faces.
It's nonpositively curved (NPC) if the link of any vertex is a flag simplicial complex.





An NPC cube complex is CAT(0) if it's simply-connected.



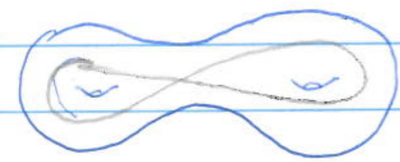
cube complexes come equipped with nice (immersed) co-dim 1 subspaces (hyperplanes)

Def: Let G be a group. A subgroup $H \leq G$ is separable if $\forall g \in G \setminus H$, there is a finite-index subgroup $G' \leq G$, with $H \leq G'$, $g \notin G'$

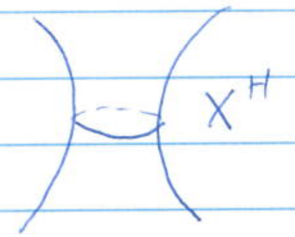
Topological Interpretation:
X compact complex, $G = \pi_1(X)$

X^H cover of X correspond to H.
$$\begin{array}{c} X^H \\ \downarrow p \\ X \end{array}$$

$S^1 \xrightarrow{f}$



$H = f_*(\pi_1(S^1))$



fig

H is separable in G iff for any compact $A \subseteq X^H$, there is an intermediate finite cover
$$X^H \to \hat{X} \xrightarrow[\text{finite}]{} X$$ in which A embeds.

separability lifts immersions to embeddings

Def: A group G is LERF if every finitely generated subgroup is separable.
A hypebolic group G is QCERF if every QC subgp is separable

Terminology:
1. A hyperbolic group G is CUBULATED if it acts properly and compactly on a CAT(0) cube complex.
2. A CUBULATED hyperbolic group G is virtually special if there is a finite-index subgp $G' \le G$ s.t. X/G' is a special cube complex
   (X: CAT(0) cube complex)

Theorem [Hanglund-Wise] A CUBULATED hyperbolic group is virtually special iff it is QCERF.

Theorem [Hruska-Wise] Suppose G is a (torsion-free) hyperbolic group and H is a separable QC subgp. There is a finite-index $G' \leq G$ s.t. $H' = G' \cap H$ is Malnormal in G' (and QC)

(Malnormal) Quasi-convex Hierarchies

Three moves:

(A) Replace G with finite-index $G' \leq G$

(B_M) If $G = A *_C B$ with C QC, malnormal then consider A, B $G = A *_C$

(B_Q) If $G = A *_C B$ (or $G = A *_C$) with C QC, consider A, B.

Def: A malnormal quasiconvex virtual [MQVH] hierarchy for a hyperbolic group G is sequence of moves (A) (B_M) so that we eventually get to (finitely many copies of) $\{1\}$.

A QVH uses moves (A) and (B_Q)

Theorem [Hsu-Wise, Haglund-Wise]
If a hyperbolic group has an MQVH, then it is virtually special.

Thm (Wise) A hyperbolic group with a QVH is virtually special.

Malnormal Special Quotient Thm (Wise)
Suppose G is hyperbolic and virtually special, and that $M \leq G$ is malnormal, QC, for <u>many</u> finite-index subgps $M' \leq M$, the quotient $\pi: G \to G/\langle\langle M' \rangle\rangle = \bar{G}$ is hyperbolic and virtually special.

Sketch pf of MSQT $\Rightarrow$ (QVH $\Rightarrow$ VS)

(Assuming (MQVH $\Rightarrow$ VS))

Key case: A, B hyperbolic, VS, $G = A *_C B$ is hyperbolic, C is QC.

By Hruska-Wise, if C were separable in G, it would be malnormal in a finite-index subgroup G'. The Bass-Serre tree for G induces a graph of groups decomposition for G' with malnormal QC subgroups.

$\Rightarrow$ MQVH for G' $\Rightarrow$ G' is VS.

So we need to prove C is separable. There is a malnormal subgroup QC $D \leq C$. $D' \trianglelefteq D$.

Idea: Given $g \in G \setminus C$. Longenough Dehn fillings, $G/\langle\langle D' \rangle\rangle$ have $\varphi : G \to \bar{G} = G/\langle\langle D' \rangle\rangle$,

1. $\bar{G}$ is hyperbolic
2. $\bar{C} = \varphi(C)$ is QC and $\cong C/\langle\langle D' \rangle\rangle$
3. $\bar{g} = \varphi(g) \notin \varphi(C)$
4. $\bar{A} = \varphi(A) \cong A/\langle\langle D' \rangle\rangle$ is QC, VS by MSQT

   $\bar{B}$ similarly.

$\bar{G} = \bar{A} *_{\bar{C}} \bar{B}$

(Gained: $\bar{C}$ is closer than C was to being malnormal).

$\square$