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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu **Email/Phone:** pengcheng.xu@okstate.edu

Speaker's Name: Vlad Markovic

Talk Title: Criterion for Cannon's Conjecture

Date: 03 / 18 / 2013 **Time:** 3 : 30 **am / pm (circle one)**

List 6-12 key words for the talk: Cannon's conjecture; surface subgroup theorem

Please summarize the lecture in 5 or fewer sentances: The speaker discussed the criterion for Cannon's conjecture.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Monday
4th talk

Criterion for Cannon's Conjecture

speaker: Vlad Markovic

Cannon's conjecture: Let G be a hyperbolic group (acts effectively on ∂G) whose boundary is homeomorphic ~~I have~~ to S^2,

$\partial G \simeq S^2$ (G acts by orientation preserving homeomorphism on ∂G)

Then G is a Kleinian group, that is $G \approx \pi_1(M^3)$ if G torsion free.

$\mu: G \to \text{Homeo}(\partial G) \quad \ker \mu = \text{trivial}$

Hypobotic Thm: M^3 irreducible, a toriadal $|\pi_1(M^3)| = \infty$ is hyperbolic

Criterion for Cannon: Suppose G is hyperbolic, $\partial G \approx S^2$, then G is a Kleinian group iff it is cubulated by surface groups.

To understand cubulation:

Lemma (Bergeron - Wise) A hyperbolic group is cubulated iff every two points in ∂G can be separated by a QC codimension 1 subgroup of G

$\boxed{\text{Codimension 1, } H \subset G}$

Conjecture (Gromov) Every one ended hyperbolic group has a surface subgroup

To prove the Criterion:

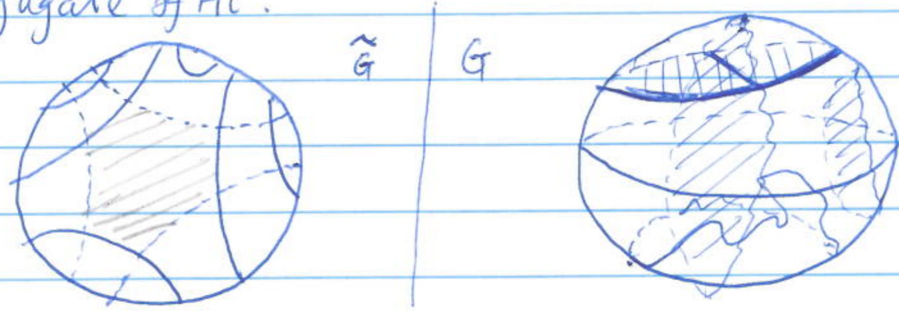
Theorem (Agol): If G (hyperbolic group) is cubulated, then QC subgroups of G are separable.

Theorem (M.) Suppose $\partial G \simeq S^2$, G hyperbolic, If G contains enough surface subgroups, then $\tilde{G} \simeq \pi_1(M^3)$, M^3 - three mfld.
$\tilde{G} < G$, finite-index. (M^3 is Haken)

1. By Scott's core thm, M^3 is compact.
2. M^3 is Haken $\xRightarrow{\text{Thurston}}$ M^3 admits a complete geometrically finitely hyperbolic structure.
   a) $G \simeq \pi_1(M^3) \simeq \Gamma$, $\Gamma < Isom(\mathbb{H}^3)$.
   b) $M^3 \approx$ Neighbourhood$(C(\Gamma))$
3. Milnor - Svaic

Thm (Gabai, Casson - Jungreis, Tukia)
$\partial G \simeq S^1$, G-hyperbolic, then G is Fuchsian.

Lemma: Let G be hyperbolic, and $\mathcal{H}$ a collection of QC, codim 1 subgroups of G, s.t. every two points of ∂G are separated by $H \subset \mathcal{H}$, $\exists \hat{G} < G$, $H_1, \dots, H_m \subset \mathcal{H}$, H_i are malnormal in $\hat{G}$, every two points are separated by conjugate of H_i.



Def: $(K, \mathcal{U})$ is a generalized cell decomposition of $\mathbb{D}^3$, if

1. K is a closed subset of $\mathbb{D}^3$
2. $\mathcal{U}$ collection of components of $\mathbb{D}^3 \setminus K$.
   $U \subset \mathcal{U}$ $\quad U \cong \mathbb{D}^3$, $\partial \mathcal{U} \approx S^2$
3. $\forall \delta > 0$, there are finitely many U's $\operatorname{diam}(U) > \delta$.

G action on $(K, \mathcal{U})$

$\mu : G \to \mathrm{Homeo}(K) \quad \mu(g)(S^2) = S^2 \quad g \in G$.

Def: $(\mu, K, \mathcal{U})$ is G-complex if

$\exists\, \phi^{\mathcal{U}} : S^2 \to \partial U$, equivariant with respect to G.

$\mu(g) \circ \phi^{\mathcal{U}} = \phi^{\mu(g)(U)} \circ \mu(g)$ on S^2. $\mid$ $\phi^{\mathcal{U}}$ is id on $\partial U \cap S^2$

Lemma: Suppose (μ, K, U) is a free convergence G-complex and stabilizer(U) is trivial, Then $G \approx \pi_1(M^3)$.

$H < G$, malnormal, $\mathcal{H}$-conjugate of H

$\forall H < \mathcal{H}$

$B_H \subset \mathbb{D}^3$

$K_{\mathcal{H}} = S^2 \cup \left(\bigcup_{H \in \mathcal{H}} B_H \right) \qquad (\mu_H, K_{\mathcal{H}}, \mathcal{U}_H)$

G-complex.

If $a, b \in \partial U \in \mathcal{U}_H$, they are not separated ...

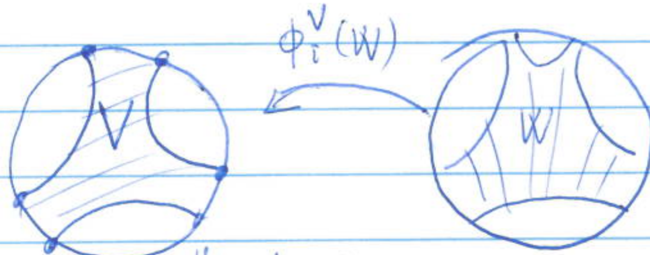
Building new G-complexes onto old.

$(\mu_i, K_i, \mathcal{U}_i, \phi_i^{\mathcal{U}_i})$, $i = 1, 2$

$\mu_1(g) = \mu_2(g)$ on S^2, $g \in G$.

Claim $\exists (\mu, K, \mathcal{U}, \phi^{\mathcal{U}})$

$\mathcal{U} = \{ U : U = \phi_i^V(W),\ V \in \mathcal{U}_1,\ W \in \mathcal{U}_2 \}$

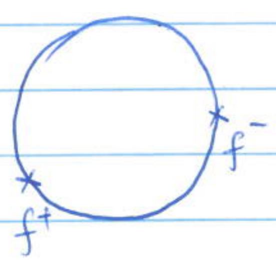


$(\mu_1, K_i, \mathcal{U}_1, \phi^{\mathcal{U}_1})$ $\phi_i^V : S^2 \to \partial V$ ( )

Lemma: $\mathrm{id} = \mathrm{Stabilizer}_{\mu}(U) = \mathrm{Stabilizer}_{\mu_1}(U_1) \cap \mathrm{stabilizer}_{\mu_2}(U_2)$

$H_1, \dots, H_k < G \Rightarrow \bigcap_{i=1}^{k} \mathrm{stabilizer}_{\mu_i}(U_i)$

$\mathcal{H}_1, \dots, \mathcal{H}_k$



$f \in G$

$f \in \mathrm{Stab}_{\mu}(U)$