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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu **Email/Phone:** pengcheng.xu@okstate.edu

Speaker's Name: Piotr Przytycki

Talk Title: Slim unicorns

Date: 03 / 20 / 2013 **Time:** 9 : 30 am / pm (circle one)

List 6-12 key words for the talk: arc graph; hyperbolicity; unicorn path;

Please summarize the lecture in 5 or fewer sentances: The speaker described unicorn path in the arc path and show that they form 1-slim triangles and are invariant under taking subpaths then deduced that all arc graphs are 7-hyperbolic

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Wednesday
1^{st} talk

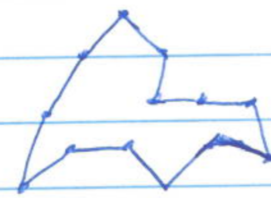
Slim unicorns

speaker: Piotr Przytycki

joint with S. Hensel & R. Webb.

Uniform hyperbolicity for arc graphs (and curve graphs)

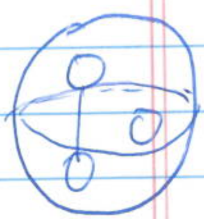
Hyperbolicity:
 Γ - connected graph
 $T \subset \Gamma$ formed of edge path



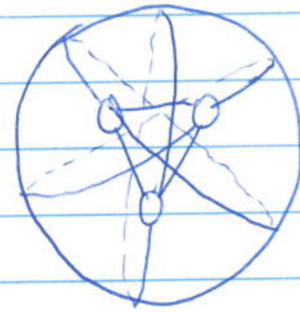
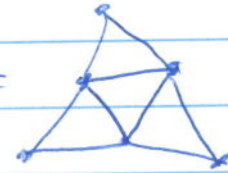
T is k-slim if each of its 3 sides is contained in k-nbhd of the other two. k-~~centered~~ centred if there is a vertex in Γ at distance $\leq k$ from all the sides.

Γ is k-hyperbolic if all of its geodesic triangles are k-centred.

Arc graph: S - compact oriented connected surface with boundary

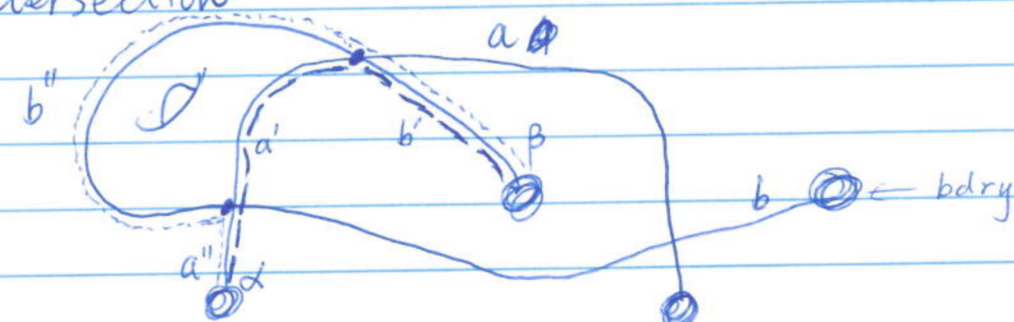



We consider arcs on S that are properly embeded and not homotopic into ∂S.
Arc graph $\mathcal{A}(S)$ is the graph whose vertex set is the set of arcs on S, up to homotopy. Two vetices are connected by an edge, if the arcs can be realized disjointly.

Example: $S =$  $\mathcal{A}(S) =$ 

Thm: [Hensel - P - Webb] $\mathcal{A}(S)$ is 7-hyperbolic

Unicorn paths: Assume vertices a, b not adjacent in $\mathcal{A}(S)$. Suppose a, b are realized as arcs on S with minimal intersection



A unicorn arc is an embedded arc of the form $a' \cup b'$ where $\alpha \in a' \subset a$, $\beta \in b' \subset b$

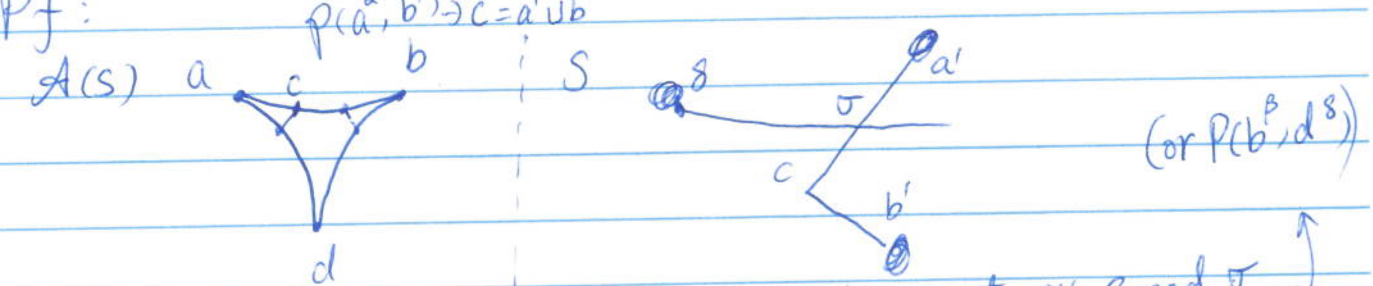
Order on unicorn arcs $a' \cup b' \leq a'' \cup b''$ if $a' \supseteq a''$ or equivalently $b' \subseteq b''$ (not at the same time)

Remark: Let $(c_1, \ldots, c_n)$ be the order set of the unicorn arcs, then $(c_0 = a, c_1, \ldots, c_n, c_{n+1} = b)$ is an edge path in $\mathcal{A}(S)$

unicorn path $P(a^{\alpha}, b^{\beta})$

Lem 1: a, b, d vertices of $\mathcal{A}(S)$, α, β, δ endpoints. Then the triangle $P(a^\alpha, b^\beta)$, $P(b^\beta, d^\delta)$, $P(a^\alpha, d^\delta)$ is 1-slim and 2-centred.

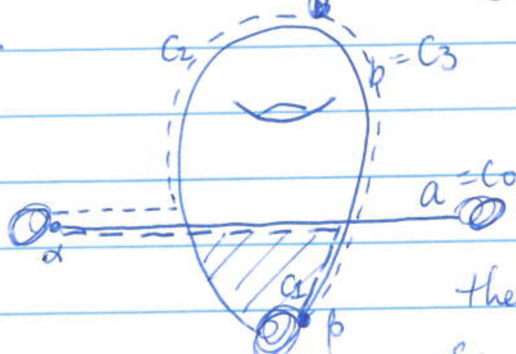
Pf:



σ - first intersection point with C and σ determines a unicorn arc on $P(a^\alpha, d^\delta)$

We realise a, b, d on S as arcs ~~pair~~ pairwise with minimal intersections. □

Lem 2. Let $c_i < c_j \in P(a^\alpha, b^\beta)$. Then $P(c_i^\alpha, c_j^\beta) \subset P(a^\alpha, b^\beta)$ except in the case where $j = i+2$ and c_i, c_j are adjacent.

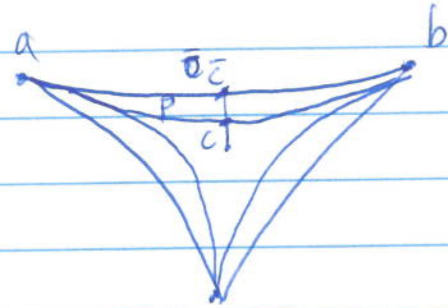


there is a homotopy to make c_1 disjoint with $c_3 = b$.

Pf: Except for this configuration, c_1 has minimal intersection with b. □

Proof of the Thm:

Consider a geodesic triangle $T \subset \mathcal{A}(S)$. Choose endpoints α, β, δ and consider unicorn paths



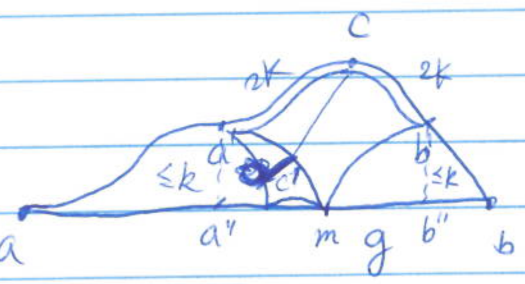
$p = P(a^\alpha, b^\beta)$, $P(a^\alpha, d^\delta)$, $P(b^\beta, d^\delta)$

By Lem 1, the triangle formed by unicorn paths is 2-centred.

It suffices to show

Proposition $\forall c \in \mathbb{P}$, $\forall g$ geodesic from a to b, $\exists\, \bar{c} \subset g$ s.t. $|c\bar{c}| \le 6$.

Pf: Let $c \in P$ be the farthest from g at distance k.



Take $a'b' \subset P$ maximal subpath containing c at distance $\le 2k$ from c.

The concatenation $a'a''b''b'$ has length $\le 8k$.

m - midpoint of $a'a''b''b'$

Consider paths $P(a'^\alpha, b'^\beta) \overset{\text{Lem 2}}{=} a'b'$

$P_1 = P(a'^\alpha, m^\mu)$ $P_2 = P(b'^\beta, m^\mu)$

By Lem 1, $\exists\, c' \in P_1 \cup P_2$ adjacent to c.

Continue. finally, $\exists\, \bar{c} \in a'a''b''b'$ at distance $\le \lceil \log_2 8k \rceil$ from c

If $\bar{c} \notin a''b''$, then $\lceil \log_2 8k \rceil \ge k$ by triangle inequality for $c, a, \bar{c}$.

If $\bar{c} \in a^n b^n$, then $\lceil \log_2 8k \rceil \geq k$ by definition of k

$\lceil \log_2 8k \rceil \geq k$ implies $k \leq 6$ $\square$