MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu **Email/Phone:** pengcheng.xu@okstate.edu

Speaker's Name: Yi Liu

Talk Title: Bounded quasi-Fuchsian subsurfaces in closed hyperbolic 3-manifolds

Date: 03 / 21 / 2013 **Time:** 11 : 00 am / pm (circle one)

List 6-12 key words for the talk: surface subgroup; Kleinian group; good pants homology

Please summarize the lecture in 5 or fewer sentances: The speaker discussed principles of constructing quasi-Fuchsian subsurfaces in closed hyperbolic 3-manifolds via Kahn-Markovic pants gluing. He showed that the surface can be manipulated to be connected, having desired boundary up to covering and desired relative homology class.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Thursday 2nd talk

Bounded quasi-Fuchsian subsurfaces in closed hyperbolic 3-manifolds speaker : Yi Liu

Kahn - Markovic construction revisited.

Constructing surfaces in closed hyperbolic 3-manifolds.

A sample problem:

M^3 closed hyperbolic, L a disjoint (union) ~~collection~~ of finitely many simple closed curves in M. Then for any $\alpha \in H_2(M, L; \mathbb{Q})$ is there a connected π_1-injectively immersed quasi-Fuchsian subsurface $j : F \looparrowright M$ with $j(\partial F) \subset L$ and $[F]$ is a multiple of α?

Three aspects.

Algebraic

Topological

Geometric

Strategy:

- Find a suitable collection of good pants.
- Matching up nicely along common cuffs.

DATA

μ : a measure of good pants $\rightsquigarrow$ ① Out put Connected qF subsurface

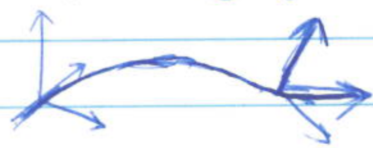
Conditions in terms of ② boundary operators

Basic techniques:

Geometry of ∂ - framed segments (bdry)

$\mathbb{H}^3$

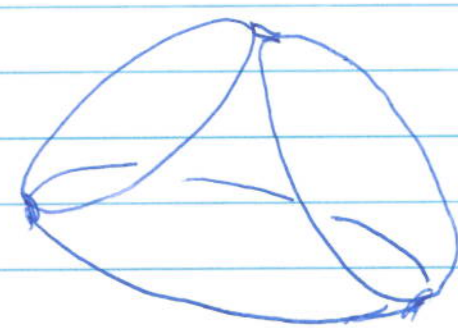
In a hyperbolic closed 3 manifold
for sufficiently good (R, ε)



- $m(\Pi_{R,\varepsilon})$ - measures on oriented (R,ε) good pants
- $m(\Gamma_{R,\varepsilon})$ - measure on oriented (R,ε)-good curves.
- $m(|\Gamma_{R,\varepsilon}|)$ - measure on unoriented (R,ε)-good curves
- $m(N^1_{\Gamma_{R,\varepsilon}})$ finite Borel, compactly supported, measures on $\coprod_{\gamma \in \Gamma_{R,\varepsilon}} N^1_\gamma$

$$\begin{tikzcd} & m(N^1_{\Gamma_{R,\varepsilon}}) \arrow[d, "\text{Total}"] & \\ m(\Pi_{R,\varepsilon}) \arrow[ur, "\partial^\#"] \arrow[r, "\partial"] \arrow[dr, "\partial^b"'] & m(\Gamma_{R,\varepsilon}) \arrow[d, "\text{Net}"] \\ & m(|\Gamma_{R,\varepsilon}|) \end{tikzcd}$$

unoriented curve



$\partial \Pi = 6$ feet

Q 1: Under what conditions can we output a connected qF surface?

A 1. • ubiquitous ($\partial\mu$ is supported on the entire $\Gamma_{R,\varepsilon}$)
• irreducible ($\nexists$ $\mu = \mu_1 + \mu_2$ where μ_1, μ_2 are supported on disjoint subset of $\Gamma_{R,\varepsilon}$)
• (R,ε) - nearly evenly distributed
• rich $((\partial\mu)(\gamma) > \frac{1}{5}(\partial^b\mu)(|\gamma|))$

Q2: Homology from good pants?

A2: $|\mathcal{L}| \subset |\Gamma_{R,\varepsilon}|$

$$ZM(\Pi_{R,\varepsilon}, |\mathcal{L}|) := \{\mu \mid \partial^b \mu \text{ is supported on } |\mathcal{L}|\}$$

(1)

$$0 \to BM(\Pi_{R,\varepsilon}, |\mathcal{L}|) \to ZM(\Pi_{R,\varepsilon}, |\mathcal{L}|) \xrightarrow{[\cdot]} H_2(M, |\mathcal{L}|, \mathbb{R}) \to 0$$

is a short exact sequence.

(2). $\exists\ \mu_0 \in BM(\Pi_{R,\varepsilon}, |\phi|)$ which is ubiquitous, irreducible, nearly evenly distributed and rich.

Hence any $\mu \in ZM(\Pi_{R,\varepsilon}, |\mathcal{L}|)$ can be adjusted to have the same properties by adding a multiple of μ_0.

(1) (1.1) $|\mathcal{L}| = |\phi|$

(1.2) using good pants homology $\Omega_{R,\varepsilon} \xrightarrow[\sim]{[\,]} H_1(M)$