The Mathematical Education of Teachers II Elementary and Middle Grades

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Elementary Teachers

“A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher.”

Elementary Teachers

"Thus, this report recommends that before beginning to teach, an elementary teacher should study in depth, and from a teacher's perspective, the vast majority of K–5 mathematics, its connections to prekindergarten mathematics, and its connections to grades 6–8 mathematics."

Essential Grades K–5 ideas

CCSS domains at K–5:

- Counting and Cardinality
- Operations and Algebraic Thinking
- Number and Operations in Base Ten
- Number and Operations – Fractions
- Measurement and Data
- Geometry

Operations and Algebraic Thinking

CCSS Table 1: Common types of $+-$ word problems

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

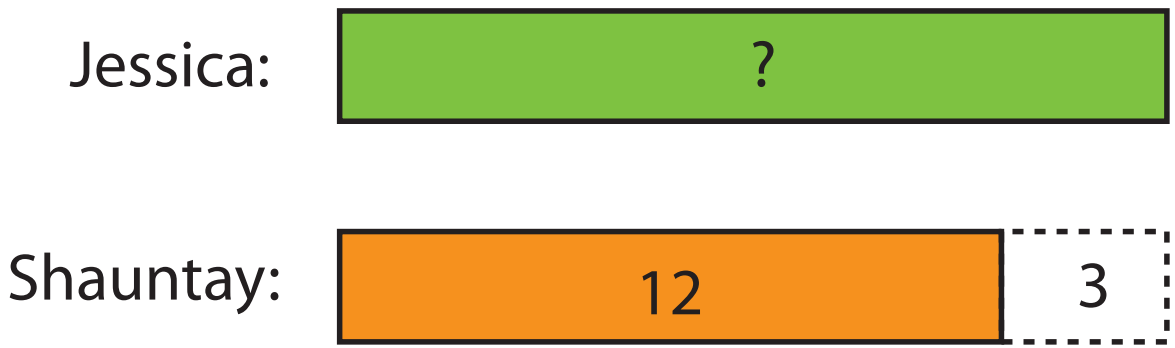
	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than

Compare, bigger unknown problem

Shauntay has 12 cards. Jessica has 3 more cards than Shauntay. How many cards does Jessica have? (“More” version)

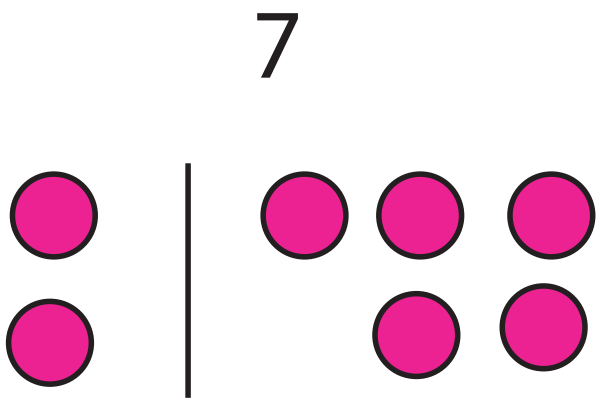
Shauntay has 12 cards. That is 3 fewer than Jessica has. How many cards does Jessica have? (“Fewer” version)

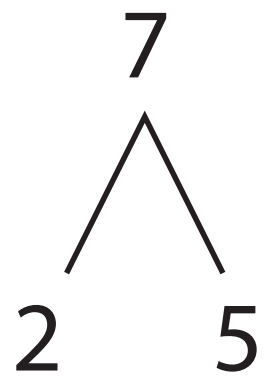


Note: students who rely only on keywords may mistakenly *subtract* 3 from 12 in the “fewer” version.

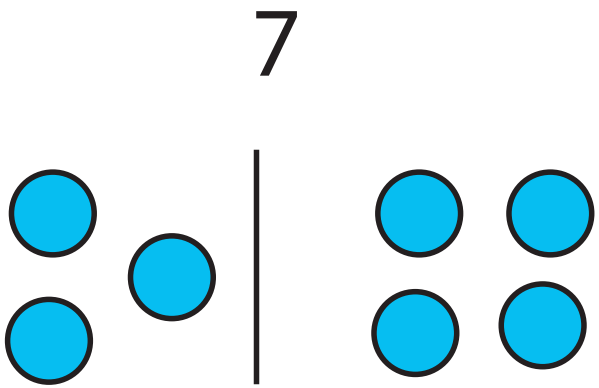
Take apart, both addends unknown

There are 7 marbles in 2 boxes. How many marbles can be in each box?





$7 = 2 + 5$



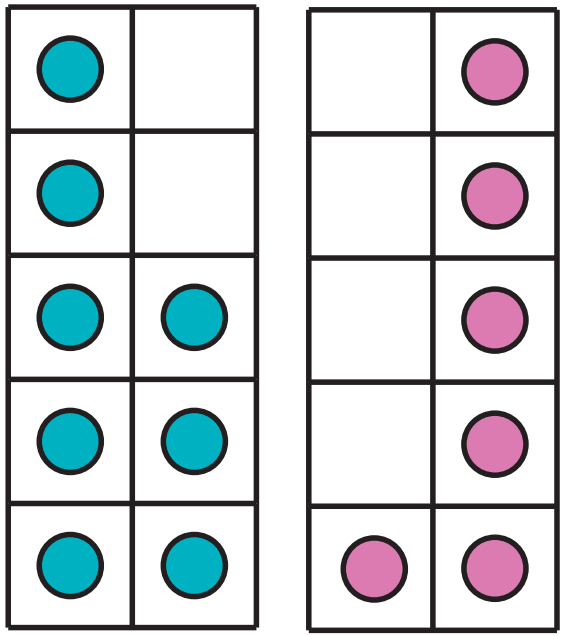
$7 = 3 + 4$

Using strategies based on properties of operations

CCSS Critical area 1, Grade 1:
Students “use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20.”

A “making a ten” method

$8 + 6$



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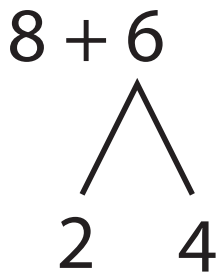
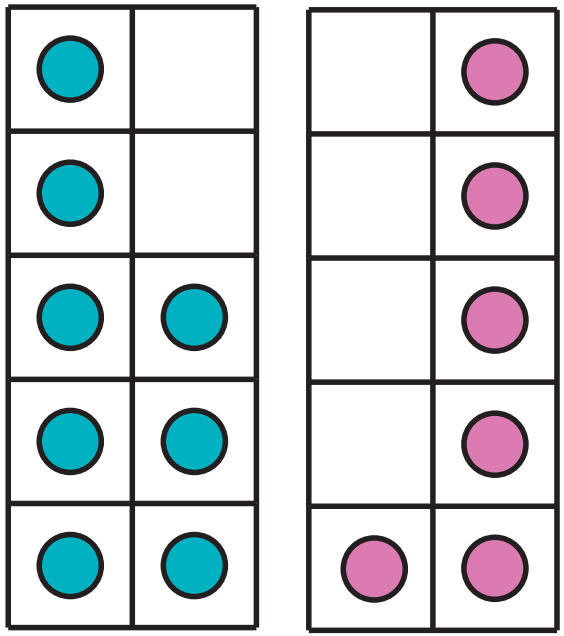
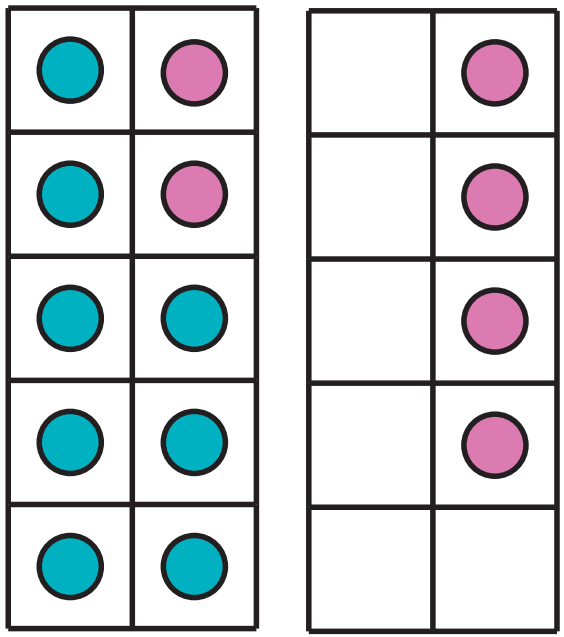




A “making a ten” method

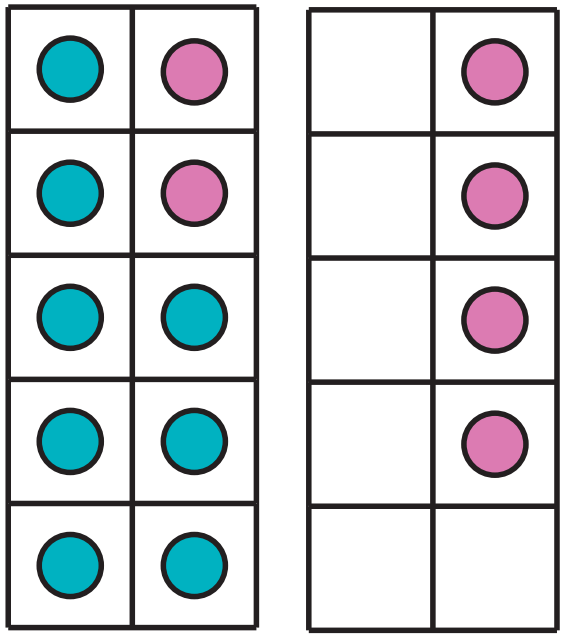
$$8 + 6 = 8 + (2 + 4)$$

2 4



A “making a ten” method

$$8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 14$$



Some subtraction methods within 20

Subtractions such as $14 - 9$ and $12 - 3$:

- View subtraction as unknown addend addition and count on to find the unknown addend
- View subtraction as unknown addend addition and make-a-ten with the unknown addend
- Subtract from ten
- Subtract down to ten first

MET2 Recommendations 1 and 2

Recommendation 1 Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach.

Recommendation 2 Coursework that allows time to engage in reasoning, explaining, and making sense of the math that prospective teachers will teach is needed to to produce well-started beginning teachers. . . .

Elementary teachers — at least 12 hours on fundamental ideas of elementary mathematics.

Professional development for elementary teachers

"The mathematics of elementary school is full of deep and interesting ideas, which can be studied repeatedly, with increasing depth and attention to detail and nuance. Therefore, although prospective teachers will undertake an initial study of elementary mathematics from a teacher's perspective in their preparation program, practicing teachers will benefit from delving more deeply into the very same topics."

Middle Grades Teachers

Essential Grades 6 – 8 ideas

CCSS domains at Grades 6 – 8:

- Ratio and Proportional Relationships
- The Number System
- Expressions and Equations
- Functions
- Geometry
- Statistics and Probability

Ratio and Proportional Relationships

CCSS 6.RP.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

A proportion problem

Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?

Reasoning with strip diagrams

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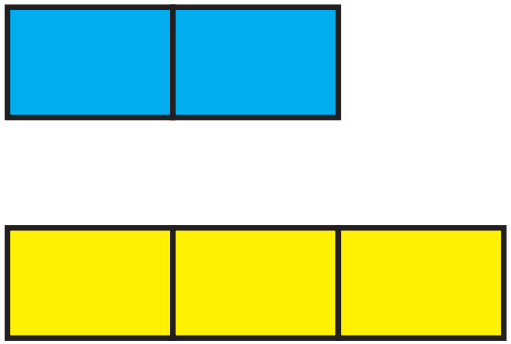

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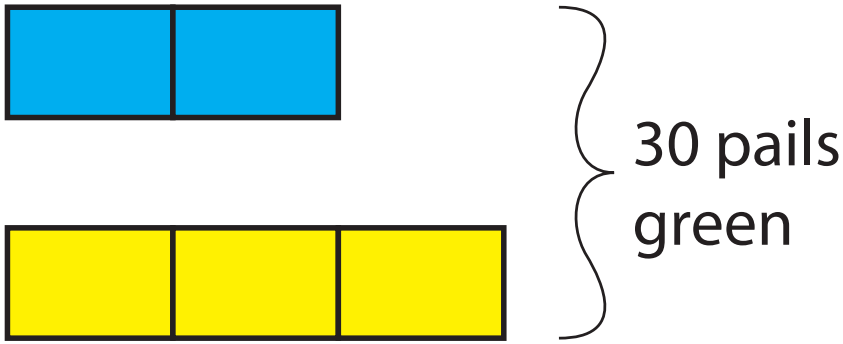

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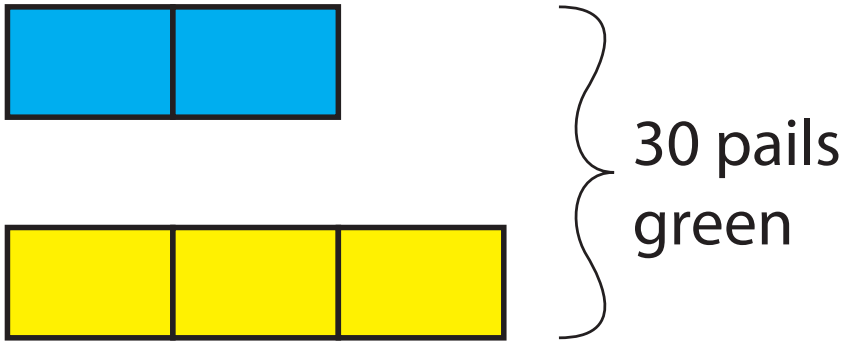

5 equal parts make 30 pails

Reasoning with strip diagrams

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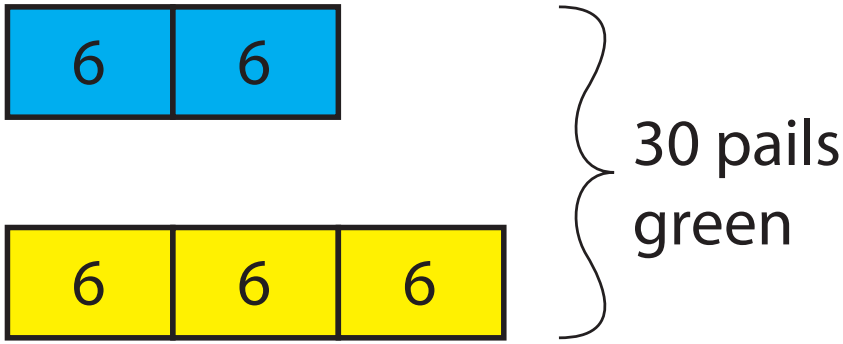

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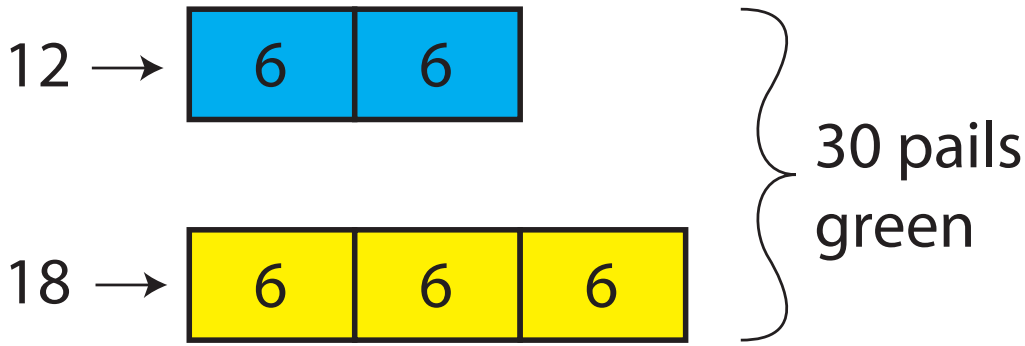
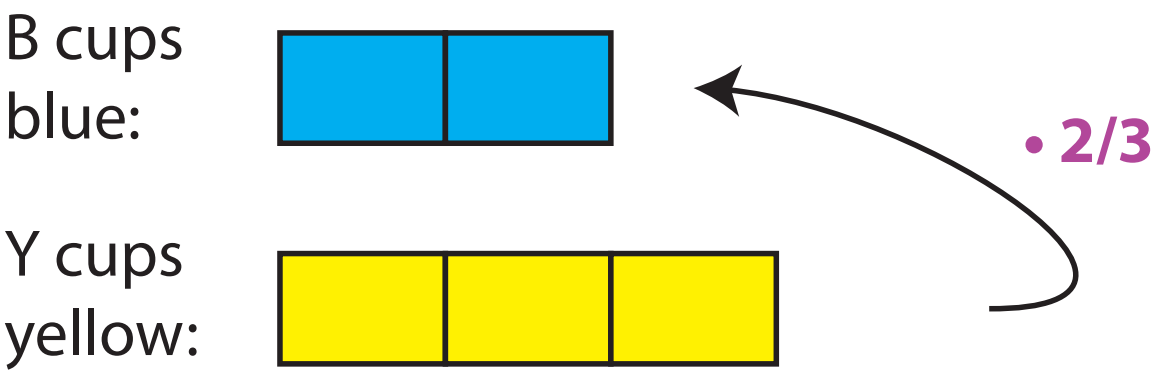


5 equal parts make 30 pails

Reasoning with strip diagrams

Ratio: 2 parts blue to 3 parts yellow

Values of the ratio: **2/3** and **3/2**



$$B = (2/3) \cdot Y$$

Reasoning with strip diagrams

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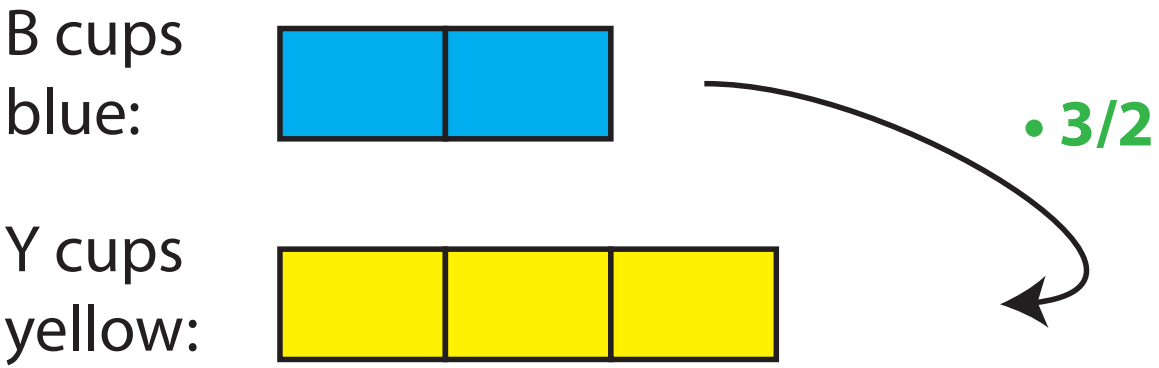

$$Y = (3/2) \cdot B$$

Middle Grades Teachers

"Because the middle grades are "in the middle," it is critical that middle grades teachers be aware of the mathematics that students will study before and after the middle grades."

CCSS 3.NF.1: "Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b."

Middle grades teachers need at least 24 hours of mathematics that includes 15 hours on fundamental ideas of school mathematics appropriate for ML teachers.

Middle Grades Teachers

Recommendation 2 ii:
“Prospective middle grades (5–8) teachers of mathematics should be required to complete at least 24 semester-hours of mathematics that includes at least 15 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.”

Middle Grades Teachers

At least 15 of the semester-hours should consist of mathematics courses designed specifically for future middle grades teachers that address the essential mathematical ideas of the middle and elementary grades.

The remaining 9 hours should be carefully selected to strengthen prospective teachers' knowledge of mathematics and the connections between one grade band and the next.

Middle Grades Teachers

"In no case should a course at or below the level of precalculus be considered part of these 24 semester-hours."
Clarification:

- NOT college algebra, trigonometry, mathematical modeling, and other such courses that precede pre-calculus;
- The 15 hours SHOULD be courses specifically designed for teachers to study elementary and middle grades mathematics in depth;
- The 15 hours should NOT include a course such as linear algebra with a few days or a few weeks or a few moments per class period attending to issues of middle school mathematics education;
- The remaining 9 hours might include a statistics course beyond the introductory level, a calculus course, a discrete mathematics course, or additional courses designed for teachers, such as a calculus for teachers course.

Recommendations on roles for mathematicians in teacher education

- Teacher education is important
- Collaboration with mathematics education faculty
- More mathematics faculty need to become deeply involved in PreK – 12 mathematics education
- We should recognize the need to improve mathematics teaching at all levels
- Mathematics education will be strengthened by the growth of a mathematics education community that includes mathematicians

Thank you!

The Mathematics Teaching Community: